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Philoſophia Britannica:
Nicholas OR, Brown Junr.
A NEW and COMPREHENSIVE
SYSTEM
OF THE
Newtonian PHILOSOPHY,
ASTRONOMY and GEOGRAPHY,
IN A
COURSE of Twelve LECTURES,

With NOTES;
CONTAINING
The PHYSICAL, MECHANICAL, GEOMETRICAL, and
EXPERIMENTAL PROOFS and ILLUSTRATIONS of
all the Principal Propoſitions in every Branch of
NATURAL SCIENCE:

ALSO
A particular Account of the INVENTION, STRUCTURE,
IMPROVEMENT and USES of all the conſiderable
INSTRUMENTS, ENGINES, and MACHINES;
With *new Calculations* relating to their
NATURE, POWER, and OPERATION.

The Whole collected and methodized from all the principal Authors,
and public Memoirs to the preſent Year;
And embelliſhed with EIGHTY-ONE COPPER-PLATES.

By B. MARTIN.

Quæ toties Animos veterum torſère Sophorum
Quæque Scholas fruſtra rauco Certamine vexant,
Obvia conſpicimus, Nubem pellente Matheſi. HAL. in NEWT. Prin.

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LECTURE V.

Of the Equilibrium *and* Motion *of* Fluids *in general. Of the* Rise *of* Fluids *in* Reservoirs *and* Pipes *of* Conduit. *Of the* Running *of* Rivers. *Of the* Origin *of* Fountains *and* Springs. *Of* Perennial, Intermitting *and* Reciprocating Springs. *The Nature of the* Recurved Syphon *and* Tantalus-Cup. *The* Theory *of* Jet d'eau's *their* Adjutages, Pipes, *&c.* Theorems *of the* Velocities, Momenta, Distances, *&c. Of* Spouting Fluids. *Various* Calculations *relating to the Motion of Fluids through Pipes applied to the* Animal Oeconomy, Cataracts, *&c. Of the Properties and Uſe of the* Crane *or* Syphon. *The whole* Theory *of* Pump-Work *at large. The Nature of the* Common Pump *explain'd. The Structure of a* New Pump *without Friction. The Nature and uſe of a* Mercurial-Pump, *working with* Quicksilver *inſtead of a* Piston, *explain'd.* Archimedes's Screw *explain'd. The Na-*

ture and Structure of NEWSHAM'S WATER-ENGINE, *of* NEWCOMEN'S FIRE-ENGINE, SAVERY'S FIRE-ENGINE, *and* PAINE'S IMPROVEMENT *of* FIRE-ENGINES *explain'd. Of the* ÆOLIPILE. *The* THEORY *of the* TIDES *at large explained.*

BY HYDRAULICS we are to underſtand the Science of the Motion of Fluids, and the Conſtruction of all Kinds of Inſtruments and Machines relating thereto; alſo the Nature of Springs, the Theory of the Tides, *&c.* are uſually explain'd under this Head.

THE *Motion of Fluids*, viz. their *Deſcent* or *Riſe* below or above the common Surface or Level of the Source or Fountain is cauſed either (1.) By the natural Gravity or Preſſure of the Fluid contained in the Reſervoir or Fountain; or (2.) By the Preſſure or Weight of the Air on the Surface of the Fluid in the Reſervoir, when it is at the ſame time either taken off, or diminiſhed, on ſome Part in Aqueducts, or Pipes of Conduit. (3.) By the Spring or elaſtic Power of compreſſed or condenſed Air, as in the common Water-Engine. (4.) By the Force or Preſſure of Piſtons, as in all Kinds of forcing Pumps, *&c.*

&c. (5.) By the Power of Attraction, as in the Caſe of Tides, *&c.* Of all which in Order.

I. THE moſt natural Motion of Fluids is that ariſing from the Force of their own Gravity, by which thoſe Parts which ſtand higheſt preſs upon others below them, till by that means they riſe to the ſame horizontal Level. Thus Water in a Fountain A B C D by its Preſſure raiſes that in the Aqueduct F G H to the ſame Height I K L M, in every Direction or Poſition of the Duct G H or G N; unleſs the Orifice of the ſaid Duct be below that Level, in which Caſe the Water will continually flow from the ſame. The Reaſon hereof is evident from the Principles of *Hydroſtatics*, where it was ſhewn, *that the Preſſure of Fluids was in Proportion to the Altitude only, and not according to the Quantity thereof*; and therefore the Effect or Riſe of the Fluid in the Duct muſt be equal thereto. Or thus; *The Velocity of the Fluid in the Duct at* L, *is to that in the Fountain at* I K, *as the Quantity of the Fluid in the Fountain is to that in the Duct in given Altitudes*; whence (from the Principles of *Mechanics)* the *Momentum* of the Fluid in each will be equal, and conſequently

Pl. XV. Fig. 5.

ſequently an *Equilibrium* will enſue at equal Altitudes K and L in both. (LXIX.)

(LXIX.) 1. Since the Particles of Fluids are to be conſider'd as ſmall ſolid Bodies, they muſt obſerve the ſame Laws of Motion, and have their *Momenta* determined in the ſame Manner, as Bodies of larger Magnitude; that is, *by the Maſſes or Quantities of Matter drawn into their Velocities.*

Pl. XV. Fig. 5. 2. Now 'tis evident the Velocities of the Fluid in the Reſervoir A D, and in the Duct S L, muſt be as the Quantities of Matter, or of the Fluid, contain'd in equal Altitudes in each inverſely; that is, as the Cylinder T V W L to the Cylinder I C D K. For when all the Fluid in the Cylinder I C D K is paſs'd into the Tube S H, it will fill a Space above V W in that Tube, or make a Cylinder equal to the former. Now ſince all Cylinders are as their Baſes and Altitudes jointly; if B be the Baſe, and H = the Height or Altitude of the Cylinder I C D K; and *b*, *h*, the Baſe and Altitude of the equal Cylinder in the Tube above V W; then becauſe $B \times H = b \times h$, we have $H : h :: b : B$, but the Altitudes H and *h* are the Spaces paſs'd thro' in the ſame Time, and will therefore repreſent the Velocities of the moving Fluid; but *b* is to B as the Cylinder T W to the Cylinder I D; (for Cylinders of equal Altitudes are as their Baſes) therefore $H : h :: TW : ID$. Or the Velocities are as the Quantities of the Fluid contain'd under equal Altitudes inverſely.

3. Or it may be otherwiſe eaſy to underſtand that the Water can riſe no higher in the Tube S H than in the Reſervoir A D, becauſe (ſuppoſing the Duct every where of an equal Bigneſs, and therefore E Q = T L) 'tis plain, if we continue F E to O, and R Q to P; that then only that cylindric Body of the Fluid in the Reſervoir which ſtands over the Orifice of the Duct E Q, that is, only the Cylinder of Water O E Q P can affect or preſs upon the Water in the Duct; for all the Fluid (beſides this Cylinder) ſtands over, and therefore preſſes upon the Bottom of the Reſervoir, as C E and Q D,

HENCE we have Conduits often ſupplied with Water from Springs which lie above them; and Cocks to ſupply the Inhabitants of a Town with Water by Pipes of Conduct proceeding from a Reſervoir in a Si-

and ſo by equal Re-action has all its Force deſtroy'd. The Water therefore will deſcend through the Orifice E Q, till by its Preſſure it has raiſed a Quantity in the other Leg S H equal to the Cylinders O Q + Q F, and therefore to an equal Height; and thus the Preſſure at F R and S G being equal, there muſt be an Equilibrium.

4. That it ought to riſe to the ſame horizontal Level M Z in the inclined Tube S N, whether that Tube be leſs, having an elliptic Baſe whoſe longeſt Diameter is S X; or whether it be equal to the Tube S H, as having the ſame Baſe; or, laſtly, if it be greater, as having a circular Baſe on the Diameter S G; I ſay, that in every Caſe it ought to riſe to the ſame Level Z M is evident, becauſe there can be no Equilibrium till the Preſſure of the Fluid in each Tube, or on each Side the Baſe S X, or S G, be equal; that ſo, being contrary, they may deſtroy each other. But there can never be an equal Preſſure on each Side the common Baſe, unleſs there be an equal perpendicular Altitude of the preſſing Fluid on each Side, becauſe (as we have ſhewn, *Annot.* LIV.) the Preſſure of Fluids on any given Area or Baſe is ever proportional to the Altitude of the Fluid, and that only.

5. Hence it appears, that an Inſtrument like this in the Figure may ſerve very well to take a Level for a ſmall Diſtance, or to draw a Line truly horizontal; for the two Points I and M will always be in a true horizontal Level or Line. But for this Purpoſe the Tube S G ſhould be at leaſt $\frac{1}{4}$ of an Inch Diameter, to avoid the riſing of the Fluid by Attraction; it ſhould alſo move very nicely by a Joint fixed on to the Part F S.

tuation

tuation above the higheſt Part of the Town (LXX.)

Pl. XVI. Fig. 8. (LXX.) 1. Let A C D B be a Reſervoir with a Duct or Conduit-Pipe F K L, and at L turn'd off from the Perpendicular to give the Water by a Stop-Cock into a Ciſtern for the Uſe of a Family. It is plain the Water will iſſue out from L with the ſame Force as it would be made to riſe to the Height L O, were the Tube or Pipe continued upright to O; for on one Side the common Baſe K, the Preſſure (when the Cock is ſhut) is proportional to the Altitude E K, but on the other it is only L K; the Difference therefore, $E\,I = L\,O$, is the Force of Preſſure by which the Water is protruded from the Cock at L. In the ſame Manner it is ſhewn, that the Force by which it is made to ſpout from the Cocks at M and N is proportional to the Altitudes H E and G E, or M O and N O, of the Fluid in the Reſervoir above them.

2. The *Running of Rivers* is upon the ſame Principle as the Deſcent of Bodies on inclined Planes; for Water no more than a Solid can move on an horizontal Plane, the Re-action of ſuch a Plane being equal and contrary to Gravity entirely deſtroys it, and leaves the Body at reſt; here we ſpeak of a Plane of ſmall Extent, and ſuch as coincides with the curved Surface of the Earth. But if we conſider a large Extent or long Courſe of Water, then we ſhall find that ſuch Water can never be at Reſt, but when the Bottom of the Channel coincides every where with the curved Surface of the Earth.

Fig 9. 3. Let A D F be the curved Surface of the Earth, C its Centre, C D, C E two right Lines drawn from thence, and E G a Tangent to the Earth in the Point D. Then 'tis plain if B D were a Channel of Water, the Water could not run, or move, becauſe they are every where at an equal Diſtance from the Centre C, and therefore equally affected by Gravity. But if there be any Place above the Surface of the Earth as E, where Water can be ſound, 'tis evident that Water

HENCE alſo the Deſcent of Water in Rivers, Streams, and Canals, from Springs

can deſcend in a Channel to any Part of the Earth's Surface between B and D, becauſe every Point in the Line E D is nearer to the Centre of the Earth, and therefore below the Point or Place E; and its Velocity will be ſo much the greater as it tends to a Point nearer B, and ſloweſt of all, when it moves in the Direction of the Tangent E D.

4. Hence it appears that the Source E of all Rivers and Streams muſt be more than a Semi-diameter of the Earth C B diſtant from the Centre C. And ſince all great Rivers run to the Sea or Ocean where they diſembogue their Waters at the Point D, the Line D C is a Semi-diameter, and = 4000 Miles nearly. Alſo the Courſe of all long Rivers being in the Direction of the Tangent at the Point D, if they were repreſented by the Tangent-Line E B, then the Height of the Source E above the common Surface of the Earth at B would be eaſily found. Thus, ſuppoſe E D were the River *Niger* in *Africa*, whoſe Source is more than 3000 Miles from the Sea; but put E D = 3000, and ſince C D = 4000, we ſhall have C E = 5000, and C E — C B 1000 = B E = the Height of the Source. But ſince we know of no Mountains above three or four Miles high, it is plain the River *Niger*, and all ſuch long Rivers, are ſo far from moving in a Tangent, that their Courſe muſt be very nearly of the ſame Curvature with the Earth's Surface, and inſenſibly diſtant from it.

5. Since Bodies move on Planes ever ſo little inclined, except ſo far as they are prevented by Friction, and ſince the Friction of the Particles of Water among themſelves is inconſiderable, it follows that the Water ſituated on a Plane ever ſo little inclined will commence a Motion; and if the Plane be conſiderably inclined, and the Quantity of Water great, its Velocity will be proportional, and its *Momentum* ſuch as will ſoon begin to wear away the Earth, and create itſelf a Courſe or

and Sources above the common Surface of the Earth; and the Breaking out of Springs

Channel to glide in. In Rivers that are made, it is uſual to allow the Fall of one Foot in 300.

6. If we allow the ſame Declivity to Rivers which make their own Way, then we find their Height at their Source above the common Surface of the Sea, as in Example of the *Niger* thus: As $300 : 1 :: 5280 : \frac{5280}{300} =$ the Height at one Mile, or 5280 Feet. Then again ſay, as $1 : \frac{5280}{300} :: 3000 : \frac{5280 \times 3000}{300} = 5280 \times 10$ = 10 Miles. From whence it is evident, that the Continents and Iſlands ought to be much above the Surface of the Sea, to give a neceſſary Deſcent and Courſe to the Waters through them.

Pl. XVI. Fig. 10. 7. Let A B C D be the Section of a Reſervoir, and B C I K the Section of a Canal of Water ſupplied from thence, and A B N the horizontal Line. Now ſince the Particles of Water are governed by the common Laws of Gravity, the Velocity of a Particle at any Part of the Bottom of the Canal, as F or H, will be the ſame as it would acquire by falling through the perpendicular Altitude O F or L H, (by *Annotat.* XXVII *Art.* 9.) that is, as $\sqrt{OF}$ to $\sqrt{LH}$ (by *Annotation* XXVI. *Art.* 4.) Hence the Velocity of the Stream is accelerated.

8. For the ſame Reaſon the Velocity of a Particle at the Bottom of the Stream H is to the Velocity of a Particle at the Top G, as $\sqrt{LH}$ to $\sqrt{MG}$; conſequently the Stream moves with a greater Celerity at Bottom than at Top.

9. The Quantity of Water which paſſes through the Section of the Stream H G is the ſame that paſſes through the Section of the Reſervoir B C in the ſame Time. The ſame may be ſaid of any other Section F E; therefore the Quantity of Water paſſing by any

at the Bottoms and on the Sides of Hills, from Ciſterns and Reſervoirs in the inter-

-

two Sections of the Stream F E and G H in the ſame Time is the ſame.

10. Since there runs the ſame Quantity of Water by G H as by F E in the ſame Time; and ſince the Velocity at G H is greater than at F E; and laſtly, ſince the Breadth of the Canal is ſuppoſed to be every where the ſame; therefore it follows, that the Depth G H muſt be leſs than the Depth F E, and ſo the Depth of the Stream muſt continually decreaſe as it runs.

11. As the Stream proceeds, the Depth H G decreaſing, the Lines M G and L H will approach nearer to an Equality, and therefore the different Velocities of the Water at Top and Bottom will approach much faſter to an Equality, as being proportionate to the Square Roots of thoſe Lines.

12. This Approach to an Equality is much farther promoted by the upper Parts being continually accelerated by the lower, and the lower Parts retarded continually by the ſlower Motion of the Waters above, incumbent and preſſing upon them.

13. Since the Difference of the deſcending Velocities, or the relative Velocities, are greateſt near the Beginning or Head of the Stream, the Waters will there fall or deſcend with the greateſt Impetuoſity, or cauſe the loudeſt Noiſe.

14. In the Courſe of Rivers, the accelerated Velocity is quickly reduced to an equable or uniform Velocity, by the Reſiſtance it meets with from the Bottom and Sides of the Channel, which Reſiſtance will be as the Squares of the Velocities, and therefore ſoon become ſo great as to equal the accelerating Force, and be communicated to the middle Parts of the Stream, cauſing the Whole to move uniformly.

15. Hence in Rivers the Motion of the Water is ſloweſt at the Sides and Bottom of the Channel, becauſe there the Reſiſtance begins, and is communicated to all other Parts.

nal Parts above them, which receive their Waters from *Rain*, *Dew*, *condenſed Vapours*, *melted Snow*, &c. diſtilling or percolated through the Pores or Creviſes and Chaſms of the upper Part of the Earth (LXXI.)

16. In different Parts of the ſame River the uniform Velocity is greateſt where the Bottom of the Channel has the greateſt Inclination or Declivity, becauſe the relative Gravity of the moving Particles is here greateſt.

17. In thoſe Parts of the River where the Velocity of the Stream is leaſt, the Depth of the Water is the greateſt, and *vice verſa*: Becauſe equal Quantities paſs through unequal Sections of the River in the ſame Time. Hence alſo it follows, that the *Momentum* of running Water muſt be every where the ſame, or a given Quantity.

(LXXI.) 1. Various have been the Theories, or rather Hypotheſes, relating to the Origin of Fountains, many of which have favour'd ſo little of Philoſophy, that they ſcarce deſerve to be mentioned or confuted. Thoſe who pretend to derive the Waters of Springs from the Waters of the Sea by ſubterranean Ducts, ſeem either wholly ignorant of the hydroſtatic Laws of Fluids, or reſolved to maintain a Theory by meer dint of Hypotheſis.

Pl. XVI. Fig. 11.

2. For let E C A F be a Part of the Earth's Surface covered with Water at A; ſuppoſe A B C a ſubterranean Duct, by which the Waters deſcend to the internal Parts of the Earth towards B; if B be the loweſt Point, or that neareſt the Centre of the Earth, the Water will be forced to riſe from B to C by the Preſſure of the Water in the Part A B. But when the Water is arrived to C it will there ſtop, and an Equilibrium will neceſſarily enſue (from what we have ſhewn in *Annotat.* LXX.) Conſequently no Water can riſe above C to the Top of the Mountain D by the Force of Preſſure; and we can ſhew no other Power in Nature to drive it thither; and therefore ought to ſuppoſe none.

BY the Preſſure of the Fluid it may eaſily be convey'd over Valleys and Hills

3. They who advance the Capillary Hypotheſis, or ſuppoſe the Waters riſe from the Depths of the Sea through the porous Parts of the Earth, as it riſes in Capillary Tubes, or Tubes of Sand or Aſhes, ſeem not to conſider one principal Property of this Kind of Tube, or this Sort of Attraction; for though the Water riſe to the Top of the Tube or Sand, yet will it riſe no higher, becauſe it is by the Attraction of the Parts above that the Fluid riſes, and where that is wanting it can riſe no farther. Therefore, tho' the Waters of the Sea may be drawn into the Subſtance of the Earth by Attraction, yet can it never be raiſed by this means into a Ciſtern or Cavity to become the Source of Fountains.

4. The true Principles which ſupply the Waters of Fountains or Springs, are undoubtedly *melted Snow*, *Rain-Water*, and *condenſed Vapours*. Several have attempted to ſolve the *Phænomena* by Snow or Rain only; but others making an Eſtimate of the Quantity of Rain and Snow, that falls in the Space of a Year, to ſee if it would be equal to that which the Rivers diſcharge annually into the Sea, found that it was much ſhort of that Quantity.

5. But that which was moſt extraordinary was, that they found by their Experiments, that the Rain and Snow which fell in one Year would not produce more than $\frac{2}{3}$ of what was raiſed in Vapour; for by Experiment it was found, that the Rain and Snow that fell in a cylindric Veſſel, raiſed a Column of Water about 19 Inches high; whereas the Water raiſed in Vapour was yearly about 32 Inches in Altitude. This great Deficiency of 13 Inches plainly indicated another Way, by which the Waters circulated from and to the Sea.

6. This (among many other Things) was left to the Diſcovery of that ſagacious Naturaliſt Dr. *Halley*; who being on the Tops of the Mountains in the Iſle of St. *Helena* (in South Lat. 16°) making his Obſervations

by bended Pipes or Syphons, from Ponds or Reſervoirs in a higher ſituation; which

for a Catalogue of the Southern Stars, (about 800 Yards above the Level of the Sea,) found that Quantity of Vapour, which there fell in Dew, was ſo great, as very much impeded his Obſervations, by covering his Glaſſes over in 6 or 7 Minutes, even when the Sky was clear.

7. Upon this he was induced to determine by Experiment, the Quantity of Vapours raiſed from the Surface of the Sea, as far as it ariſes from Heat; for other Cauſes, as Wind, and the Warmth of common Air, contribute to exhale the Water, as the ſame Gentleman found by making the Experiment in a cloſe Room, where 8 Inches Altitude of Water was exhaled in one Year; and 'tis well known from common Obſervation, that a ſtrong Wind will carry off a great Quantity of Water in a few Hours time, as in the late *January* Storm ſeveral ſhallow Ponds had all their Water blown away by the Wind in one Day. Alſo, we find by Experience, that one windy Day will make the Ways dryer after Rain than 3 or 4 Days Sun-ſhine.

8. But the Doctor's Deſign was to ſee what Quantity was evaporated by the Heat of the Sun only. For this Purpoſe he inſtituted the Experiment in the following Manner: He took a Veſſel of Water, and made it ſalt to the ſame Degree with Sea Water, by means of the Hydrometer. In this he placed a Thermometer, and by a Pan of Coals he brought the Water to the ſame Degree of Heat with that of the Air in the hotteſt Summer.

9. This done, he affixed the Veſſel of Water, with the Thermometer in it, to the End of a Pair of Scales, and nicely counterpoiſed it with Weights in the other; then at the End of two Hours he found by the Alteration made in the Weight of the Veſſel, that about the 60th Part of an Inch of the Depth of the Water was gone off in the Vapour, and therefore in 12 Hours, one Tenth of an Inch would have been evaporated. *Note*,

the Ancients being ignorant of, were at vaſt Pains and Expence very often to effect. Hence the running of Water

12 Hours only are here allow'd for the Time of Exhalation, that being the Length of the Day at a Mean; and in the Night, as much, if not more Water returns by Dew than is exhaled.

10. Upon this Suppoſition, every 10 ſquare Inches of the Surface of the Water yields in Vapour, *per Diem*, a cubic Inch of Water, which weighs 253$\frac{1}{3}$ Grains *Troy*; therefore every ſquare Foot will yield $\frac{1}{2}$ a Pint Wine Meaſure; every Space of 4 Feet ſquare, a Gallon; and every ſquare Mile 6914 Tons. A Degree ſquare (reckoning 69 *Engliſh* Miles to a Degree) will produce 33 Million of Tons; and if the *Mediterranean* be 40 Degrees long, and 4 broad (the narrow Parts compenſating for the broader, which is the leaſt that can be ſuppoſed, then will there be in its Surface 160 ſquare Degrees, which will evaporate *per Diem*, 5280 Millions of Tons in the Summer Time.

11. The *Mediterranean* receives Water from the following nine great Rivers, *viz.* The *Iberus*, the *Rhone*, the *Tyber*, the *Po*, the *Danube*, the *Neiſter*, the *Boriſthenes*, the *Tanais*, and the *Nile*; all the reſt being of no great Note, and their Water inconſiderable. Each of theſe nine Rivers (together with all the ſmaller) are ſuppoſed to bring as much Water to the Sea, as is equal to ten Times the Water of the *Thames*, at leaſt.

12. In order to eſtimate the Quantity of Water, which paſſes daily through the *Thames*, the Doctor aſſumes the Breadth of the River at *Kingſton* Bridge (where the Flood ſeldom reaches) to be 100 Yards, and the Depth 3; ſo that the Section of the Channel is 300 ſquare Yards, and allowing the Velocity of the Water to be at the Rate of 2 Miles *per* Hour, there will run in 24 Hours, the length of 48 Miles, or 84480 Yards; therefore 84480 × 300 = 25344000 cubic Yards, which make 20300000 Tons which the River *Thames* yields *per Diem*.

13. Now each of the above nine Rivers being ſuppoſed to bring ten times as much as the *Thames*, will

through the *bended Syphon* accounts for the odd Phænomena of *intermitting and reci-*

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yield 203000000, or 203 Millions of Tons, and therefore all the nine will produce 1827 Millions of Tons; which is but little more than $\frac{1}{3}$ of the Quantity (5280 Millions of Tons) evaporated every Day from the Sea. The $\frac{2}{7}$ nearly of this prodigious Quantity of Water, the Doctor allows to Rains which fell again into the Seas; and to the Rain imbibed by the low Parts of the Earth, and ſpent in Vegetation in general.

14. The Quantity of Water exhauſted from the *Mediterranean*, being ſo much greater than what is return'd by the Rivers, is the Occaſion of the Water's ſetting in from the Ocean, to ſupply the Deficiency, by a continual Influx or Stream, which before this Diſcovery was quite unaccountable. Alſo, hence it appears, why the *Caſpian* Sea, tho' it receives the Waters of many large Rivers, is yet never liable to overflow; becauſe the Waters, brought in by the Rivers, are exhauſted by the Sun, Wind, *&c.* Yea, the Difficulty is on the other Hand, ſince the Exhalation is ſo much greater than the Supply by the Rivers, why the Sea does not appear diminiſhed, or its Waters continually waſting? which Thing has never been obſerved.

15. The prodigious Quantity of Vapours raiſed by the Sun's Heat, and otherwiſe, being carried by the Winds over the low Lands to the very Ridges of Mountains, as the *Pyrenean*, the *Alps*, the *Appennine*, the *Carpathian*, in *Europe*; the *Taurus Caucaſus*, *Imaus* and others in *Aſia*; *Atlas*, the *Montes Lunæ*, or Mountains of the Moon, with other unknown Ridges in *Africa*; I ſay, the Vapours being compelled by the Stream of Air to mount up with it to the Top of thoſe Mountains, where the Air becoming too light to ſuſtain them, and condenſed by Cold, they precipitate in Water, and gleet down by the Crannies of the Stone; and Part of the Vapour entering into the Caverns of the Hills, the Waters thereof gather as in an *Alembic*, into the Baſons of Stone it finds, which being once fill'd, all the Over-

procating

RPJCB

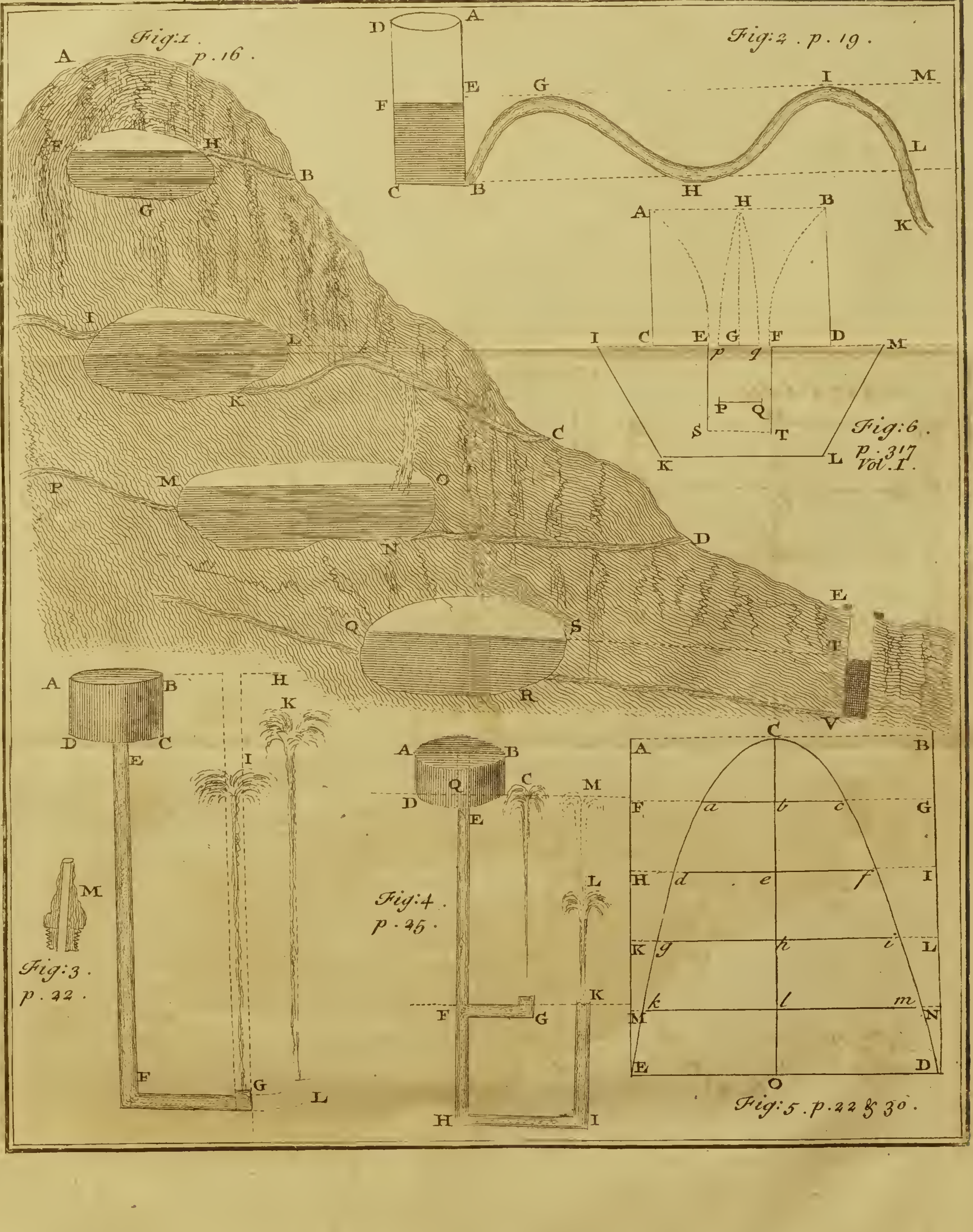

Fig: 1. p. 16.

Fig: 2. p. 19.

Fig: 3. p. 22.

Fig: 4. p. 25.

Fig: 5. p. 22 & 30.

Fig: 6. p. 317 Vol. I.

procating Springs; and alſo in that Inſtrument call'd the *Tantalus-Cup*. The Riſe of

plus of Water that comes thither, runs over by the loweſt Places, and breaking out by the Sides of the Hills, forms ſingle Springs.

16. Many of theſe Springs running down by the Vallies between the Ridges of the Hills, and coming to unite, form little Rivulets, or Brooks; many of theſe again meeting in one common Valley, and gaining the plain Ground, being grown leſs rapid, become a River; and many of theſe being united in one common Channel, make ſuch enormous Streams as the *Rhine*, the *Rhone*, the *Danube*, &c. And it may almoſt paſs for a Rule, that the Magnitude of a River, or the Quantity of Water it diſcharges, is proportional to the Length and Heights of the Ridges from whence the Fountains ariſe.

17. This beautiful Account of the Origin of Springs and Rivers has been received with univerſal Applauſe and Satisfaction in the learned World. I ſhall now add a Word or two concerning the different Sorts of Springs which we find in divers Parts of the Country. But one Thing I ſhall firſt remark, and that is, it has been aſſerted, and often taken for granted, that *there are Springs of Water upon the very Tops, Summits, or higheſt Parts of Mountains*; which Poſition is contrary to the Hydraulic Law of Fluids, by which they riſe to the Level of the Fluid in the Reſervoir, but no higher; and not only that, but is undoubtedly falſe in Fact: For I have myſelf examined moſt of the remarkable Hills in the *Weſt*, *South*, and *Eaſt* Parts of *England*, but could never diſcover any ſuch Thing; though I often found them on the Sides, not far below the Tops, but low enough to account for their being there. And Mr. *Caſwell* carefully examined the Tops of the Mountains in *Wales* (at the Deſire of a Friend) and could find no Spring there. I ſhall therefore conclude there never was, nor can be any ſuch Thing in Nature.

Water in Wells alſo is owing to the Weight of that in the ſubterranean Ciſterns from

18. The ſeveral Sorts of Springs obſerved are (1.) *common Springs*, which either run continually, and then they are called *Perennial Springs*; or elſe run only for a Time, or at certain Times of the Year, and then they are called *Temporary Springs*. (2.) *Intermitting Springs*, or ſuch as flow and then ſtop, and flow and ſtop again, by regular Alternations or Intermiſſions. (3.) *Reciprocating Springs*, whoſe Waters riſe and fall, or flow and ebb, by regular Intervals, or Reciprocations the Surface. All which are produced after the following Manner.

Plate XVII. Fig. 1. 19. Let A B C D E repreſent the Declivity of a Hill, whoſe Section, from Top to Bottom perpendicularly, is ſhewn in the Figure; in which let F G H be a Cavern or Baſon near the Top which collects the Water gleeting through the Crannies, and has a drain or Duct leading from H to the Side of the Hill at B. 'Tis evident when the Water riſes to the Drain H, it will deſcend through it to B, where it will break out in Form of a Fountain or Spring, and will continue thus, ſo long as the Reſervoir is ſupplied with Water above the Level F H, and after that it will become dry. Thus we ſee a Spring may be generated near the Top of the higheſt Hills or Mountains. Of this we have a notable Inſtance near the Top of *Landſdown Hill* by *Bath*.

20. There is one odd *Phænomenon* of ſuch Springs that is ſometimes obſerved, and that is, *that they run in dry Weather, and are dry in wet Weather*; to account for which, we need only obſerve, that while the Weather continues wet, the Waters are gathering into the Baſon or Reſervoir, till there is a ſufficient Quantity to run over and make the Spring play, by that Time the Weather is altered and become dry; during the dry Weather, the Remains of the laſt Rains (for theſe Springs proceed chiefly from Rain-Water) are continually feeding the Reſervoir, and by that means ſupplying the Spring, but by the Time theſe are all ſpent,

the Weather again alters to wet, and the Spring ceaſes of courſe, till it meets a freſh Supply from the preceding Rains, and ſo on.

20. The INTERMITTING SPRINGS may be thus accounted for: Let I K L be a Cavity in the Mountain, to which at I there is a feeding Stream that brings the Water from other Parts, and at K, on the lower Part, there goes a Duct K *k* C, which is of a curved or crooked Form, and conveys the Water to the Side of the Hill at C, where it breaks out in a Spring. Here 'tis evident, that as the Water riſes in the Cavern, it will riſe in the Duct at the ſame Time, till the Surface of the Water I L be level with the higheſt Part *k* of the Duct, after which the Water will deſcend from *k* to C, which Point C, being lower than the Orifice of the Duct K, will exhauſt the Cavern of all its Water, and then the Spring will ſtop till the feeding Stream I repleniſhes the Ciſtern to the ſame Level I K, and then the Fountain will play again; this ſuppoſes the Duct K C to carry off more Water than the other at I brings to the Ciſtern; and then every Thing follows from the Nature of the *bended Syphon* as explained in the next *Annotation*.

21. The RECIPROCATING SPRINGS are occaſioned after this Manner. Let M N O be a Cavity ſupplied by a feeding Stream P M, and alſo by a Syphon K *k* O, which brings Water from another Cavity above, as I K L; the Duct N D carries the Water to the Side of the Hill at D, and there makes a conſtant Spring, by Virtue of a conſtant Supply of Water, by the Drain P M. The Water at D will alſo flow and ebb alternately, for when the Syphon K *k* O works, the Surface of the Water M O will be ſuddenly raiſed, and preſs upon the Water at N with a greater Force, by which Means it will iſſue out at D with a greater Velocity, and raiſe the Surface if confin'd; but when the Syphon intermits, or ceaſes, the *Momentum* of the Water at D is not ſo great as before, and then the Spring will ſink or decreaſe.

22. In like Manner we account for the riſing of Water in Wells. Thus ſuppoſe a Well be ſunk at the Foot of the Hill at E, to ſuch a Depth E V, as will

bring the Diggers to an Eruption of a Spring at V, whoſe Water is brought by the Duct R V (or many of them) from a Cavity Q R S in the Hill, (or otherwiſe from a Pond, a River, the Sea, *&c.*) 'tis evident the Water in the Well will riſe from the Bottom V, to an Altitude V T, where the Surface of the Water at T is upon a Level with that in the Reſervoir Q S, and thus conſtitutes a Well.

23. If thoſe Reſervoirs of Water in the Body of Mountains be ſituated where Mineral Ores abound, or the Ducts or feeding Streams run through Mineral Earth, 'tis eaſy to conceive the Particles of Metal will mix with, and be abſorb'd by the Water, which being ſaturated therewith, becomes a *Mineral Spring or Well.* If Salt, Sulphur, Lime-ſtone, *&c.* abounds in the Strata, through which the Water paſſes, it will then be ſaline, ſulphureous, Lime-Water, *&c.* If Sulphur and Iron ſhould both abound in the Parts of the Hill, whence the Waters come, the Waters will partake of the Warmth or Heat, which is occaſioned by the Mixture of two ſuch Subſtances in the Earth, where they are found. Now, though every Thing may not happen preciſely in the Manner, as here repreſented, yet that it is in ſome Way analogous to it, I believe no Perſon will doubt, who has been at the *Peak* in *Derbyſhire*, or at *Wookey-Hole* in *Somerſetſhire*, and ſeen the wonderful Caverns, Receptacles and Streams of Water, which Nature has there furniſhed in the Bowels of the Mountains.

SCHOLIUM.

24. I ſee but one Objection which can be made to the foregoing noble Theory of Dr. *Halley*, and that is, that a Body of Water heated to the ſame Degree with the hotteſt Summer Air will certainly evaporate more from a given Space in a given Time than the deep Waters of the Sea, whoſe Surface only is affected by the Heat of the Summer Air; and therefore in all probability the Doctor's Quantity of Water raiſed in Vapour was much too great. To which I anſwer, (1.) The Rivers are ſuppoſed to return much more Waters to the Sea than they really do, for no one can imagine there

whence they are ſupplied by proper Canals or Springs (LXXII.)

there can be 90 Times the Water of the *Thames* brought to the *Mediterranean* by all its Rivers. (2.) The Doctor accounts for Springs by the Quantity of Vapour exhaled by the Sun alone, whereas if we abate near half this Quantity, and take in what is raiſed by the Wind and Agitations of the Sea, together with the large Supplies from Rain, Melted Snow, &c. we ſhall undoubtedly find ſufficient Matter for the Sources of all the Rivers and Springs. (3.) As the Vapour is exhaled from the Surface of the Water only, it will be proportional to the Heat in the Surface only, which muſt be nearly the ſame for the moſt Part of the Day with the contiguous Air, and therefore the Caſe is not ſo widely different from the Doctor's Experiment as might be at firſt imagined.

(LXXII.) 1. The Nature of the *bended Tube or Syphon* (of which we have made ſo much Uſe in the preceding *Annotation*) will eaſily appear by conſidering it as follows. A B C D is a Veſſel, into which is inſerted at the Bottom a bended Pipe B G H I K to repreſent the Syphon. Now if Water be poured into the Veſſel, it will as it riſes in the Veſſel riſe alſo into the Pipe or Duct, till the Water in the Veſſel be upon a Level F E with the upper Part G of the curved Tube; what more is poured in after this will run over the Bend at G down the lower Part G H; and from the loweſt Point H it will, by the ſame Principle as before, be forced into the riſing Part H I, where at the Bend I it will again run over, and deſcend through the Part I K, and at K it will run out, as being lower than the Level of the Bottom of the Veſſel C L.

Plate XVIII. Fig. 2.

2. That the Pipe muſt neceſſarily be full to the ſame Altitude every where as the Water ſtands in the Veſſel, by the Preſſure of that in the Veſſel, is evident from what we have ſhewn in *Annotation* LXX. And ſince it can riſe no higher in the Veſſel than in the Pipe, and in the Pipe it can riſe no higher than the bended Parts

FROM the Preſſure of the ſuperior Fluid we have the Invention of a *Natural Foun-*

G or I, therefore it can riſe no higher in the Veſſel than thoſe bended Parts of the Pipe. Again, becauſe the Preſſure of the Air upon the Water at F E in the Veſſel, and upon the Orifice at K is equal, therefore if the Orifice K were in the ſame Level with the Bottom of the Veſſel C L, the Water, having the ſame Altitude E B and M L, would not run out at the Orifice L, but remain *in Equilibrio* with that in the Veſſel.

3. But, laſtly, ſince the Orifice K is below the Level, the Water at K will have a greater Altitude M K, and therefore a greater Preſſure, or *Momentum*, and ſo will prevail againſt the equal Preſſure of the Air, and run out; and will continue to run out, till all the Water in the Veſſel be exhauſted, becauſe the Preſſure of the Air upon the Surface F E will force the Water into the Pipe to prevent a *Vacuum*, which muſt otherwiſe happen in ſome Part thereof, which is abſurd or contrary to Nature.

4. For ſuppoſe the Water were not to enter at B as faſt as it runs out at K, there muſt happen a Separation of the Water in ſome Part of the Pipe, as at I for Inſtance; then the Water running out at K will cauſe the ſeparated Part to deſcend from I towards L, and ſince nothing can get into the Space between the ſeparated Surfaces of Water at I and L, all that Space muſt be a pure *Vacuum*, and therefore the Water muſt be at reſt in the Pipe between B and I, though the Preſſure of the whole Atmoſphere acts upon the Part at B, and nothing at all at the Part I to reſiſt or counteract it, which is contrary to the Nature or Definition of a Fluid, as has been abundantly ſhewn in *Annot.* LIII. and all the ſubſequent ones.

5. This bended Syphon is uſually called a TANTALUS, from the Semblance of the Experiment made with a Man in a Glaſs to the Caſe of *Tantalus* in the Fable, who, according to the Poets, was ſentenced for his horrid and unnatural Crime of killing, boiling, and

tain, which playing from the ſmall Orifice of an Adjutage at the Bottom, will ſpout

ſerving up his own Son at a Feaſt of the Gods, to ſtand in a River of Hell up to his Chin in Water, which yet he is not permitted to ſip to quench his eternal Thirſt. Now this *Tantalus* is the Man in the Glaſs, with a Syphon concealed in his Body, beginning in the Bottom of one of his Feet, and aſcending to the Upper Part of his Breaſt; there it makes a Turn, and deſcends thro' the other Leg on which he ſtands, and from thence down through the Foot of the Glaſs, where it runs out, and cauſes the Water to ſubſide in the Glaſs, as ſoon as it riſes to the Height of the Syphon, or to the Chin of the Man; and thus aptly expreſſes the Fable.

6. This Syphon may very eaſily be concealed alſo in the Handle of a common Pint Pot or Cup, and then it is called the TANTALUS-CUP: For the Top of the Handle being a little below the Top or Rim of the Cup, and the loweſt Part of the Handle or Syphon below the Bottom or Orifice by which it communicates with the Cup, will cauſe the Cup to have the Property of emptying itſelf when fill'd to the Brim; as is eaſy to underſtand from what has been ſaid above.

SCHOLIUM.

7. The Nature of this Syphon, or the hydraulic Law of Fluids, on which it depends, was not underſtood by the Ancients; for though we read ſurpriſing Accounts of the *Roman* Aquæducts and Pipes, by which they convey'd Water to the Diſtance ſometimes of five or ſix Miles, yet we find no mention made of any other than horizontal and deſcending Pipes or Ducts, and never of any riſing or aſcending. If the Water was to be convey'd where a Valley interpoſed, they with great Expence and Labour conſtructed a Water-Courſe over the ſame, ſupporting it with vaſt Piles and Arches ſometimes from the Depth of an 100 Feet. Whereas a ſingle Pipe laid over the Vale would have done the

very beautifully nearly to the Height of the Water from the Fountain; but it cannot quite reach that Altitude, by reaſon of the Air's Reſiſtance (LXXIII.)

Buſineſs moſt effectually. If a Mountain interpoſed, we find they would ſometimes carry their Aquæduct through the very Body of it by Dint of Labour, which ſurely could not have been their Choice, had they known that a Pipe laid over the Top coming from a Reſervoir of a proper Height would have anſwered the ſame End. Such Reſervoirs indeed ſuppoſe the Uſe of Pumps to raiſe the Water into them; and Pumps they had of various Kinds, but could not apply them for want of knowing the Theory. They who would acquaint themſelves with the Magnificence of the *Romans* in regard to their Water-Works and Aquæducts, may conſult *Frontinus*, and others who have wrote on this Subject.

Plate XVIII. Fig. 5. (LXXIII.) 1. The Theory of Natural Fountains, or *Jet d'Eau's*, depends on the following Principles. It has been ſhewn, that Water coming from a Reſervoir A B C D, through the Pipe E F G H, will riſe from the loweſt Part G to the ſame Altitude H, in the Part G H, as is upon a Level with the Surface of the Water A B in the Reſervoir: And alſo that it thus roſe from the Point G by a Force of Preſſure proportional to the Altitude of the Water in the Reſervoir, which is equal to the Altitude G H. *Annot.* LXIX.

2. Now I preſume 'tis very evident that the Tube G H itſelf can contribute nothing towards the Water's riſing in it; but, on the contrary, it rather impedes the Aſcent by the Friction it occaſions to the Particles which move againſt the internal Surface thereof. Therefore if the Part G H be taken away, the Water would riſe to the ſame Height H, excepting ſo far as it is obſtructed by other concurring Incidents; for in all Fountains the Height G I is ſomewhat leſs than G H, the Height in the Tube, for the following Reaſons.

FROM this Preſſure of Fluids, as the Cauſe, we can eſtimate the various *Mo-*

3. *Firſt*, The Air's Reſiſtance is an Obſtruction to the Jet, and diminiſhes its Height; and ſince we know that the Reſiſtance of Fluids is proportional to the Squares of the Velocity, (*Annot.* LXVIII. *Art* 4.) and the Deficiency of Height H I is proportional to the Reſiſtance; therefore a Jet that plays with a double Velocity will have that Deficiency four Times as great; and with three Times the Velocity, nine Times as great; and ſo on.

4. The ſecond Impediment is the Friction againſt the Sides of the Hole, and the Adjutage at G; and ſince this is in proportion to the Quantity of Surface in the Hole, it will be greater in a ſmall Hole than a large one, in reſpect of the Body of the Jet, or ſpouting Water; becauſe the Jet will increaſe in Magnitude with the Square of the Diameter of the Hole, whereas the Reſiſtance will increaſe only with the Diameter ſimply. Or a Hole of twice the Diameter emits four Times the Water, and gives but twice the Reſiſtance. This makes a ſmall Jet riſe to a leſs Height than a large one, from the ſame Reſervoir.

5. A third Impediment ariſes from hence, that ſince all the Particles ſet out from G with an equal Velocity, and this Velocity is continually diminiſhing, it follows that the Velocity of the inferior Parts is greater than the Velocity of the Parts above them, and therefore muſt in ſome Degree ſtrike againſt them; by this Impulſe (ſince Fluids move every way) the Particles will be urged ſideways, and the Column of the Jet become wider, and conſequently ſhorter than it would otherwiſe be.

6. The fourth Cauſe why Jets do not riſe to the Height of the Reſervoirs, is becauſe the Water upon the Top of the Jet does not immediately run off, but ſpreading into a Head, lies with its Weight upon the aſcending Water below, and hinders it from riſing ſo high as it would do. This will appear by inclining

menta or Forces of ſeveral Jets ſpouting through the Sides or Bottoms of Veſſels at

the Jet a little, that the upper Water may not bear upon the riſing Stream, for the Jet will then play higher, but be leſs beautiful.

7. If the Hole of the Adjutage G be leſs than $\frac{1}{4}$ of an Inch in Diameter, the Force of the Attraction of Coheſion will extend itſelf through the Body of the Jet at the Hole, and greatly obſtruct its Aſcent; hence the Riſe of Jets in Models, for Experiments cannot be expected ſo high as others, becauſe of the very ſmall Holes made uſe of in them. Whence all Jets of larger Fountains (ſuch as are in Gentlemens Gardens) ought to exceed $\frac{1}{4}$ of an Inch in Diameter, and that in Proportion to the Height of the Reſervoir, as will be ſhewn hereafter.

8. If the Conduct-Pipe E F G be not of a proper Size to ſupply Water, as faſt as it can be expended at the Adjutage G, the Jet will likewiſe be checked, and will not riſe to the full Height. To aſcertain the Proportion of the Conduct-Pipe to the Bore of the Adjutage, is the Buſineſs of Experiments; and what has reſulted from thence, in regard to this Matter, will be ſhewn in a Table by and by.

9. And one Thing more is neceſſary to be known, that the Jet may play the higheſt poſſible, *viz.* that the Part of the Conduct-Pipe at the Adjutage does not turn up at a Right Angle, but with a gentle eaſy Curve; that is, not as at G, but as at L, where the Jet plays to a greater Height at K. The upright Part at G directly reſiſts the Water coming from F, where the Curve at L cauſes the Impulſe of the Water againſt it to be very oblique; and therefore a leſs Part of its *Momentum* will be deſtroyed, and conſequently the greater remaining Force will throw the Jet the higher.

10. From what has been ſaid in *Art.* 4. it appears (*cæteris paribus*) the Hole in the Adjutage ought to be made in a thin Plate of Braſs, and not through the Bore of a Tube of any Length, becauſe of the Quan-

different Depths below the Surface: For the *Momentum* of Fluids, as well as ſolid

tity of Surface in ſuch an Adjutage-Piece, which muſt greatly retard the Jet, and diminiſh the Height. This is a Fault which moſt People have run into for want of an experimental Knowledge of this Science. For the ſake of Beauty they ſcrew on ſuch a conical Piece as repreſented at M, of an Inch in length, inſtead of a Plate of Braſs, whoſe Thickneſs need not exceed the 40th Part of an Inch for very large Jets, and whoſe Friction will therefore be 40 Times leſs.

Plate XVIII. Fig. 4.

11. If the Jet be play'd from G, the Velocity with which it ſets out from the Adjutage, will be as $\sqrt{FQ}$, and throw it to its full Height G C at once; but if the Pipe deſcends below F G to H I, and thence returns upwards again to the ſame Level F K, and the Jet be play'd from K, the Velocity with which it ſets out from K at firſt will not be the ſame as the Velocity from G, that is, it will not be as $\sqrt{FQ}$, or $\sqrt{QH - FH}$, but it will be as $\sqrt{QH} - \sqrt{FH}$, which is leſs, and therefore the Height at firſt will be only K L, from whence by Degrees it will riſe to M upon the ſame Level with C, ſo that its laſt Height K M will be equal to the Height G C of the Jet from G.

12. The Truth of the laſt Article is thus demonſtrated. The Water in the Legs Q H and I K are to be conſider'd as two Non-elaſtic Bodies, whoſe *Momenta* (being as the Preſſure) are as the Altitudes Q H and I K; and ſince both the Velocities and Quantities of Matter moved in a given Time, are as the Square Roots of the Altitude (as will be ſhewn in the next Annotation) therefore we have $QH = \sqrt{QH} \times \sqrt{QH}$, and $IK = \sqrt{IK} \times \sqrt{IK}$, and ſince the Difference of the *Momenta*, divided by the Sum of the Bodies, gives the Velocity after the Stroke (*Annotation* XXIII. *Art.* 4.) which we will call x, therefore

Bodies, is *compounded of the Quantity of Matter and Velocity*; but in ſpouting Fluids,

$$\frac{\sqrt{QH}\times\sqrt{QH}-\sqrt{IK}\times\sqrt{IK}}{\sqrt{QH}+\sqrt{IK}}=x=\sqrt{QH}$$

$-\sqrt{IK}$, (because $\sqrt{QH}\times\sqrt{QH}-\sqrt{IK}\times\sqrt{IK}=\overline{\sqrt{QH}+\sqrt{IK}}\times\overline{\sqrt{QH}-\sqrt{IK}}$; which is the Velocity of the Water at firſt ſetting out from K.

13. But ſince Fluids act not by a ſingle Impulſe, as Solids do, but conſtantly; the Velocity of the Water at K will not be equable at firſt, but accelerated for ſome Time, that is, till all the Water in the Leg I K ſhall be brought to move with the ſame Velocity with that in the Leg Q H when it comes to be uniform: For as the Velocity in the Leg I K is accelerated, ſo by its Re-action does it retard the Motion or diminiſh the Velocity of the Water in the Leg Q H; but as it cannot totally deſtroy the Motion of Q H, that which remains muſt be equable or uniform; and what that is will eaſily appear if we only conſider, that that Part of the Motion which is deſtroy'd in Q H by the Re-action of the Leg I K muſt be equal to its own *Momentum*, which is as I K; (becauſe Action and Re-action are equal) therefore the remaining *Momentum* of Q H will be Q H — I K = Q H — F H = Q F, wherefore the laſt Velocity will be as $\sqrt{QF}$, and therefore equal to that at G, and ſo carry the Water as high; that is, the Jet will riſe from L to M, as high as from G.

14. To give an Example: Suppoſe Q H = 25 Feet, F H = I K = 9; then Q F = (Q H — H F =) 16; therefore the firſt Velocity will be to the laſt Velocity as $\sqrt{QH}-\sqrt{IK}:\sqrt{QH-IK}\ (=\sqrt{QF}::\sqrt{25}-\sqrt{9}=5-3=2:\sqrt{25-9}=\sqrt{16}=4$. Conſequently the Height of the Jet K L is to the Height at laſt K M, as 2 to 4. And thus you carry on the Calculation for any Length of the Leg I K whatſoever.

the Quantity of the Fluid iſſuing through the ſame Hole in the ſame Time is always as the Celerity of its Motion, as is eaſy to concieve: Whence the *Momenta of ſpouting Fluids are proportional to the Squares of the Velocities, or Quantities of Matter iſſuing out in a given Time:* But ſince the *Momenta* are the Ef-

15. I ſhall here ſubjoin a Table of the Height to which Jets will riſe in Feet and decimal Parts from Reſervoirs of an Height of five Feet to 150 Feet.

Ref.	*Jet.*	*Ref.*	*Jet.*	*Ref.*	*Jet.*	*Ref.*	*Jet.*	*Ref.*	*Jet.*	*Ref.*	*Jet.*
5	4,91	22	20,58	39	34,95	56	48,24	73	60,71	90	72,48
6	5,88	23	21,46	40	35,74	57	48,99	74	61,42	91	73,15
7	6,84	24	22,33	41	36,55	58	49,74	7	2,13	92	73,82
8	7,8	25	23,20	42	37,35	59	50,49	76	62,84	93	74,49
9	8,74	26	24,06	43	38,14	60	51,24	77	63,54	94	75,16
10	9,68	27	24,92	44	38,93	61	51,99	7	64,24	95	75,83
11	10,62	28	25,78	45	39,75	62	52,73	79	64,94	96	76,40
12	11,55	29	26,63	46	40,53	63	53,47	80	65,64	9	77,15
13	12,48	30	27,48	47	41,31	64	54,20	81	66,33	98	77,81
14	13,40	31	28,32	48	42,09	65	54,93	82	67,02	99	78,47
15	14,31	32	29,16	49	2.87	66	55,6	83	67,71	100	79,12
16	15,22	33	30.	50	43,65	7	56,39	84	68,40	110	85,58
17	16,13	34	30,83	51	44,42	68	57,12	85	69,08	120	91,86
18	17,03	35	31,6	52	45,19	69	57,84	86	69,76	130	97,99
19	17,93	36	32.47	53	45,96	70	58,56	87	0,47	14	103,97
20	18,82	37	33,29	54	46,72	71	59,28	88	71,14	150	107,87
21	29,70	38	34,11	55	47,48	72	60.	89	71,81		

16. By various Experiments that have been made by Mr. *Mariotte*, Dr. *Deſaguliers*, and others, it has been found, that if the Reſervoir be five Feet high, a Conduct-Pipe $1\frac{3}{4}$ Inch Diameter will admit a Hole in the Adjutage from $\frac{1}{4}$ of an Inch to $\frac{3}{8}$ of an Inch; and ſo on as in the following Table:

fect of Preſſure, it is evident, *the Velocity, or Quantity of ſpouting Fluids is ever pro-*

Height of Reſervoir.	*Diameter of the Adjutage.*	*Diameter of the Pipes of Conduct.*
5 Feet	$\frac{1}{8}$ to $\frac{3}{4}$	1 $\frac{3}{4}$ Inch.
10	$\frac{1}{4}$ to $\frac{1}{2}$ Inch	2 Inches.
15	$\frac{1}{2}$ an Inch	2 $\frac{1}{4}$ Inches.
20	$\frac{1}{2}$ an Inch	2 $\frac{1}{2}$ Inches.
25	$\frac{1}{2}$ an Inch	2 $\frac{3}{4}$ Inches.
30	$\frac{1}{2}$ to $\frac{3}{4}$ of an Inch	3 Inches, or 3 $\frac{1}{2}$.
40	$\frac{3}{4}$ of an Inch	4 $\frac{1}{2}$ Inches.
50	$\frac{3}{4}$ of an Inch	5 Inches.
60	1 Inch	5 $\frac{3}{4}$ Inches.
80	1 $\frac{1}{4}$ Inch	6 $\frac{1}{2}$ Inches.
100	1 $\frac{1}{4}$ or 1 $\frac{1}{2}$ Inch.	7 or 8 Inches.

17. Here the Jet is ſuppoſed to be within 100 or 150 Feet of the Reſervoir; but if the Conduct-Pipe much exceeds this Length, it muſt be of a larger Diameter than what is here aſſigned. Thus for Jets from $\frac{3}{4}$ of an Inch to thoſe of an Inch, and from Reſervoirs from 40 to 90 Feet Height, if the Diſtance be from 150 Yards to $\frac{1}{4}$ of a Mile, the Diameter of the Pipe ſhould be of 6 Inches; from $\frac{1}{4}$ of a Mile to two Miles, it muſt be of 7 Inches; and from 2 Miles to 5, it muſt be of 8 Inches Diameter for the ſame Jets.

18. If it be required to keep any Number of Jets playing, whoſe Adjutages are given in Diameter, by one common Conduct-Pipe, we muſt find the Diameter of an Adjutage equal to all the given ones. Thus if there be four Adjutages of $\frac{3}{4}$ of an Inch Diameter each, then the Square of $\frac{3}{4}$ is $\frac{9}{16}$, which multiplied by the Number of Adjutages 4, makes $\frac{36}{16}$; the Square Root of which is $\frac{6}{4} = 1\frac{1}{2} =$ the Diameter of the Adjutage equal to all the four ſmall ones. A Pipe of Conduct of 10 Inches Diameter will ſupply all the Jets, as being a little more than ſix times as great as the Diameter of the one large Adjutage now found. After

portional to the Square Root of the Preſſure or Altitude of the Fluid (LXXIV.)

this Manner the Dimenſions of a Conduct-Pipe may be found for any other Number of Adjutages.

(LXXIV.) To repreſent theſe Propoſitions in Symbols may render them more perſpicuous, and, perhaps, more elegant. Therefore let Water ſpout from the Side of a Veſſel at two different Depths B and E from the Surface of A; the *Momentum* of the ſpouting Fluid will be as the Force which protrudes it, and this Force is every where proportional to the Altitude of the Fluid, therefore the *Momentum* of the Water at B and E will be as A B, and A E, which let be denoted by m and M. Then let v and V be the Velocities with which the Water ſpouts; and q and Q the Quantities of Water iſſuing out in the ſame Time from the Adjutages B and E, then we ſhall have $m = vq$, and $M = VQ$. And therefore $\frac{m}{q} = n$, and $\frac{M}{Q} = V$.

Pl. XV. Fig. 6.

2. Now the Quantity of Water ſpouting from any Hole in a given Time muſt neceſſarily be as the Velocity with which it flows; for the greater the Velocity, the greater will be the Number of Particles which come to the ſame Point in the ſame Time, or the greater will be the Quantity of Water which paſſes by the Orifice of the Adjutage, as is very eaſy to conceive; therefore $q : Q :: v : V :: \frac{m}{q} : \frac{M}{Q}$; conſequently $\frac{M}{Q} \times q = \frac{m}{q} \times Q$; or $Mqq = mQQ$; wherefore $m : M :: qq : QQ$ ($:: vv : VV$, as is evident from the Proportionality of the Roots.) Therefore alſo $q : Q :: v : V :: \sqrt{m} : \sqrt{M} :: \sqrt{AB} : \sqrt{AE}$.

3. Hence if $AB : AE :: 1 : 4$, the Velocity at E will double the Velocity at B, and the Quantity of Water which flows from E will be twice as great as that which flows from B in one Minute, or any given Time. If A E be equal to 9 A B, the Velocity and

FRtM hence it appears, *that the Velocity of a Fluid ſpouting at any Depth below the*

Quantity of Water will be three times greater at E than at B, and ſo on.

4. Hence 'tis very evident the Velocity of a Spouting Fluid, and that of an heavy Body, which it acquires by falling from the Surface of the Water A to the Adjutage B or E *(in Vacuo)* will be both the ſame, as being both equal to $\sqrt{AB}$ or $\sqrt{AE}$. See *Ann:t.* XXII. *Art.* 4.

5. Hence alſo, if A B D E be a Veſſel of Water whoſe Surface is A B, then a Parabola D C E deſcribed Plate XVIII. Fig. 5. on the Bottom D E and Height of the Water O C, will expreſs by its Ordinates ac, df, gi, km, E D, the Velocities with which the Water will ſpout from the Veſſel through Holes G, I, L, N, D, made in the Side upon a Level with thoſe Ordinates reſpectively; becauſe theſe Ordinates are as the Square Roots of the Abſciſſæ C b, C c, C h, &c. that is, as the Square Roots of the Altitude of the Fluid above the ſaid Holes. Thus if $Cl = 4Cb$, then $ac : km :: \sqrt{Cb} : \sqrt{Cl} : 1 : 2 ::$ Velocity of the Water ſpouting from G to the Velocity of that ſpouting from N. And ſo in every other Place.

6. Hence, if A B the Axis of a Parabola be equal Pl.XVII Fig. 1. to $\frac{1}{2}$ B C its greateſt Semi-ordinate, the Point B will be the *Focus*, and B C the *Semi-parameter* of the Parabola. Now ſince Water, or any heavy Body, falling from the Height A B, or Water ſpouting at the Depth A B below the Surface, will acquire at B a Velocity that in an uniform Motion will carry it over a Space equal to 2 A B or B C; and ſince in one Second of Time a Body falls through 16 Feet; therefore if we divide A B into 16 equal Parts, and B C into 32, and Ordinates be drawn through every Point of Diviſion in the Axis A B, and Perpendiculars to the ſame Point B C, then there will be exhibited at one View the *uniform Velocities* acquired by the ſeveral Falls or Jets of Water, from all Heights to 16 Feet.

Surface, is equal to the Velocity a heavy Body would acquire by falling from the ſame Height; becauſe the Velocity, as was obſerved, is always as the Square Root of the Space deſcended through. If therefore on the Altitude of the Fluid A K, as a Diameter, we deſcribe the Semicircle A F K; and from any Point therein, as I, we draw the Perpendicular I H, that ſhall be pro- Pl. XV. Fig. 6.

7. Theſe Velocities are expreſſed by the reſpective Ordinates, and their Meaſure in Feet ſhewn by the Perpendiculars in the Scale B C on the Baſe. Thus, for Example, a Fall of 4 Feet gives an uniform Velocity of 16 Feet *per* Second; a Fall of 9 Feet gives 24; and ſo of the reſt. Alſo the Time of the Falls, being proportional to the Ordinates, might have been expreſſed by another Scale of 60 Thirds (equal to one Second) in the Baſe B C; but to avoid Confuſion, I have given the Scale of Time upon the Curve of the Parabola: So that any Queſtion relating to the *Time*, *Velocity*, or *Height* of a Fall or Jet of Water leſs than 16 Feet, may be readily ſolved by Inſpection, and that to an Exactneſs ſufficient for the niceſt Caſe in Practice.

8. After the ſame Manner may a Parabola be conſtructed for all Heights to an 100 Feet, but that I leave to the Reader, that which I have given being nearly ſufficient for all common Occaſions. Such a Scale has been greatly wanting in Books of this Sort, and ſuperſedes the Neceſſity of thoſe large Tables which we find in ſome voluminous Treatiſes of *Hydroſtatics*; as thoſe of M. *Belidor*, &c. Note alſo, the ſeveral Ordinates of this Parabola expreſs the Lengths of a Cylinder or Column of Water iſſuing from an Adjutage at any of thoſe Depths in one Second of Time, as being proportional to the Velocity at each reſpective Place.

portional to the Diſtance to which the Fluid will ſpout from an Adjutage at H: For the *Velocity* will be as the Square Root of A H, and the *Time* as the Square Root of H K; whence the Product of theſe two will expreſs both the Space paſs'd over by the projected Body, and alſo the Line H I, as is well known to Geometricians.

HENCE it follows, that *a Fluid will ſpout from a Hole or Adjutage* D *in the Centre of the Semicircle, or middle Point of the Altitude* A K, *to the greateſt horizontal Diſtance* K M *poſſible*; becauſe the Perpendicular F D is the greateſt that can be drawn to the Diameter A K. Alſo 'tis evident, that *from two Holes* B *and* H, *equally diſtant above and below the middle Altitude* D, *the Jets of Water will be made to the ſame horizontal Diſtance* K N; becauſe the Perpendiculars to theſe two Points, *viz.* C B and I H, are equal.

MOREOVER, the horizontal Diſtance K M, to which the Water ſpouts from D the Centre, is equal to the Diameter or Altitude A K, or twice D F. For ſince the Velocity of the Jet at Q is equal to that acquired by falling through the Height A D, or D K, it will, as being uniform, carry the Fluid in an horizontal Direction over

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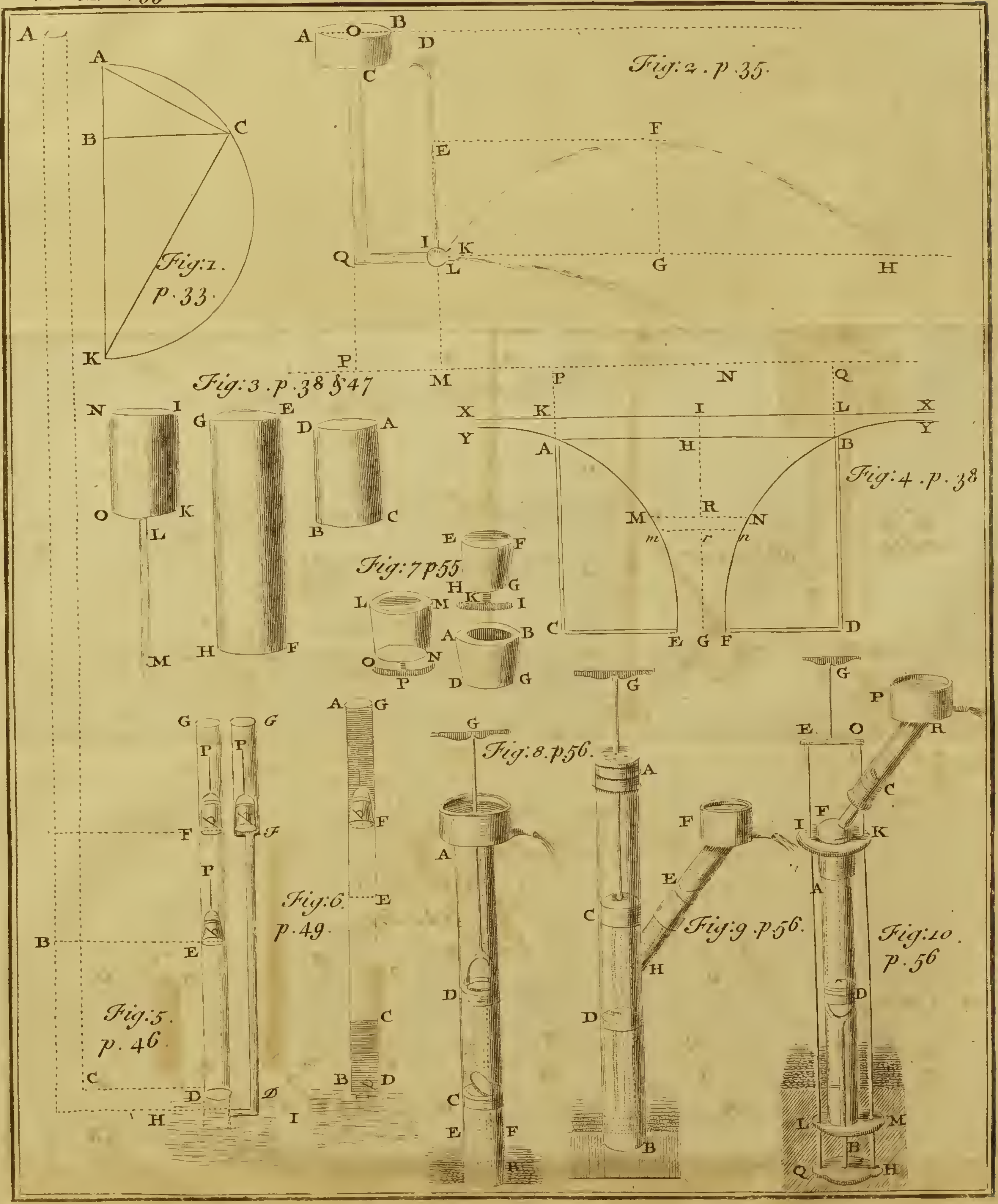
Fig:1. p.33.
A
B
C
K
Fig:2. p.35.
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Fig:3. p.38 §47
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Fig:4. p.38
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Fig:5. p.46.
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Fig:6. p.49.
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Fig:8. p.56.
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Fig:9. p.56.
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Fig:10. p.56
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over twice the Space D K or D F in the ſame Time: And therefore, ſince the Diſtances of Jets from D and B are as D F to B C, and the Diſtance of the Jet from D is equal to twice D F, the Diſtance of the Jet from B will be alſo equal to twice B C, or K N = 2 B C (LXXV.)

(LXXV.) 1. The Velocity of the Fluid ſpouting from B we have ſhewn to be as $\sqrt{AB}$, and the Time it takes in going to the horizontal Line will be as Pl. XV. $\sqrt{BK}$, becauſe that would expreſs the Time if it were Fig. 6. to deſcend to K in the Perpendicular B K, by *Annot.* XXVI. 4. And though it goes not in the Perpendicular, but is projected from B with an Impetus in an horizontal Direction, it will not by that means be any longer arriving to the horizontal Line, but only go in a different Tract, as is evident from what has been ſaid in *Annotation* XXIV. Therefore the Time of its coming to the Ground at N will be as $\sqrt{BK}$, that is, as the Square Root of the Altitude of the Spout B K.

2. Now the Spaces which ſpouting Fluids paſs over in an horizontal Direction muſt be conſider'd as uniform, becauſe it proceeds from a ſingle Impulſe; for a Particle of Water, after it has made its Exit from the Orifice of the Adjutage, is no more affected by the Preſſure or Impulſe of the confined Water, and conſequently its Velocity in the Direction B C will be uniform. But in all uniform Motion the Spaces deſcribed are as the Rectangle under the Times and Celerities, or as $\sqrt{AB} \times \sqrt{BK}$; but this Rectangle is alſo as the Line B C.

3. For let there be drawn A C and C K, 'tis evident the Triangle A C K is ſimilar to the Triangle B C K, Pl. XIX. becauſe the Angles A C K and C B K are right, and Fig. 1. the Angle C K B is common to both; therefore A C : C K :: B C : B K. In the ſame Manner we ſhew the

I Shall only obſerve farther in regard to ſpouting Water, that the Jet will go farther through the bare Hole in the Veſ-

Triangle ACK is ſimilar to the Triangle ACB; therefore alſo $AC : CK :: AB : BC$. Conſequently, $AB : BC :: BC : BK$; therefore $AB \times BK = BC^2$. And ſo $BC = \sqrt{AB \times BK} = \sqrt{AB} \times \sqrt{BK}$.

4. Now becauſe the Space the Fluid deſcribes according to the Horizon, and the Line BC, are both proportional to the ſame Thing, (*i. e.* to $\sqrt{AB} \times \sqrt{BK}$) therefore they muſt alſo be proportional to each other; and ſo the horizontal Diſtance of any Jet, as B, will be in Proportion to the Line BC; and the Jet from D will be proportional to the Line DF, and ſo on.

5. If now D be taken into the Centre of the Circle AFK, the Velocity at D would (as being uniform) carry it through twice the Space $AD = 2DF = DP$ in the horizontal Direction or Line DP, as is evident from *Annot.* XXVI. 7. becauſe the Particles of a Fluid obſerve the ſame Laws of Motion with larger Bodies. Conſequently, though the Fluid goes not in the Direction or Line DP, it will go to the ſame Diſtance from the Perpendicular DK on the Horizon; becauſe $DK = AD$, or the perpendicular Spaces, and conſequently the Times of the Fall, are equal. Whence, becauſe the horizontal Diſtances deſcribed by Jets from D and B are as DF and BC, and the horizontal Diſtance of the Jet from D is $2\,DF = DP = KM$, therefore the horizontal Diſtance of the Jet from B will be equal to $2BC = KN$.

6. From what has been hitherto ſaid, 'tis eaſy to obſerve that the Motion of a ſpouting Fluid is every way ſimilar to that of a projected Solid. The Path of the Fluid is a Parabola, becauſe it is impell'd by two Forces, one horizontal, the other of Gravity in the Perpendicular, in the ſame Manner as was ſhewn of the projected Solid. The Impetus of the Jet B, D, H, E, is the Height of the Reſervoir AB, AD, AH, AE, the greateſt horizontal Random KM is that from the Jet

ſel at P, Q, or R, than through the Adjutage P B inſerted in thoſe Holes, and farther ſtill in Proportion as the Adjutage

D, directed to the middle Point F of the Semicircle; and any two Jets equally diſtant, as B, H, go to the ſame Diſtance K N on the Horizon. If the Adjutage be horizontal, the Jet will be a Semi-parabola; if oblique, it will be a whole Parabola. All which Particulars were obſerved to be the Properties of the Motion of ſolid Projectiles; a Jet of Water being every Way the ſame as a Diſcharge of ſmall Shot.

P. XIX. Fig. 2.

7. But to apply this more particularly: Let ABCQL be a Fountain and Conduct-Pipe with an Adjutage moveable on a Joint, to be ſet in any Poſition; as perpendicular, as at I; oblique, as at K; or horizontal, as at L. In the firſt Caſe, a Jet will play upright to D; the Height of which Jet I D is the *Impetus*, or the Meaſure of the Force of the Machine to project the Fluid. If the Adjutage be placed obliquely, as at K, the Jet of Water will be the Parabola K F H; but if placed horizontally, as at L, it will form the Semi-parabola L N. Now in this latter Caſe it will be $LM : MN :: MN \; \frac{MN^2}{LM} =$ *Latus Rectum* of the Parabola L N when compleated.

8. But it has been ſhewn, that the *Impetus* I D, or Force by which the Parabola is deſcribed, is equal to *one fourth Part* of the *Latus Rectum*, (*Annot.* XXXIII. 12.) therefore it will always be $ID = \frac{MN^2}{4LM}$; conſequently, $4LM : MN :: MN : ID$. So that if we ſee a Jet of Water at any Time, and would know the Height of the Reſervoir whence it comes, we can eaſily find it by the Proportion above, becauſe the three firſt Terms are always given by Meaſurement.

9. By this Means alſo we can eſtimate the Force by which any Fluid is projected, where no Reſervoir or Preſſure of Fluids is concern'd, but the Action of Solids; as the Force by which the Urine is protruded; the

is ſhorter and the Bore larger. On the contrary, if a Pipe be inſerted perpendicu-

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Force of Reſilition in the Blood ſpouting from Veins and Arteries in Bleeding; the Force by which the Chyle aſcends from the *Receptaculum* by the Thoracic Duct to the Left Subclavian Vein, or by which the Blood aſcends from the loweſt Parts of the Body to the Heart. In ſhort, nothing in the Animal Oeconomy can be well underſtood but by an Acquaintance with the Hydraulic Laws of Fluids: For an animal Body is the moſt curious and compounded Hydraulic Machine in Nature.

10. A ſignal Example of the Uſe of *Hydraulics* in this Reſpect I ſhall produce from the admirable Experiment of Dr. *Hales* in his *Hæmaſtatics.* The Doctor cauſed a Mare to be tied down alive, and laid open the Left Crural Artery, in which he inſerted a Braſs Tube, and on that fix'd a Glaſs Tube; and then cutting the Ligature on the Artery, the Blood roſe in the Tube in an Inſtant about 4 Feet, and continued to riſe till it attained the Height of 8 Feet and 3 Inches. In a Gelding he tried the ſame Experiment, and the Blood roſe to the Height of 9 Feet 8 Inches: This was in the ſame Artery as before. In another Mare he fix'd a Pipe and Tube into an Inciſion of the Left Jugular Vein, and the Blood roſe to $4\frac{1}{2}$ Feet; and then he fix'd it in the Left Carotid Artery, where the Blood roſe to 9 Feet and 6 Inches. In an Artery of a Doe it roſe to 4 Feet 2 Inches; in a Dog, to 6 Feet 8 Inches; in a Sheep, to 6 Feet $5\frac{1}{2}$ Inches; and he conjectures that it will very probably riſe in a Man to the Height of 7 Feet and 6 Inches.

11. Now to eſtimate the Force with which the Left Ventricle of the Heart (of the firſt Mare) propell'd the Blood, the Doctor found by means of injected Wax, that the Capacity of the Ventricle was 10 Cubic Inches, and the Surface on the Inſide was equal to 26 Square Inches; alſo the Area of the greateſt Section of the Ventricle from *Apex* to Baſe was 6,83 Square Inches: Conſequently the whole Preſſure upon the internal Sur-

larly into the Bottom of a Veſſel, it will diſcharge itſelf ſooner through it, than

face of the Ventricle was equal to $26\times114=2964$ Cubic Inches. (See *Annot.* LV. 10, 11.) Alſo the Weight or Preſſure downwards was equal to $6,83\times114=778,63$ Cubic Inches, that is, to the Height multiplied by the Area of the greateſt Section of the Ventricle.

12. Now a Cubic Inch of Blood weighs 267,7 Grains; wherefore $2964\times267,7=113,22$ Pounds *Avoirdupois,* allowing 7008 Grains to a Pound; which therefore is the Force of Preſſure upon the Surface of the Ventricle when dilated. Again, $6,83\times114=29,7$ Pounds = the Force of Weight, or the *Momentum* of the Blood which the muſcular Fibres of the Ventricle muſt reſiſt in ſupporting ſuch an Altitude of Blood, on ſuch an Area of Baſe.

13. The Velocity of the Blood entering the *Aorta* is thus computed: The Ventricle each Syſtole throws out 10 Cubic Inches of Blood, and the Area of a tranſverſe Section of the *Aorta* at the Heart is 1,036; therefore $\frac{10}{1,036}=9,64$ Inches = the Length of the Cylinder of Blood form'd in paſſing through the *Aorta*'s Orifice at each Syſtole of the Ventricle. The Ventricle contracted, or the Pulſe in the Horſe beat 36 times in a Minute, or 2160 times in an Hour; but $9,64\times2160=20819,4$ Inches, or 1734,9 Feet, would paſs in an Hour, if the Time of the Contraction of the Ventricle were equal to the Time between two Pulſes: But it is but a third Part of that Time; therefore three times 1734,9 Feet, or 5204,7 Feet, will be the Velocity *per* Hour, or one Mile nearly, or 86,7 Feet *per* Minute.

14. But this Velocity is abated by the Diviſions and Subdiviſions of the Artery into numberleſs Branches, ſo that in the fine Capillaries it muſt be nearly deſtroyed, did not other Powers take place to continue and augment it; ſuch as the Re-action of the elaſtic Coats of the Arteries, the Attraction of Coheſion, and perhaps an electrical Virtue ariſing from conſtant At-

through the bare Hole, and ſtill quicker as the Pipe is longer and larger (LXXVI.)

trition, and extended through all the tubular Compages.

Pl XIX. Fig. 3.

(LXXVI.) 1. Let A B be a Veſſel fill'd with Water, it will run out at the Bottom through a Hole with a Velocity at firſt proportional to the Height A C, which will conſtantly decreaſe with the Altitude till the Veſſel be emptied. If another Veſſel N K, equal to the former, has a Tube inſerted at the Bottom, as L M, the Water will begin to run out at M with a Velocity proportional to LM; the Veſſel NK will therefore be exhauſted ſooner than the Veſſel A B. If another Veſſel E H be of an equal Diameter with the Veſſel N K or A B, and equal in Length to the Veſſel and its Pipe, the Surface of Water in each will begin to deſcend with equal Velocities; but that in E H will ſoon become greater than that in N K, becauſe of an equal Altitude of the Water, and a freer Exit at the Bottom, as having no narrow Tube to retard it.

Fig. 4.

2. The Theory of Water running through a Hole E F in the Bottom of a Veſſel ABDC is of a curious Conſideration, and may be thus explain'd. A B is the Surface of the Water, or Diameter of the Orifice of the Veſſel; and ſince the Water muſt move from the Surface to the Hole EF, it muſt move in a Body of a particular Figure AMEFNB, which is call'd the *Cataract* of deſcending Water. Now the Nature of the Curve A M E or B N F is thus determined.

3. The *Momentum* of moving Water is every where the ſame, (*Annot.* LXX. 17.) *viz.* at A B, M N, or E F; alſo the Velocity at MN or EF is as $\sqrt{HR}$ or $\sqrt{HG}$, (*Annotat.* LXXIV.) If we now take MN as an Ordinate to the Curve A M E, and put $MR = y$, and $HR = x$; then ſince the Quantity of Water flowing by M N in a given Time is as the Area of the Section or Square of its Diameter, *viz.* MN^2, which is as yy, and alſo becauſe the Velocity is as $\sqrt{HR} = \sqrt{x}$, therefore the *Momentum* of the Water will be $yy \times \sqrt{x}$. And becauſe this is every

II. THE ſecond Cauſe of the Riſe or Motion of Fluids is the *Preſſure of the Air* on

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where the ſame in the moving Cataract, the Quantity $y^2 \times \sqrt{x}$ is given or conſtant; and ſo $y^2 \sqrt{x} = 1$, and $\sqrt{x} = \frac{1}{yy}$, whence $x = \frac{1}{y^4}$; which ſhews the Curve AME, is an *Hyperboloid* of the fourth Order, whoſe Aſymptotes are X I parallel to the Horizon, and I G perpendicular thereto.

4. To find the Quantity or Magnitude of the Cataract we proceed thus; let *m n* be drawn infinitely near MN, then will $Rr = \dot{x}$, and becauſe $x = \frac{1}{y^4}$, we have $\dot{x} = -\frac{4x\dot{y}}{y} = -\frac{4\dot{y}}{y^5} = Rr$. The Area of the Circle upon the Diameter M N is pyy, but $pyy \times \dot{x} = pyy \times -\frac{4\dot{y}}{y^5} = -\frac{4p\dot{y}}{y^3} = -4py^{-3}\dot{y} = 2py^{-2} = \frac{2p}{yy} =$ C = the Cataract: But becauſe $x = \frac{1}{y^4} = \frac{1}{y^2} \times \frac{1}{y^2}$; therefore $xyy = \frac{1}{yy}$, conſequently $2xpyy = \frac{2p}{yy} =$ Cataract; but $2x = 2IR$, and pyy is the Area of the Circle M N. Therefore the whole Cataract from any Section to the Aſymptote XX, is equal to a Cylinder whoſe Baſe is the Section of the Cataract, and its Length twice the Height of the Cataract.

5. Now 'tis very evident from what has been ſaid of the Nature of the Curve Y A M E, that as ſoon as the Water begins to run out at E F, there will be at that Inſtant a Velocity in all Parts of the Cataract even in thoſe of the Surface A B. For let A = Area of the Circle upon AB, or Orifice of the Veſſel. and a = Area of the Hole E F; V = Velocity of the Water in paſſing through the Hole, and v = the Velocity at the Surface AB. Then becauſe it is A v = a V; we have V : v :: A : a; therefore whatever Proportion a has to A, the ſame Proportion will there be between v and V. So

the Surface of that in the Fountain or Reſervoir. Thus if a SYPHON or *Crane* be

that till A becomes infinite, or till A B coincides with XX, there will always be ſome Degree of Velocity in the Surface of the Water. But when A is infinitely greater than a, v will be infinitely leſs than V, that is nothing at all.

6. Since the Aſymptote XIX (or the infinite Surface of the Water) is the higheſt Part of the infinite Cataract, and where the Velocity commences, it is evident the Velocity will be in all Parts below as at A B, E F, as $\sqrt{IH}$ and $\sqrt{IG}$; conſequently the Velocity of the Water is different from that of Bodies falling from the Height of the Fluid, for of ſuch a Body the Velocity at E F would be as $\sqrt{HG}$, and at A B it would be nothing.

7. Let $GH = H =$ the Height of the Water in the Veſſel, and $IH = z$, then will $H + z = IG$. And ſo $V : v :: A : a :: \sqrt{IG} : \sqrt{IH} :: \sqrt{H+z} : \sqrt{z}$; or $V^2 : v^2 :: A^2 : a^2 :: H + z : z$; whence by Diviſion of Proportion, we have $H+z : H :: V^2 : V^2 - v^2 :: A^2 : A^2 - a^2$; conſequently $H + z = IG = \frac{V^2 H}{V^2 - v^2} = \frac{A^2 H}{A^2 - a^2}$.

8. According to Sir *Iſaac Newton* Bodies fall *in Vacuo* at the Rate of $193\frac{1}{2}$ Inches, or $16\frac{1}{9}$ Feet, the firſt Second, which Velocity becoming uniform will carry a Body over twice the Space in one Second, *viz.* $32\frac{2}{9}$ Feet. But uniform Velocities are as the Spaces deſcribed by them in the ſame Time: and the laſt acquired Velocities of falling Bodies are as the Square Roots of the Spaces deſcended through; therefore $32\frac{2}{9} : V :: \sqrt{16\frac{1}{9}} : \sqrt{IG}$ or $\sqrt{\frac{A^2 H}{A^2 - a^2}}$. Wherefore $\frac{32\frac{2}{9}\sqrt{IG}}{\sqrt{16\frac{1}{4}}} = V = 8{,}02773\sqrt{IG}$, or $8{,}02773\sqrt{\frac{A^2 H}{A^2 - a^2}}$ Feet, or $96{,}33276\sqrt{\frac{A^2 H}{A^2 - a^2}}$ Inches *per* Second.

immerſed with the ſhorter Leg in Water, and the Air ſucked out of the Inſtrument,

9. When A is very great in reſpect of a, then a^2 may be rejected as inconſiderable; and ſo the Expreſſion will be reduced to $802773\sqrt{H}$ Feet, or $96,33276\sqrt{H}$ Inches, for the Velocity of the effluent Water *per* Second. But (as Sir *Iſaac Newton* found, and is eaſy to obſerve) this Velocity is not ſo great in paſſing thro' the Hole in Experiments, as the Theory gives it: The Reaſon is, becauſe the Vein of effluent Water contracts itſelf, and grows narrower to the Diſtance of about a Diameter of the Hole below it; where, by meaſuring, Sir *Iſaac* found the Diameter of the Vein to that of the Hole as 21 to 25; whence the Area of a Section of the Vein there will be to the Area of the Hole as 441 to 625, that is, as 1 to $\sqrt{2}$; conſequently, the Velocity of the Vein is to the Area of the Hole as $\sqrt{2}$ to 1 in that Part, becauſe of equal *Momenta* in both Caſes.

10. Hence, if $IG = 48$ Inches, and the Diameter of the Hole 1 Inch; then $48+1=49$ Inches will be the Height from the Place where the Velocity is greateſt. Now the natural Velocities acquired by Water deſcending freely, from the Heights of 48 and 49 Inches, are as 69 to 70, and therefore nearly equal: Conſequently, the great Inequality of 1 to $\sqrt{2}$, or the Diminution of the Velocity in the Hole, muſt be owing to the Retardation occaſioned by the lateral Water reſiſting in ſome meaſure its perdendicular Deſcent.

11. Hence the Velocity of the Water in the Hole is very nearly equal to that which would be acquired in deſcending through $\frac{1}{2}IG$; becauſe the Velocity acquired by deſcending thro' $\frac{1}{2}IG$ is to that acquired by deſcending through IG as 1 to $\sqrt{2}$. Whence the Velocity in the Hole is to be diminiſhed in the Ratio of 1 to $\sqrt{2} = 1,414$. Thus, as $1,414 : 1 :: 8,02773 : 5,6773$; and as $1,414 : 1 :: 96,33276 \cdot 68,1278$; ſo that the true Velocity of the Water in the Hole is $5,6773\sqrt{H}$ Feet, or $68,1278\sqrt{H}$ Inches *per* Second.

the Fluid will aſcend into the vacuous Space by the Preſſure of the Air on

12. We obtain an Expreſſion of the Velocity alſo in other Terms, thus: The Quantity of (Q) of Water diſcharged is greater in Proportion to the Area of the Hole (a), the Velocity (V), and the Time (T) which it ſpends. Therefore $Q = TVa$; and ſo $V = \frac{Q}{Ta} = 68,1278\sqrt{H}$; and conſequently $Q = 68,1278\ T a \sqrt{H}$. Or, becauſe $a = 0,785398\ d\ d$, therefore $Q = 53,507476\ d d T \sqrt{H}$ Cubic Inches of Water. And becauſe a Cubic Inch of Water weighs $253\frac{1}{3}$ Grains, the Weight of the Water diſcharged will be $W = 13555,32\ d d T \sqrt{H}$.

13. Suppoſe a Veſſel 10 Inches ſquare, 4 Feet or 48 Inches high, and having an Hole at the Bottom 1 Inch ſquare. Then will $A = 100$, $a = 1$, and $H = 48$; alſo $A^2 = 10000$, $a^2 = 1$, and ſo $\frac{A^2}{A^2 - a^2} = \frac{10000}{9999} = 1,0001$, which may be rejected as inconſiderable. Wherefore $68,1278\sqrt{H} = 472$ Inches (*ferè*) the Velocity of the iſſuing Water *per* Second. $Q = 100 \times 48 = 4800$ Cubic Inches: Whence $\frac{4800}{472} = \frac{Q}{V} = T = 10$ Seconds, the Time in which ſuch a Veſſel would diſcharge a Bulk of Water equal to its own Bulk, if kept continually full all the while.

14. Now becauſe, when a is given, $Q = TV$; and when Q is given, $TV = 1$, and $T = \frac{1}{V}$, and $V = \frac{1}{T}$; it is plain all theſe Caſes are the ſame with thoſe of Bodies moving uniformly, as in *Annot.* XXII. And ſince V is a given Quantity when it increaſes or decreaſes in a given Ratio, therefore the above Theorems will be equally adapted to the uniformly accelerated and retarded Motion of Bodies, whether ſolid or fluid. From whence we can eſtimate the Time in which the above.

the Water in the Veſſel, and fill the whole Cavity of the Tube; and becauſe there is a greater Column or Body of Water in the longeſt Leg, it will preponderate and deſcend through it; and will keep flowing out, till the Veſſel is exhauſted to the Orifice of the ſhorter Leg (LXXVII.)

mention'd Veſſel full of Water will be diſcharged by its own Gravity only.

15. Thus it is evident, that the Column of Water that ſtands over the Hole will deſcend alone through it in ½ a Second, becauſe Bodies fall at the Rate of 4 Feet in ½ a Second, and that is the Height of the Column. If the Veſſel contains any Number of equal Columns beſides, the Time of their Evacuation will be proportional; that is, Q will always be as T, becauſe V in all Caſes (uniformly decreaſing with the ($\sqrt{H}$) Square Roots of the Altitudes) is a given Quantity; and becauſe there are 100 ſuch Columns in the Veſſel, it will require 100 Half-Seconds, or 50 Seconds, for the Time of the Diſcharge. This would be the Time, were the Water to ſet out with the Velocity of the Deſcent from the Height of 4 Feet: But ſince the initial Velocity is leſs than that in the proportion of 1 to $\sqrt{2}$, (*Art.* 9.) and ſince the Times are inverſely as the Velocities when Q or the Quantity of Water is given, (*Art.* 14.) therefore as $1 : \sqrt{2} = 1{,}414 :: 50'' : 70{,}''7$ = the true Time of the Diſcharge.

(LXXVII.) 1. It has been already ſhewn, that when a *Syphon* is firſt fill'd with, and then immerſed in Water, it is the Preſſure of the Air that cauſes it to continue full, and that it runs out of the longer Leg by the Preponderancy of Water in that Leg. (*Annot.* LXXII.) I ſhall only add here, that when the Air is drawn out by the Mouth-Piece, the Preſſure of the Air on the Surface of Water in the Veſſel cauſes it to riſe into and fill the Syphon; to evince which, we need on-

THE Effect of the COMMON PUMP is from the ſame Principle: For the Bucket or Sucker, raiſing a Column of Water above it in the Barrel, makes a vacuous Space, or greatly attenuates the Air in the Barrel and Pipe below it; whence the Water muſt neceſſarily riſe in the Pipe, by the ſuperior Preſſure of the Air on the Surface of that in the Well. The Water aſcending in the Pipe, paſſes through the Valve in the Bottom of the Barrel, which ſhutting upon the Depreſſion of the Sucker, the Water is thereby forced through the Valve of the Sucker into the Ciſtern of the Pump, and diſcharged through the Spout. All which is clearly ſhewn by the Model of a Pump in Glaſs (LXXVIII.)

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ly put a Syphon, while working, under the Receiver of an Air-Pump; for upon exhauſting all the Air, the Syphon will ceaſe running.

2. Or if the Syphon be equicrural, or has both the Legs of an equal Length, the Water will not run thro' it, becauſe the Force of Water againſt the Air at each Orifice is equal. The ſame Thing will be effected with a Syphon of unequal Legs, though the ſhorter Leg be immerſed in Water, if the Syphon be ſo inclined that the Orifice of both Legs be on the ſame horizontal Level. I need not after this give the Reaſon why the Water will not run out of the ſhorter Leg, if the longeſt be placed in the Water.

(LXXVIII.) 1. I ſhall here give the Theory of *Pump-Work*, and the Structure of ſuch Pumps as are found to be moſt uſeful for the various Purpoſes of Life. The Theory depends in a great Meaſure upon thoſe

III. IV. But the moſt conſiderable of all Water-Engines is that call'd *New-*

Properties of the inverted Syphon, or Fountain, deſcribed in *Annot.* LXXIII. *Art.* 11, 12, 13, 14. Let ABCDEFG repreſent the inverted Syphon there deſcribed: Only here the Part ABCD repreſents a Column of Air, and DG the lower Part of the Pipe of a Pump immerſed in the Water of the Well H I. Let P be the Piſton of the Pump at E in its loweſt Situation, and at F in its higheſt. Now as both theſe Parts communicate with the Water, one by preſſing on it, the other by opening into it, they may be looked upon as communicating with one another.

2. Now A B C, the Column of Air, would by its Weight or Preſſure force up a Column of Water into the Pipe D G to the Height of 33 Feet, were the Air exhauſted from the ſaid Pipe, and continued to that Height; (the Weight of a Column of Air being equal to the Weight of ſuch a Column of Water of the ſame Baſe, as will be ſhewn hereafter.) If now the Piſton P be thus thruſt down to E, meeting the Water there, and from thence it be raiſed to F with an uniform Motion, the Water will riſe from E to follow the Piſton with a variable Velocity; the leaſt of which is as $\sqrt{AC} - \sqrt{DE}$, and the greateſt as $\sqrt{AC - DE} = \sqrt{AB}$, (by *Annotation* XXIV. 12.)

3. If in lifting up the Piſton the Velocity of Water be leſs than that of the Piſton, it will not be able to follow it, but will leave a void Space between them, which will increaſe more and more as the Velocity of Water becomes leſs than that of the Piſton. The Conſequence of this will be, that a Part of the Stroke of the Piſton will be loſt; and not only that, but the Piſton, when the Water leaves it, will riſe very hard, as having the Weight of Water upon it, and the Air of greater Denſity above than below; whence the Equilibrium, which ought to be in Pumps, is deſtroyed, and the Balance againſt the Workman. Now as this can happen where the Diameter of the Sucking-Pipe is equal to that of the Pump-barrel, much ſooner

ſham's Engine, from the great Improvements which Mr. *Newſham* made in it ;

will it happen when the Pipe is leſs than the Barrel, becauſe the Water riſing through a leſs Paſſage will be no longer in filling the Pump-Barrel, and therefore quit the Piſton, and leave the greater void Space between.

Pl. XIX. Fig. 5.

4. On the contrary, if the leaſt Velocity of Water riſing in the Pipe be greater than that of the Piſton, there will be no void Space ; and the Pump-Barrel may be made in proportion as much wider than the Pipe as the Velocity of Water is greater than that of the Piſton. Now that this may be the Caſe, we ſhall ſhew by Calculation what Diameters the Barrel and Pipe ought to have compared with the Velocity of the Water and Piſton. Let A repreſent the leaſt Altitude of the Atmoſphere $AC=31$ Feet of Water; $B=DF$ the higheſt Elevation of the Piſton above the Surface of the Water H I, which let be 16 Feet. And let the greateſt Velocity of the Piſton which can well be given to a Pump be that of four Feet in a Second $=v$; and $V=$ the leaſt Velocity of Water that riſes in the Pipe ; $D=$ the Diameter of the Barrel ; and $d=$ Diameter of the Pipe.

5. Now here we have $\sqrt{A}-\sqrt{B}=V=$ the leaſt Velocity of Water ; and the Fall which will produce that Velocity is the Square of that Expreſſion, *viz.* $A+B-2\sqrt{AB}$, that is, $31+16-2\sqrt{31\times 10}=2$ Feet 6 Inches, the Height of the Fall required. Whereas by the common Way of taking the Square of $\sqrt{A-B}$, *viz.* $A-B$ for the Height, we have 15 for the Fall, which extraordinary Error muſt be of very bad Conſequence in Practice.

6. Here the Velocity $\sqrt{A}-\sqrt{B}=\sqrt{31}-\sqrt{16}=5{,}6-4=1{,}6$ *per* Second. The Velocity of the Water at the Bottom of the Pipe D is as $\sqrt{A}=5{,}6$; that alſo muſt be the Velocity of the Piſton at D, that the Water may follow it ; whence the Piſton moving with the ſame Velocity at F, where the Velocity is but 1,6,

the Nature and Effect of which will be eaſily underſtood from a perpendicular

we have 5,6—1,6=4 Feet of void Space, therefore 4 Parts in 5,6 of ſuch a Stroke would be loſt or ineffectual. We may here obſerve by the Way, that ſince the Velocity of the Water at D is 5,6, and the greateſt Velocity which can be given to the Piſton (without damaging the Machinery) is but 4; therefore a Piſton working at the lower End of the Pipe or Barrel will always have Water more than enough to prevent any void Space or Loſs of Labour.

7. But ſince we find ſo great a Void in the Pipe at F of the ſame Diameter with the Piſton, 'tis evident if we contract all the Parts below F into a ſmall Pipe as *F D*, and let the Part F G remain as it was for the Barrel for the Piſton to play in, as at *F G*, that then the Water will riſe into the Barrel *FG* with a greater Velocity than before, in proportion as the Pipe is leſs; conſequently *if the Bore of the Pipe F D be to that of the Barrel FG, as the Velocity of the Piſton* P, *or the Water in the Barrel, is to the Velocity of the Water in the Pipe*, there will always be a Sufficiency of Water to prevent a *Vacuum* in the Barrel. Which Rule in Symbols is thus expreſſed; $DD : dd :: V : v$; whence $D^2 v = d^2 V$ for a general Canon; any three of which Quantities being given, the fourth may be found.

8. Thus, for Example, ſuppoſe D, v, and V were given to find d, we have $d = \frac{D\sqrt{v}}{\sqrt{V}}$. Let the Diameter of the Barrel D = 6 Inches. And ſuppoſe the Piſton gives 20 Strokes in a Minute, each a two Foot Stroke, ſpending as much Time in its Aſcent as Deſcent, then will the Motion of the Piſton be 80 Feet *per* Minute, or $1\frac{1}{3}$ *per* Second; whence $v = 1\frac{1}{3}$. Laſtly, to obtain the Value of U, we muſt fix on the Length of the Pipe *F D*, which let be 16 Feet, then the higheſt Elevation of the Piſton will be 18 Feet (if it comes to the Bottom of the Barrel, as it ought to do;) wherefore an Height of Water of 18 Feet in the Pump *G D* acts Pl. XIX. Fig. 3.

Section of a Model, with ſome Variation which I have made of the Structure there-

againſt the Weight or Height 31 Feet in the Leg A C. Now the Velocity V of the Water in the Pipe *FD* being uniform, or conſtantly the ſame, we muſt find what Difference of uniform Velocities will be generated by Falls from 31 and 18 Feet Heights. Thus $\sqrt{16\frac{1}{9}}$: 32 : : $\sqrt{31}$: 43 nearly; and $\sqrt{16\frac{1}{4}}$: 32 :: $\sqrt{18}$: $32\frac{3}{4}$; whence the Difference of theſe uniform Velocities will be $43 - 32\frac{3}{4} = 10\frac{1}{4} =$ 10 Feet 3 Inches *per* Second; therefore $V = 10\frac{1}{4}$. Wherefore $\frac{D\sqrt{v}}{\sqrt{V}} = d = 2\frac{1}{10}$ Inches for the Diameter of the Pipe *F D*.

17. The Reaſon why we make no Uſe of the Expreſſion $\sqrt{AC} - \sqrt{DF}$ in this Caſe, is becauſe this gives only the Difference of the inſtantaneous Velocities, or the leaſt Velocity with which the Water at F can begin its Motion upwards, whereas we here want to find what the conſtant and uniform Motion of the Water will be, or how much it will ſupply every Second uniformly, which is done by the Method above. For ſince a Fall of $16\frac{1}{9}$ gives an uniform Velocity of 32 Feet *per* Second, a Fall of 31 Feet will give 43; thus a Fall of 18 Feet will give $32\frac{3}{4}$, and their Difference muſt be that of the Water at F.

18. If we know the Velocity of the Piſton v, the Diameter of the Barrel D, and the Diameter of the Pipe d, we ſhall find the Velocity of the Water in the Pipe $V = \frac{vDD}{dd}$; or thus in Words, *Multiply the Square of the Diameter of the Barrel by the Velocity of the Piſton; divide the Product by the Square of the Diameter of the Pipe, the Quotient will be the Velocity ſought from the Water in the Pipe.* This Velocity when found, muſt be taken from 43, the Remainder $43 - V$ will be the uniform Velocity produced by a Fall from the higheſt Situation of the Piſton to the loweſt Surface of the Water in the Well, and which is found by ſaying, as 32 :

of. The Motion of the Water in this Engine is effected by the *Preſſure of the*

$\sqrt{16\frac{1}{9}}$ :: 43—V : $\sqrt{DF}$; whence DF the higheſt Elevation of the Piſton will be known.

19. When we know D, V, and *d*, we find *v* the Velocity of the Piſton by this Theorem $\frac{V\,dd}{DD} = v$; that is, in Words, *Multiply the Square of the Diameter of the Pipe by the Velocity of the Water in it, and divide the Product by the Square of the Diameter of the Barrel, the Quotient is the Velocity of the Piſton required.*

20. Having given the Velocity of the Water in the Pipe V, the Diameter of the Pipe *d*, the Velocity of the Piſton *v*, to find the Diameter of the Barrel D, we have this Theorem $D = \sqrt{\frac{V\,dd}{v}}$; that is, *Multiply the Square of the Diameter of the Pipe by the Velocity of its Water, and divide that Product by the Velocity of the Piſton; the Square Root of the Quotient is the Diameter of the Barrel ſought.*

Pl. XIX. Fig. 6.

21. If a Piſton works in a Barrel or Pipe AD of the ſame Diameter from Top to Bottom; then we can find the Height to which the Water will riſe at every Stroke of the Piſton playing at any Diſtance above the Surface of the Water B D. Thus let E be the loweſt Part to which the Piſton comes, and let ED=12 Feet Diſtance from the Water, and ſuppoſe its Length of Stroke EF=4 Feet. Then when the Piſton is at E, the lower Part of the Pipe ED is fill'd with Air of the ſame Denſity or Spring with the common Air, which call S; when the Piſton is raiſed from E to F, ſo that FD=16, two Effects will follow; one is, the Air will now expand itſelf into a larger Space beneath the Piſton, and have its Spring weakened; the other is, that ſince the Spring of the Air in the Pipe is leſſen'd it will preſs leſs on the Surface of the Water in the Pipe than the external Air preſſes on that without, the Water will riſe ſo far into the Pipe, till its Weight, together with

Atmoſphere, by the Force of Piſtons, and *by the Spring of condenſed Air,* in the fol-

the diminiſh'd Spring of the Air, be equal to the Preſſure of the external Air.

Pl. XIX. Fig. 6.

22. Suppoſe the Water riſes to c; call its Height $CD = x$; then the Space into which the Air is expanded will be $FC = 16 - x$. Let its Spring thus diminiſhed be call'd s; then the Spring of the Air being inverſely as the Spaces which it takes up, we have $S : s :: 16 - x : 12$; therefore $\frac{12S}{16-x} = s$. The Weight of a Column of Water 32 Feet high is equal to the Preſſure of the Atmoſphere, that is, equal to S; whence $32 : S :: x : \frac{Sx}{32} =$ the Weight of the Water x. But this added to the weaken'd Spring of the Air is equal to S; that is, $\frac{12\,S}{16-x} + \frac{Sx}{32} = S$; and dividing by S, we have $\frac{12}{16-x} + \frac{x}{32} = 1$; or $384 + 16x - x^2 = 512 - 32x$; or $x^2 - 48x = -128$, and by adding on each Side the Square of 24 (to compleat the Square) we have $x^2 - 48x + 576 = 576 - 128 = 448$, and extracting the Root, $x - 24 = \pm\sqrt{448} = \pm 21{,}166$. Wherefore $x = 2{,}834 = DC$, the Height of the Water required.

23. This Height taken from $12 = ED$; leaves $EC = 9{,}166$, and calling the whole Height to which the Water will riſe by the firſt and ſecond Stroke together x; we ſhall have this ſecond x determin'd by the Equation $\frac{9{,}166}{16-x} - \frac{x}{32} = 1$, inveſtigated and reſolved as before. For ſince the Water CD cannot recede (by reaſon of a Valve below) when the Piſton a ſecond Time comes to E, the Valve in the Piſton will be opened, and the Part EC again fill'd with common Air; whence $S : s :: 16 - x : 9{,}166$, &c. as before. The Equation reſolved gives $x = 5{,}098$, from which taking the firſt $x = 2{,}834$, there remains 2,264, for the

lowing Manner, *viz.* When the Piſton R is raiſed, a *Vacuum* would be made in the

Height it riſes the ſecond Stroke. In the ſame Manner we calculate that the Water riſes,

Firſt Stroke	2,834	2,834
Second	2,264	5,098
Third	2,025	7,123
Fourth	2,043	9,166
Fifth	2,413	11,579
Sixth	3,620	15,199

24. But to ſave the Trouble of a Calculation every Time we would find x for the various Heights of the Piſton, let us raiſe a general Theorem for all Caſes; thus let $a =$ the greateſt Elevation of the Piſton, $b =$ leaſt, $a - b = p =$ the Length of the Stroke, and $h =$ Height of a Column of Water equal to the Weight of the Atmoſphere; then the foregoing Theorem will become $\frac{b}{a-x} + \frac{x}{h} = 1$, and ſo $ax - x^2 + bh = ah - xh$; or $ax + hx - xx = ah - bh = ph$; putting $\frac{a+h}{2} = s$, we have $2sx - xx = ph$; and changing the Signs we have $xx - 2sx = -ph$; and compleating the Square, $xx - 2sx + ss = ss - ph$; and ſo $x - s = \pm\sqrt{ss - ph}$; and finally $x = \pm\sqrt{ss - ph} + s$.

25. Now this general Theorem finds not only x; but any of the other Quantities, having the reſt given. Thus ſuppoſe the higheſt Elevation of the Piſton be 16 Feet $= a$, $h = 32$; and I would raiſe the Water the firſt Stroke 10 Feet: *Quære* p the Length of the Stroke? Here $x = 10$, and $s = \frac{a+h}{2} = 24$; then above we had $2sx - xx = ph$, whence $p = \frac{2sx - xx}{h} = 11,9$, or near 12 Feet for the Length of the Stroke required.

26. Since after every Stroke or Aſcent of the Piſton, the Air in the Barrel is in a dilated State; and ſince

Barrel TV, did not the Water follow it from the inferior Canal EM (through the Valve

when the Piſton deſcends, the Water by its Tendency downwards carries down the Valve at D, and by this means cuts off the Communication of the Air at D, and ſupports the Column of rais'd Water, 'tis plain the Valve in the Bucket of the Piſton will be affected on its upper Part by the Weight or Spring of the common Air, and on the under Side by the Spring of the dilated Air only; and therefore upon the Deſcent of the Piſton the Valve of the Bucket will not riſe but compreſs the internal dilated Air into a Space gradually leſs till its Spring by that Means comes to be equal to that of the external Air, and then as the Piſton deſcends it will ſuffer no farther Compreſſion, but force up the Valve, and make its Eſcape.

27. Thus for Example, by the firſt Stroke (in *Art.* 23.) the Water roſe 2,834, which ſubſtracted from 16, leaves 13,166 for the Space into which 12 Feet of Air was dilated. Now its Spring being weaker than that of the external Air, upon the Deſcent of the Piſton, it will be compreſs'd into the former Space of 12 Feet before it can recover its former Spring, the Piſton will then have deſcended through 1,166; and ſince the Stroke is 4 Feet, and the Air will now be no farther compreſs'd, therefore $4-1,166=2,834$ will be the Quantity of Air which will make its Eſcape thro' the Valve of the Piſton; which in this Caſe is equal to the Column of Water raiſed.

28. The Piſton being now again at E, upon its Aſcent, it will permit the Column of Air $EC=9,166$ to expand itſelf into a $Space=16-5,098=10,902$; and upon deſcending it will compreſs the Air again into 9,166 Feet, before it can eſcape. To do this the Piſton muſt deſcend through the $Space=1,736$; and therefore $4-1,736=2,264$ is the Quantity of Air that will be exhauſted the 2d Stroke; and the Column of remaining Air EC will now be $9,166-2,264=6,902$ Feet. And in this Manner are the Numbers found for every following Stroke.

H)

H) which riſes through the Glaſs Tube EF immerſed in the Water of a Veſſel by

29. Hence 'tis eaſy to obſerve, that whenever the Space thro' which the Piſton muſt paſs in its Deſcent to reduce the internal Air, by Compreſſion, to a Denſity or Spring equal to the external Air; I ſay, when this Space happens to be greater than the Length of the Stroke, there can no Water riſe in the Pump; becauſe in that Caſe, the Air will be no farther able to make its Eſcape thro' the Valve of the Sucker, by reaſon of the greater Preſſure of the external Air upon it.

30. When this is the Caſe of a Pump, let x be the greateſt Height to which the Water can be raiſed in it; when the higheſt Elevation of the Piſton is a, the loweſt b, the Preſſure of the Atmoſphere h, and $a-b=p$ the Stroke of the Piſton, as before. Then when the Piſton is down, the Space in which the Air is contain'd is $b-x$; and when up, it is expanded into $a-x$; its Preſſure is then the $\frac{b-x}{a-x}$ Part of what it was before, that is of the Atmoſphere; the Weight of the Water being the other Part, *viz.* $\frac{x}{h}$; and both together being equal to the Atmoſphere, we have $\frac{b-x}{a-x}+\frac{x}{h}=1$; therefore $bh-hx+ax-x^2=ah-hx$; or, $ax-x^2=ah-bh=ph$. And changing the Signs, $x^2-ax=-ph$; and compleating the Square, $x^2-ax+\frac{1}{4}aa=\frac{1}{4}aa-ph$; wherefore $x-\frac{1}{2}a=\sqrt{\frac{1}{4}aa-ph}$; therefore $x=\pm\sqrt{\frac{1}{4}aa-ph}+\frac{1}{2}a$.

31. Hence if the Stroke of the Piſton and its Height above the Water be given, we know the greateſt Height to which the Water can riſe, and conſequently, if ſuch a Pump can be effectual. For Example, let $a=28$, $b=25$; then $p=3$, and put $h=32$; then $x=\pm 10+14$; that is, the greateſt Height to which the Water can riſe $=14-10=4$ Feet; for the other Root, $14+10=24$, is impoſſible, becauſe all Heights between 4 and 24 will be too great.

the Preſſure of the Atmoſphere on the Surface thereof. The Water of the Bar-

32. Again, ſuppoſe $a=24$, $b=20$; then, $p=4$, and $h=32$; $x=\pm 4+12$, that is, $x=12-4=8$ Feet, the greateſt Height. If $a=20$, $b=16$, $p=4$; then we ſhall have $x=\pm\sqrt{\frac{1}{4}aa-ph}+\frac{1}{2}a=\sqrt{100-128}+10=\sqrt{-28}+10$. Now the ſquare Root of a negative Quantity being impoſſible, ſhews the Caſe in Queſtion has no Place in Nature, or that here is no Space between x, and the loweſt Situation of the Piſton. Nor can there ever be any when ph is greater than $\frac{1}{4}aa$, the Water riſing to the Piſton in all thoſe Caſes. When $\frac{1}{4}aa=ph$, then $\sqrt{\frac{1}{4}aa-ph}=0$; and $x=\frac{1}{2}a$; and in this Caſe, $p=\frac{\frac{1}{4}aa}{h}$, or $\frac{aa}{4h}$. Thus if $a=16$, $\frac{1}{2}a=8$, $=x$, the greateſt Height the Water will riſe to, and $p=\frac{\frac{1}{4}aa}{h}=\frac{64}{32}=2$ Feet, the Length of the Stroke.

Note, If we make the Fluxion of the Equation $x^2-ax=-ph$, equal to 0, (*viz.* $2x\dot{x}-a\dot{x}=0$) it gives $2x=a$, or $x=\frac{1}{2}a$, when a *Maximum*, as before.

33. From this Theorem we learn this general Rule concerning Pumps, *That no Pump can raiſe Water, but when the Length of the Stroke of the Piſton exceeds the Square of the higheſt Elevation of the Piſton divided by* 128. Thus if that Elevation be 16 Feet, the Square of that is 256, which divided by 128, quotes 2; therefore the leaſt Stroke of the Piſton muſt be greater than 2 Feet. If the Elevation be 20 Feet, then $20\times20=400$ and $\frac{400}{128}=3,13$, which ſhews the Elevation of the Piſton muſt be in that Caſe greater than 3,13 Feet. If the Elevation be 12 Feet, then $12\times12=144$, and $\frac{144}{128}=1,12$, which is leſs then the leaſt Length of the Stroke of the Piſton. And theſe Things, I think, finiſh the Theory of Pumps.

Plate XX

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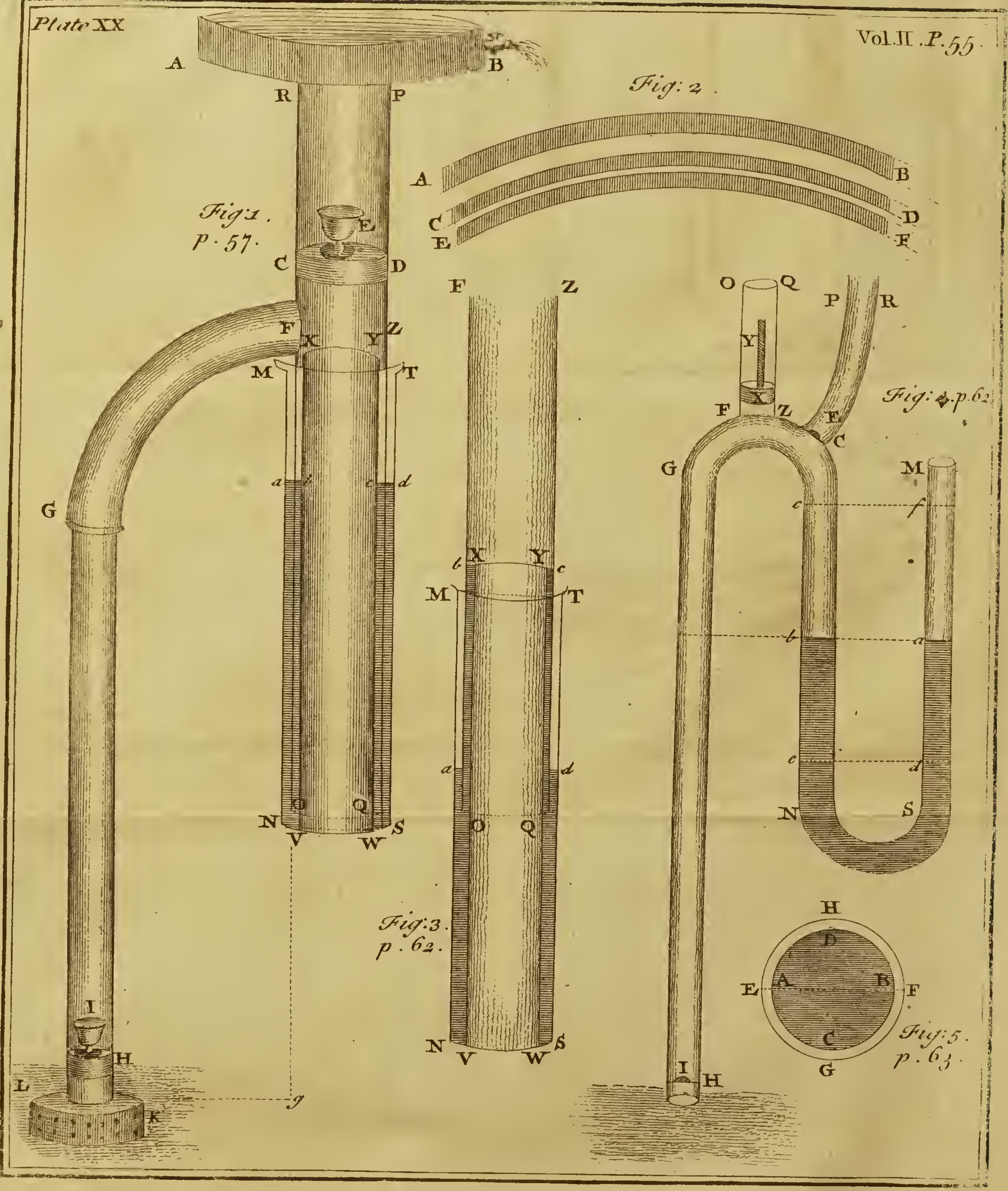

rel T V, by the Depreſſion of the Piſton R is forced through the ſuperior Canal O N,

34. I ſhall now proceed to the Deſcription of two or three moſt uſeful Pumps, but firſt give an Account of that Sort of Valves which I find are exceeding good, and ought to be uſed in Pumps and all Kinds of Water Engines where Valves are neceſſary. Let A B C D be the Bucket of a Piſton, or any other Part where a Valve is required; in the Middle there is a circular but tapering Hole from Top to Bottom, in which is fitted the tapering or conical Piece E F G H, with a Piece I K to be ſcrew'd in and out of the Bottom Part H I. It is to be ſcrew'd out, when the ſaid Solid E G is put into its Place or Hole in A C; and afterwards ſcrew'd in, when the Whole together appears as in the Figure L M N O. The Piece E G now becomes a Valve, or capable of permitting the Water to aſcend, and to prevent its Deſcent. Pl. XIX. Fig. 7.

35. That the Water puſhing againſt the Bottom of the Valve will raiſe it upwards, is evident from the conical Form thereof, and its lying in the Hole only by its own Weight; the Length of the Key at K being ſufficient to permit ſuch a Riſe of the Valve, as will admit a Space between it and the Hole for the Water to paſs as freely as required: And, that the Valve may not be thrown quite out of the Hole, the croſs Piece I is added, of a greater Length than the Diameter of the loweſt Part of the Valve.

36. If the Valve E G and its Socket A C be of Braſs, and fitted, by grinding them with Emery firſt, and Putty afterwards, with a Drill-Bow into each other, they will not only be Water-tight, but even Air-tight; and that too if but ſlightly touch'd with fine Emery or Putty; for if they are ground to a Poliſh, the Attraction of Coheſion will take place, and prevent the Valve from riſing ſo freely as it ought to do; yea, ſometimes thoſe Surfaces have been found to cohere ſo ſtrongly, that the Force of the riſing Water could not overcome it: But all this will be prevented, and every thing ſuc-

to enter by the Valve I into the Air-Veſſel *a b c d*; and the like being done alternately

ceed to one's Wiſh, if they are made as above directed, as I have learnt by long Experience and trying every Way.

Pl. XIX. Fig. 8. 37. The COMMON SUCKING PUMP (as it is call'd) is ſo far deſcribed in the Theory, that ſcarce any Thing more can be ſaid to explain it. However, I have given the Figure of ſuch a Pump, in which A is the Ciſtern, A B the Barrel or Pipe ſtanding in the Water E F, the Piſton G D with its Bucket and Valve D, and the Valve C open for the Aſcent of the Water through it.

Fig. 9. 38 A FORCING PUMP of the common Sort is thus conſtructed: A B is the Barrel ſtanding in the Water of the Well or Mine at B; G C is the Piſton; and C a ſolid Piece without any Hole or Valve, becauſe no Water is to paſs through it, as in the other. This Piece ſhould be well leather'd, to fit the Barrel very nicely, that in its Motion neither Air nor Water ſhould paſs between. At a Diſtance below, a Valve with its Diaphragm is fixed in the Barrel, as at D. Between this and the loweſt Situation of the Piſton C there goes off a Pipe at H, in which there is fixed a Diaphragm and Valve, as at E. Now the Piſton being drawn up from C towards A, attenuates the Air above D, by which means the Water ruſhes into the Space C D; then when the Piſton is forced down, as the Water cannot repaſs at D, it is forced to aſcend into the Pipe at H, and through its Valve E into the Ciſtern F, (which may be ſituated at any Diſtance above the Pump) from whence the Water will run off by the Spout.

Fig. 10. 39. A LIFTING PUMP is a Forcing Pump of another Form or Structure. A B is a Barrel fixed in a Frame I K L M, which alſo is fixed immoveable, with the lower Part in the Water to be exhauſted. G E Q H O is a Frame with two ſtrong Iron Rods, moveable through Holes in the upper and lower Parts of the by the other Barrel W X, and its Piſton S, the Air-Veſſel is by this means continually

Pump I K and L M; in the Bottom of this Frame Q H is fixed an inverted Piſton B D, with its Bucket and Valve upon the Top at D. Upon the Top of the Barrel there goes off a Part E R, either fixed to the Barrel, or moveable by a Ball and Socket, (as here repreſented at F) but in either Caſe ſo very nice and tight, that no Water or Air can poſſibly get into the Barrel, which would ſpoil the Effect of the Pump. In this Part, at C, is fixed a Valve opening upwards. Now when the Piſton-Frame is thruſt down into the Water, the Piſton D deſcends, and the Water below will ruſh up through the Valve D, and get above the Piſton; where, upon the Frame's being lifted up, the Piſton will force the Water through the Valve C up into the Ciſtern P, there to run off by the Spout. *Note*, this Sort of Pump is ſet ſo far in the Water, that the Piſton may play below the Surface of it.

40. The MERCURIAL PUMP, or that which works by Quickſilver, is the next which I ſhall deſcribe. It was the Invention of Mr. *Joſhua Hoſkins*, and brought to its preſent Perfection by Dr. *Deſaguliers*, together with his Workman Mr. *William Vreem*. As this is one of the moſt curious of modern Inventions, I ſhall be a little the more prolix in its Deſcription; which I ſhall give from a Model that I have made to explain the Nature and Manner of working in this Pump to thoſe who attend my Lectures in Philoſophy. This Model works very well, and is here repreſented in its real Magnitude in the Figure.

41. A B is a Ciſtern placed on the Top of a Braſs Tube P O Q R, open at each End P R and O Q. Towards the upper Part of the Tube at F is inſerted (or ſolder'd) a curved Pipe F G, opening into the Tube at F; and in the End G is cemented a Glaſs Tube G H, fixed below in an hollow Box L K, full of Holes for the Admiſſion of Water, into which it is immerſed. M N S T is an exterior Tube, open at the Top M T, Pl. XX. Fig. 1.

filling with Water, which greatly compreſſes the Air above the Surface of the

and cloſe at the Bottom N S; in which Bottom is firmly fixed another hollow Tube V X Y W, cloſe on the Top X Y. This inner Tube may be a ſolid Piece of Wood, as in my Model it is. This inner Tube or Cylinder of Wood goes up into the Tube Q O P R, at the ſame Time that this is inveſted by the exterior Tube M N S T, as is ſeen in the Figure. In the lower Part of the Pipe at H is a Diaphragm, and a Valve I, opening upwards for the Aſcent of Water when a *Vacuum* is made. At C D in the main Tube, above the Inſertions of the Pipe, is another Diaphragm and Valve E, opening upwards alſo to give Paſſage for the Water in the forcing Part of the Stroke.

42. The conſtituent Parts of the Pump being thus deſcribed, we are next to view the peculiar Manner of working by *Quickſilver*, which for that Purpoſe is pour'd into the exterior Tube M S, which, when applied to the Pump in its Place, will be made to riſe in two cylindric Shells; one about the Tube of the Pump outwardly; the other within, about the innermoſt Tube or Plug X Q, as repreſented in the Figure at *a*, *b*, and *c*, *d*. At the Bottom there is but one cylindric Shell, becauſe the middle Tube does not reach the Bottom, leaving the Space V O W Q.

43. From this Account 'tis eaſy to obſerve, that the Part C O Q D anſwers to the Barrel of the Common Pump, F G H to the Pipe, the inner Tube X Q to the Piſton, and the Quickſilver at Bottom to the Leathers of the common Piſton; for it prevents all Communication of the external Air and internal Part of the Barrel, where the *Vacuum* is to be made. Whence 'tis evident, that upon letting down the outer Tube M S, it carries down the inner Tube X Q at the ſame Time, and makes a larger Space in the Barrel and Pipe, in which the Air will be expanded or become rarer, and its Spring thereby weakened. In conſequence of this, the Water will riſe through the Valve at I into

Water in the Veſſel, and thereby proportionably augments its Spring; which at

the Pipe, and alſo the Mercury will riſe in the inner Shell by the Preſſure of the Air on the outer Shell, till the Equilibrium be reſtored: And the Height of the Water raiſed will be nearly 14 Times as great as that of the Mercury.

44. When the Tube S M is raiſed again, the Air will be compreſs'd within the Barrel; and, its Spring increaſing, it will act upon the Water, the Mercury of the inner Shell, and the Valve E; the Water above the Valve I it cannot move, becauſe of the Valve being ſhut below; its whole Force is therefore ſpent on the Mercury, and Valve E; it will act on the inner Shell of Mercury, and drive it down to the Level of that in the outer Shell, as at *a b*, *c d*; and then the Preſſure will be every where equal, *viz.* on the inner and outer Shell, and on the Valve E; as the Tube after this continues moving up, the Air will be farther compreſs'd, and its Springs made greater than that of the outward Air, which therefore it will overcome, and ſo thruſt up the Valve E, till ſo much has eſcaped as leaves the Remainder of an equal Tenour, or *in Equilibrio* with the Atmoſphere. The two Shells of Mercury will be upon a Level all the while the Air is going through the Valve, becauſe the Preſſure is not greater within than without.

45. By repeating the Operation a ſecond Time, the Air will be farther rarified, and the Water will again riſe in the Pipe; and thus on till the Pipe and Barrel be full as in the common Pump; and hence 'tis evident all thoſe Theorems raiſed in the preceding Articles, are equally applicable here as there. This Pump made at large for Uſe, ſhould have the following Dimenſions: The Length of the outer Tube M N = 30 Inches; of the inner Tube X O = 31 Inches; the Diameter of the inner Tube X Y or Q O = 6 Inches; the Thickneſs of the outer Tube = $\frac{1}{10}$ of an Inch,

length is ſo far increaſed, as to re-act with great Force on the Surface Y Z of the ſub-

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of the middle one $\frac{8}{100}$ of an Inch, and of the inner one, $\frac{13}{100}$ of an Inch, and the Bottom of the Tube Z O to come within an Inch of the Bottom N S of the outer Tube.

46. Theſe Dimenſions afford ſufficient Strength if the Tubes or Barrels are made of Copper, or caſt Iron, and of ſuch a Diameter, that the Diſtance between each may be $\frac{1}{2}$ the Tenth of an Inch; and this ſhould be nicely effected by having the Tubes truly turned in a Lathe. A tranſverſe Section of a Part of the Circumference of thoſe Tubes, their Thickneſſes, and Spaces between, you ſee repreſented in the *Fig.* 2. where A B is the outer, C D the middle, and E F the inner Tube. The Spaces between are ſo narrow, becauſe otherwiſe too great a Quantity of Quickſilver would be neceſſary, and yet of no Uſe; becauſe Fluids preſs only according to the Altitude, and not the Quantity.

47. If now every Part be fitted for Work, and Mercury poured into the Height of 24 Inches, as ſhewn by *a* N S *d*; and the Barrel and Pipe fill'd with Water, ſo that the whole Pump be full, and *in Equilibrio* with the Atmoſphere; if the outer Tube M S be moved down through the Space of 14 Inches, at the Commencement of the Motion, the *Equilibrium* in the Pump is deſtroyed by the greater Space which would enſue upon the Deſcent of the inner Tube X Q, and which cannot be ſuffered, becauſe of the Preſſure of the Air on the Water at H, and on the Mercury in the outer Shell at *a*, *d*. And becauſe theſe Preſſures of the Air outwardly at H and *a* are equal, but there is not an equal Preſſure inwardly on the Valve I, and the inner Shell of Mercury *b*, *c*; the Valve being preſs'd with all the Water in the Pipe above it, and which is proportional to the Altitude C *g*; and the Surface of the Mercury of the inner Shell at *b*, *c*, being preſs'd only with the Altitude of Water C *b*, 'tis plain the Water will

jacent Water; which aſcending through the ſmall Tube *ef* to the *Stop-Cock eg*, is

not ruſh in at I till the Preſſure on *b*, *c*, becomes equal to it, and that will be when the Height of the inner Shell at *b* is greater than that of the outer Shell at *a*, by near $\frac{1}{14}$ Part of the Difference of the Altitudes *bg*; and then the Preſſure being equal at *b* and I, upon the Motion of the Tube X Q downwards, the Water will be forced up through the Valve at I, and the Height of the Mercury in the inner Shell will always exceed that in the outer Shell, in ſuch Manner, that the Exceſs will be about $\frac{1}{14}$ of *bg*, or the Height of the Point *b* above the Water at H.

48. When the outer Tube is in its loweſt Situation, the Mercury in the inner Shell will be nearly at the Top X Y of the inner one, and in the outer Shell it will have but a ſmall Height as *a* O, or Q *d*. And when the Tube M S is drawn up again, the inner Tube X W will force the Water in the Pump to act upon the upper Part of the Valve I, the under Part of the Valve E, and the Surface of the Mercury at *b* in the inner Shell. The Valve I it ſhuts cloſe, the Valve E it endeavours to puſh up but cannot, till it has firſt reduced the Mercury in each Shell to a Level, when they, ballancing each other, produce an *Equilibrium* with the external Air acting equally on the Surface of the Mercury in the outer Shell at *a*, and on the Valve E. After this, as the Tube M S continues its Motion upwards, the Mercury will riſe in the outer Shell, the Preſſure now on the inmoſt being greateſt. And for every 14 Feet the Water is forced above the Surface of the inner Shell *b*, the Mercury will riſe in the outer one, one Foot.

49. Becauſe the Structure of this Pump makes the Theory of its Operation not quite ſo eaſy and obvious to thoſe who have not conſider'd it very thoroughly, (as I have found by frequent Experience in my Lectures) I have contrived to repreſent it more plainly, by a Kind of Syphon or Pump of a very ſimple Form, and

there, upon turning the Cock, ſuffered to paſs through a Pipe *b*, fixed to a *Ball and*

yet acting on the very ſame Principles as this complex Pump under Conſideration. The Syphon is H G F Z N S M, where H G F is the Water-Pipe as in the Pump; Z N repreſents the inner cylindric Tube, and M S the outer one; O F Z Q is the Barrel in which the Piſton V X moves (which is a Forcer, or without a Valve,) and C R a Conduct-Pipe to carry away the Water forced through the Valve E. In all this it is eaſy to ſee the Analogy between this Syphon and the mercurial Pump. Its Operation likewiſe is the ſame.

Pl. XX. Fig. 3, and 4 together.

50. For let Mercury be poured into the Legs M S and Z N, it will come to a Level at *a*, *b*. Now ſuppoſe the other Part of the Syphon full of Water, and the Piſton raiſed, 'tis plain the Water at H cannot open the Valve I, till a Column of Mercury be raiſed in the Leg Z N above the Surface of that in the outer Leg M S, ſufficient to balance the Weight of the Water in the Pipe H F, then will the Preſſure of the Air be equal upon the Water H, and the Mercury in the Leg M S; and as the Motion of the Piſton is continued, the Water and Mercury will continue to riſe with equal *Momenta*, and therefore the Mercury will riſe 1 Inch for every 14 Inches nearly, till the Piſton ſtops.

51. When this happens, the Surface of the Mercury in the Leg Z N will be at *c*, and that in the Leg M S at *d*; and now, if the Piſton be puſhed down again, it will cauſe the Water to ſhut the Valve I, and to act on the Mercury at *c*, and on the Valve at E, but the Preſſure of the Air at E will not ſuffer the Valve to riſe till the Preſſure there be greater within than without, which it cannot be till the Surface of Mercury in the outer Leg M S be higher than that in the inner one; wherefore, before the Valve E can open, the Mercury in each Leg muſt come again to the Level *a*, *b*; after which, the Mercury will riſe in the outer Leg M, ſo as to be always *in Equilibrio* with the Water in the Conduct-Tube R C, and the Part C *e* of

Socket, from the Orifice of which it iſſues with a great Velocity, to a very great

the inner Leg; ſuppoſing the Mercury now ſtands at *e* in that, and at *f* in the other: And then the ſaid Height of the Water will be nearly 14 times the Height *df*, or *ce*. I think all this is very eaſy to underſtand from the common Principles of Hydroſtatics, and if this be underſtood, the Nature of the Pump muſt, as being the very ſame Machine with a different Diſpoſition of its Parts.

52. From this Theory of the mercurial Pump, 'tis eaſy to infer, that in conſtructing one, the Part Z F O Q ought to be placed in the Middle of the Height from the Water H to be raiſed, to the Ciſtern A B which receives and delivers it; or more nicely, the level Surface *a b c d* of the Mercury when the Pump is full, and juſt going to work, ought to be in the middle Point of the Line P *g*; the reaſon is, becauſe in the Deſcent and Aſcent of the Tube M S, the Differences between the Altitude of the outer and inner Shells *a d* and *c b* are equal, and in each Caſe a 14th Part of the Height of the Water below or above it.

53. Suppoſe the whole Height from the Water to the Piſton be $Pg = 60$ Feet; then $aP = ag = 30$ Feet, or 360 Inches; then alſo 14)360(26 nearly, that is, the Difference of the Altitudes in the mercurial Shells will be about 26 Inches. The Place therefore where they ought to be on a Level is at leaſt 13 Inches below M, or the Quickſilver muſt never be poured in to a Height N *a* greater than about 15 or 16 Inches, or 17 at moſt. If the Height P *g* be greater than 60 Feet, the Tubes M S, &c. muſt be proportionably enlarged.

54. To compute the Quantity of Quickſilver in the Shells; let A C B D be the Area of the Top of the inner Tube X Y, and E H F G the Area of the Surface of the inmoſt Mercury Shell ſurrounding it; then becauſe $AB = 6$ Inches, we have $AB^2 \times 0,7854 = 36 \times 0,7854 = 28,274 =$ the Area of the Circle A D B C. Pl. XX. Fig. 5.

Height or Diſtance, in a ſmall continued Stream, directed every way, or to any par-

Again, ſince the Diameter E F is 6,1, the Area of the Circle E H F G will be $EF^2 \times 0,7854 = 29,224$; but $29,224 - 28,274 = 0,95$ of an Inch, the Area of the annular Surface of Mercury; wherefore $26 \times 95 = 24,7$ the cubic Inches of Mercury contained in 26 Inches Altitude of the inner Shell; which as it is leſs than the Area of the Plug 28,274; ſhews that before the outer Tube M S can deſcend one Inch, the Water will begin to riſe to H, near 30 Feet below.

55. Alſo becauſe the Circumferences of Circles are as their Diameters, and the Diameters of theſe Shells are as 6,1 to 6,36 (*Art.* 45.) therefore 6,1 : 6,36 :: 0,95 : 0,99 or 1 Inch; that is, every Inch of Altitude in the exterior Shell is 1 cubic Inch, and therefore when the Tube M S (in the forcing Stroke) has deſcended near 1 Inch, the Water will force up the Valve E, the Mercury then being about 26 Inches high in the outer Shell.

56. If the Water be but 30 Feet below the Ciſtern, the Heights of the mercurial Shells will then be but half as great as before, *viz.* 13 Inches one above the other; and then 12,35 cubic Inches of the inner Shell will equal 13 Altitude, which therefore will riſe with a leſs Deſcent than $\frac{1}{2}$ an Inch of the Tube M S, in the inner Shell, and of Aſcent in the outer Shell. Therefore if the Stroke or Deſcent of the Tube M S be 15 Inches, there will be leſs than a 30th Part of the Stroke loſt, or a 30th Part of the Labour beſtow'd in Pumping. And if the Mercury Shells were but half as thick as here ſuppoſed, the Part of the Labour loſt would be but $\frac{1}{60}$ of the Whole, which is far leſs than in any other Pump of this Kind.

57. Since the Area of the Piſton X W is 28,274 Inches, therefore every Inch of its Motion raiſes the $\frac{28,274}{282}$ Parts of a Gallon, or the $\frac{28,274}{17766}$ Parts of an Hogſhead; that is, 28,274)17766(= 628,4 Inches of Motion

RPJCB

Vol: II. P. 65. Plate XXI.

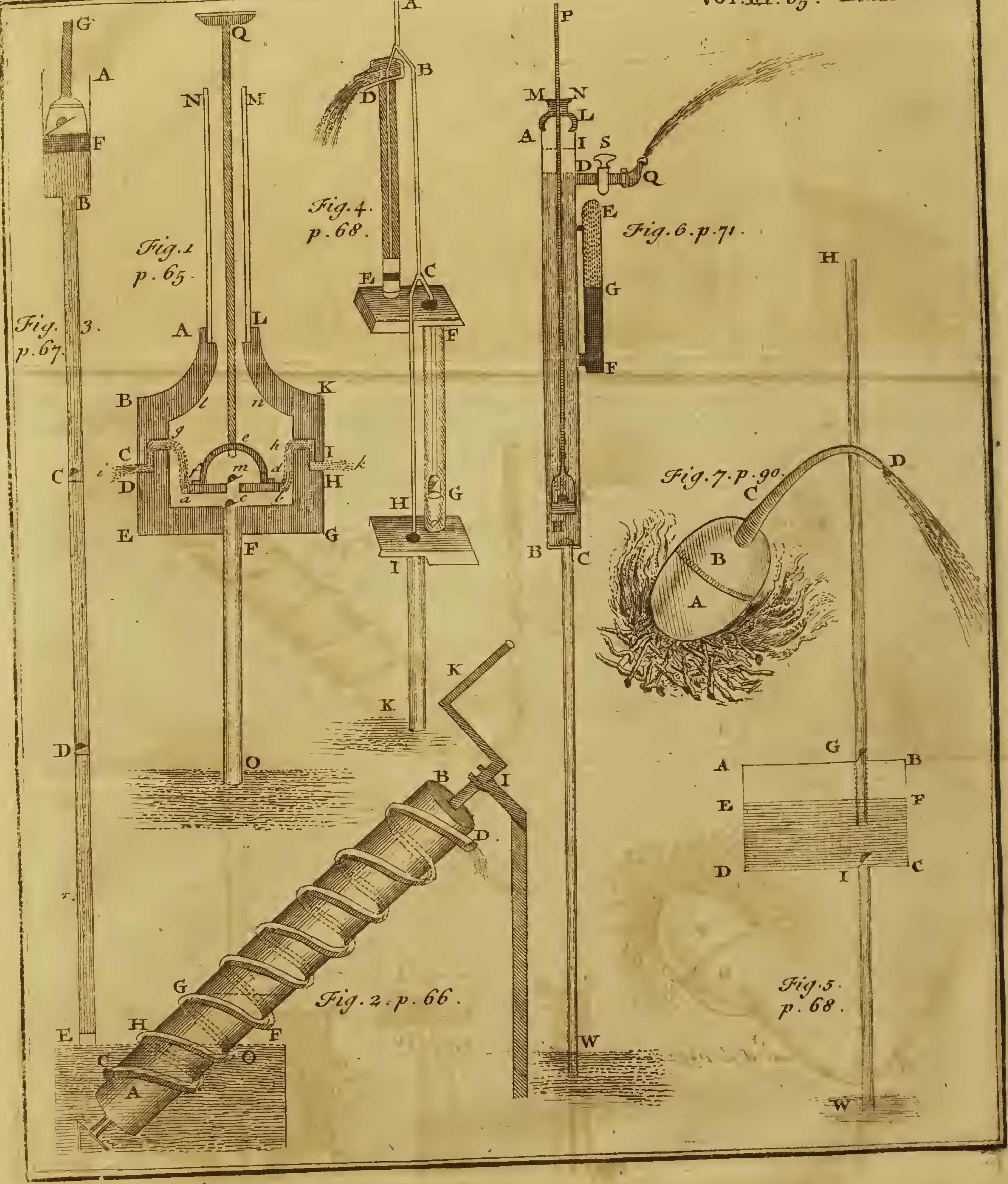
Fig. 1 p. 65.

Fig. 2. p. 66.

Fig. 3. p. 67.

Fig. 4. p. 68.

Fig. 5. p. 68.

Fig. 6. p. 71.

Fig. 7. p. 90.

Motion for an Hogſhead. Hence $52\frac{3}{10}$ Strokes, each 12 Inches, will raiſe one Hogſhead. And this, if the Weight be not greater than 10 Feet to which the Water is raiſed, may be done by an able Man in *one Minute*, working ſo as to be able to hold it all Day; as has been found by Experience. And therefore this is to be looked upon as the greateſt Effect, which Perſons can expect from any Hydraulic Engine, to raiſe Water of any Kind whatſoever, unleſs he can invent ſomething more compleat and perfect in this Kind, than the Pump we have now been deſcribing.

58. Another excellent Pump, of the lifting Sort, is the Invention of Meſſrs. *Goſſet* and *de la Deuille*, and ſet up in the King of *France*'s Garden at *Paris*. This Pump has this Peculiarity, that its Piſton works without Friction. The Form and Structure thereof is as follows: A B C D E F G H I K L is a Sort of Box incloſing the Piſton. This Box conſiſts of two Parts, the upper A B C I K L, and the under one D E F G H, which ſhut upon each other. The Piſton within is *a b*, and is a circular Piece of Wood, about the Circumference of which is nail'd a Piece of well ſeaſoned Leather, of a circular Form, and ſo wide, that when the Piſton is placed at the Bottom of the Box, the Leather may lie over the Sides thereof at D H all around; the Piſton and Leather thus placed on the upper Part, is forced down upon the Leather, and then both Parts ſcrew'd very faſt together. The Manner of which is very eaſy to apprehend from the Figure, where *i g f d b k* is the Leather going from the Piſton through the Jointure of the Box.

59. Upon the upper Part of the Piſton is fixed a circular (or any figured) Piece of Iron or Wood, denoted by *d e f*, in the Top of which, at *e*, is fixed the Rod of the Piſton Q C, by which the Piſton is drawn up towards the upper Part of the Cavity *l n*, and from thence forced down again in working the Pump. Now as the Diameter of the Piſton is leſs than that of the Cavity, it is plain that in its Motion up and down, no Friction can happen, as there are no Parts for it to rub againſt, which is occaſioned by the Contrivance of ſuſpending it on the Leather.

60. In the bottom Part is fixed a Pipe F O to bring up the Water from the Mine or Well at O, which it delivers into the Box by a Valve at *c*. In the Middle of the Piſton is likewiſe another Valve *m* opening upwards. A L M N is a Tube or cylindric Pipe, in which the Water is raiſed to a Ciſtern to run off. 'Tis eaſy to obſerve, that as the Piſton is drawn up, the Water will run in beneath, through the Valve *c*, to prevent a *Vacuum*; and alſo, that when the Piſton is forced down, the Water in the lower Parts muſt be forced up through its Valve, and when the Piſton is raiſed again, the Water above it will be forced up the Pipe A M to the Ciſtern.

61. Another Thing peculiar to this Pump is, the Shortneſs of the Stroke of the Piſton, which is compenſated by the largeneſs of its Area, and the greater Number of Strokes that may be made in the ſame Time. The only Objection to this Pump is, that it is always charged with the Weight of ſo much Water, as is equal to a Column of Water, whoſe Baſe is equal to the Area of the Piſton, and the Height equal to that of the Reſervoir above it, as is evident from what we have ſaid in explaining the *Hydroſtatic Paradox*. So that the Tube A L M N has no Advantage in being ſmall, unleſs any particular Circumſtance or Conveniency require it.

Pl. XXI. Fig. 2. 62. ARCHIMEDES's SCREW deſerves a Conſideration, not only for its Antiquity, but its Uſefulneſs in raiſing Water. It conſiſts of a long Cylinder, with an hollow Pipe, Tube or Groove coil'd about it, as repreſented in the Figure; where the Cylinder is A B, and the Tube C D open at each End. It is placed in an oblique Poſition to the Horizon with the lower End in the Water to be pump'd away, the other End being ſupported on the lower Part of the Winch I K, by which the Screw and Cylinder is turned round.

63. As ſoon as the Screw is immerſed in the Water, it immediately riſes therein by the Orifice C, to the Level of the Surface of the Water E F, and if the Point of the Helix or Spiral, which in the Beginning of the Motion is coincident with the Surface of the Water, happens not to be on the lower Side of the Cylinder,

the Water will upon the Motion of the Screw move on in the Helix, till it comes to the Point which is on the under Side, and coincident with the watery Surface; when it is arrived to that Point, which ſuppoſe at O, it cannot afterwards poſſeſs any other Part of the Spiral, than that which is upon the loweſt Part of the Cylinder, for it cannot move from O towards H or G, becauſe they are ſituated higher above the Horizon; and ſince this will ever be the Caſe, after the Surface of the Water in the Helix has attained the Point O, it is plain it muſt always be on the under Side of the Cylinder.

64. But ſince the Cylinder is in Motion, every Part of the ſpiral Screw from O to D, will by Degrees ſucceed to the ſaid under Part of the Cylinder; the Water therefore in the Helix muſt ſucceed to every Part thereof, from O to D, as it comes on the lower Side; that is, it muſt aſcend on the lower Part of the Cylinder thro' all the Length of the Pipe, till it comes to the Orifice D, where it will run out, as having nothing farther to ſupport it. Hence it appears, how much thoſe Gentlemen are miſtaken, who (affecting the Wonderful) ſay, *that the Water aſcends by deſcending*; whereas, if they would have made the moſt of the Wonder, they might have ſaid truly, *that the Water aſcends, becauſe it cannot aſcend*, i. e. that it aſcends one way, becauſe it cannot aſcend another; but then the Wonder is loſt.

65. I obſerve farther, with reſpect to Pump-Work, that no ſingle Pump, though perfectly tight, will raiſe Water by a Piſton playing at a greater Diſtance than 32 or 33 Feet; and even then the Stroke of the Piſton muſt be more than 8 Feet (for ſince $\frac{\frac{1}{4}aa}{h}=p=8$ Feet, than $a=32=h$. See *Art.* 30, 31, 33.) I ſay, no Pump will raiſe Water to a greater Height than 32 Feet by a ſingle Piſton only; nor can this be remedied by ſeveral Valves in the Pipe below the Piſton, as ſome Gentlemen have pretended.

66. For let A B be the Barrel of a Pump, G F the Piſton, B E a Pipe going from the Barrel to the Water at E, 60 Feet below it; and let B, C, D, E, be 4 Valves placed in the Pipe at 20 Feet Diſtance from each other. Pl. XXI. Fig. 3.

other. Now when the Piſton is drawn up from B to F, a *Vacuum* will be made between, but the Valve B will not riſe by the Water below it; for the Water in the Part B C cannot riſe itſelf, much leſs can that in the Part C D below it, as being preſs'd with the Weight of the Column above, and its own Weight downwards, therefore no Part of the watry Column B F can move of itſelf, or by itſelf. Now no Power can be applied any where but at E to move the whole Column; nor can the whole be moved without raiſing all the Valves at once; therefore the Power able to raiſe the Valve at E, and conſequently the Valve at B, muſt be able to overcome the Preſſure of a Column of Water 60 Feet high: But the Air can ſuſtain a Column only of 33 Feet high; therefore the Air cannot raiſe the Valve E, nor any of the reſt; therefore the Water cannot riſe in the Barrel A B of a ſingle Pump.

Pl. XXI. Fig. 4. 67. However, we may raiſe Water by a compound Pump, ſuch as is repreſented in the Figure, where A is the Rod of the Piſton, which at B is divided into two, one of which goes to the Barrel D E, and draws up the Water from the Ciſtern E F; the other Part B C goes down to the ſaid Ciſtern, where it is divided at C into two other Parts; one going to the Pump F G, ſupplies the Water to the Ciſtern E F; the other Part C H goes to another Pump I K below, and draws up the Water from K to the Ciſtern H I. Now theſe Pumps all working at once, will draw Water from any Depth, provided each Pump does not exceed 32 Feet in Height.

Fig. 5. 68. I ſhall conclude this Subject with an Account of one other Method of raiſing Water, by the *natural Agency of Heat and Cold* only. It is as follows, A B C D is a pretty large Veſſel filled with Water to the Height E F, the Space above being poſſeſs'd of Air. On the upper Part of the Veſſel is a tall Tube inſerted G H, and deſcends below the Surface of Water to K. On the nether Part is another Tube or Pipe I W. In each Tube is a Valve opening upwards, as at I and G. The Body of this Inſtrument being nicely cloſed every where, ſo that no Air can eſcape, and placed with the lower End in the Water W, and thus continued in the hot

Vol II. Plate XXII. P. 69.

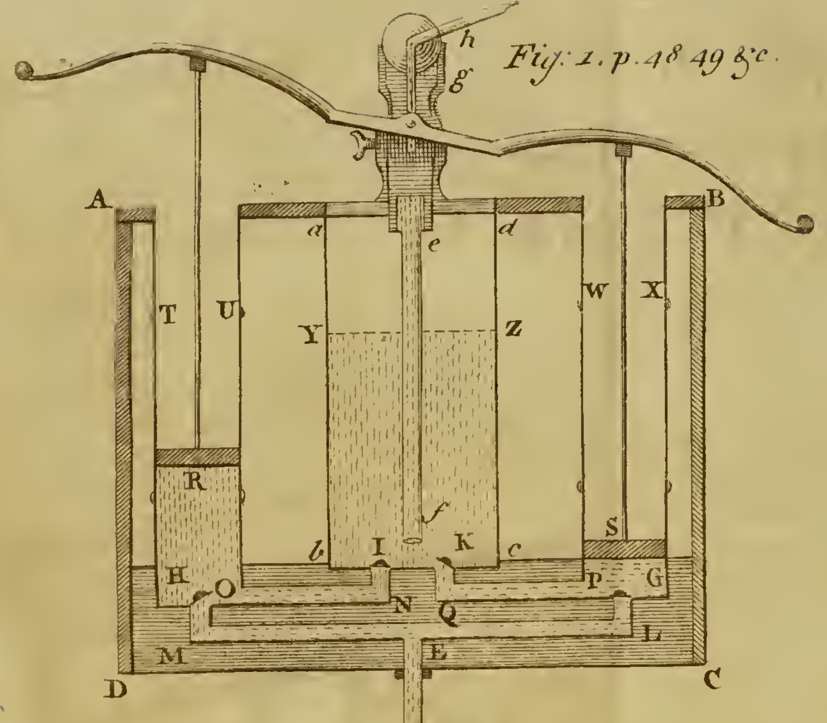

Fig: 1. p. 48 49 &c.

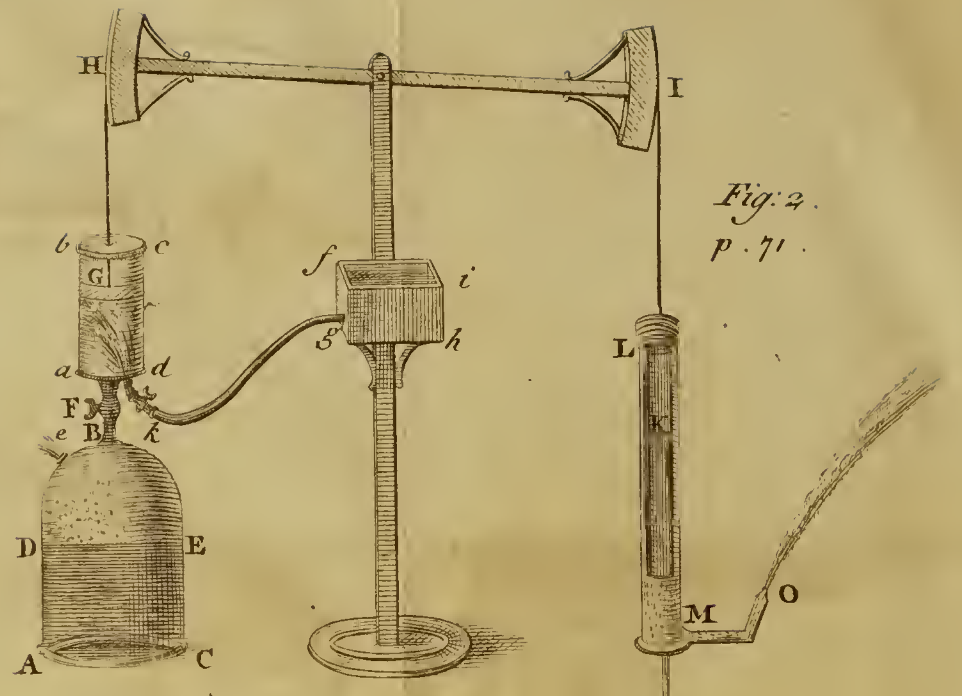

Fig: 2. p. 71.

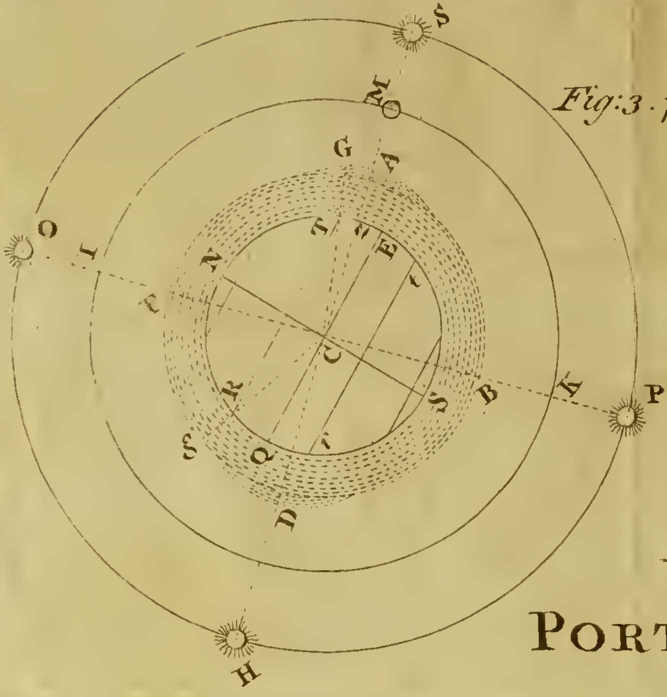

Fig: 3. p. 91

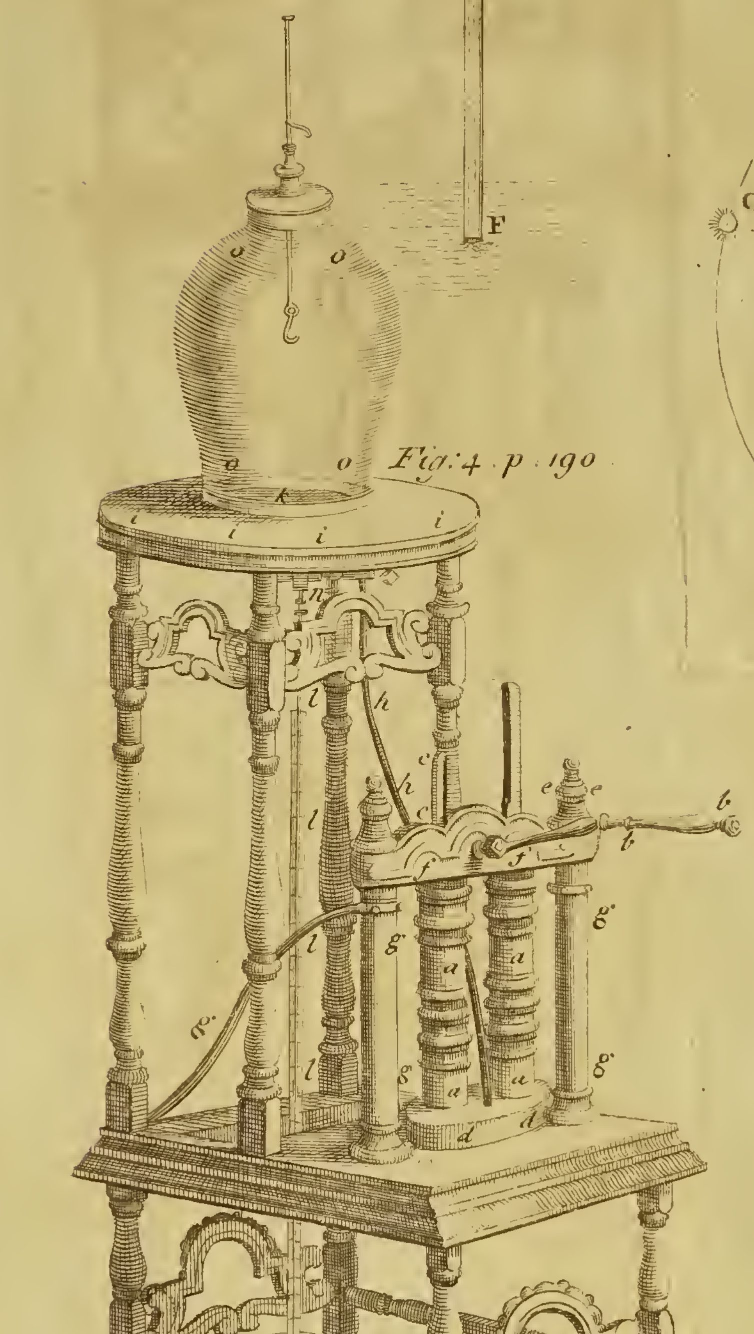

Fig: 4. p. 190

The
NEW-INVENTED
PORTABLE AIR-PUMP.

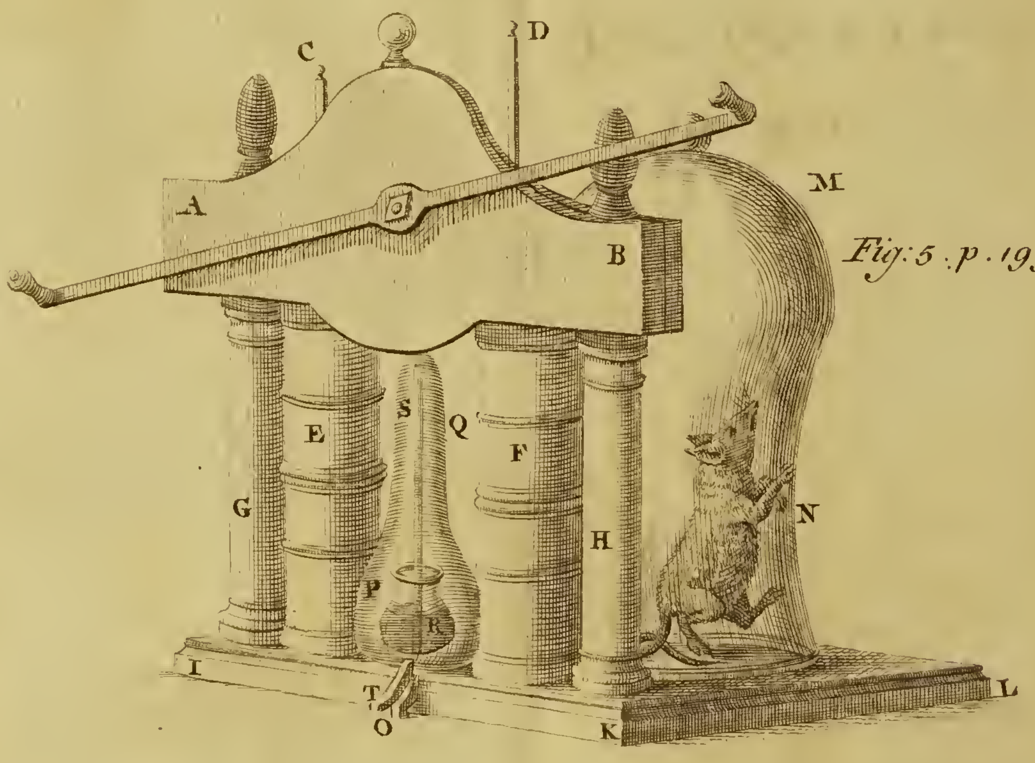

Fig: 5. p. 193.

E. Bowen sc.

ticular Place, by means of the Ball and Socket (LXXIX.)

hot Sun of a Summer's Day, the Air will be rarified by the Heat of the Sun in the upper Part, and will compreſs the ſubjacent Water, and force it up through the Valve G into the Tube G H, and by the Cold of the following Night it will be condenſed again, and then the Preſſure of the Atmoſphere will force the Water at W up the Pipe W I, to repleniſh the Veſſel each Day. And in this Manner may Water be raiſed in a conſiderable Quantity in the Summer Time, or in very hot Climes.

69. By ſuch a Contrivance as this, ſeveral curious Effects may be produced. Thus if a cylindric Cover be annexed to the top Part of ſuch a Veſſel (diveſted of its Tubes,) it may be raiſed to collect the Sun-Beams at Noon on the Top of the Veſſel, which will ſo greatly rarify the Air within, as to make it produce a conſiderable Stream of Water through a ſmall Adjutage in the Side thereof; which Stream may fall on a Wheel to play an Organ at Dinner, &c. Thus alſo, an Image may be made to weep in the Sun-Beams, or at the Approach of Fire; with other Devices of the like Kind.

Pl. XXII. Fig. 1. (LXXIX.) The greateſt Artifice of this Engine is its Contrivance to produce a *continual Stream*, which is done by Compreſſion, and Elaſticity of Air in the Barrel, called the Air Veſſel *a b c d*. For the Air, being an elaſtic Fluid, will be capable of Compreſſion in any Degrees by the Water forced in through the Valves at I, K; and ſince the Force of the Air's Spring will always be inverſely as the Space which it poſſeſſes, (as will be ſhewn) it follows, that when the Air Veſſel is half full with Water, the Air will be compreſſed into half the Space it poſſeſs'd at firſt, and therefore its Spring will be twice as great as at firſt.

2. But this Spring at firſt was equal to the Preſſure of the Atmoſphere; for if it was not, it could not have ſuſtain'd or reſiſted the Preſſure of the Atmoſphere

THERE is another Sort of *Engine* for raiſing Water, which works by Fire, in

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which ſtood over it, and conſequently could not have fill'd the Veſſel before the Water was driven in; which yet we find it did, and mantain'd an Equilibrium with the common Air. The Veſſel then being half fill'd with Water, or the Air compreſſed into half the firſt Space, its Spring will be in this Caſe equal to twice the Preſſure of the Atmoſphere; and therefore when the Stop-Cock at *p* is turned, the Air within preſſing on the ſubjacent Water with twice the Force it meets with from the external Air in the Pipe *e f*, will cauſe the Water to ſpout out of the Engine to the Height of 32 or 33 Feet, if the Friction is not too great.

3. That it muſt ſpout to that Height is evident from hence, that only one half the Force of the internal Air is ſpent in overcoming the Reſiſtance of the external Air; and this being deſtroy'd, the remaining Half of the Force will be exerted in projecting the Water, which now muſt be conſider'd as ſpouting *in Vacuo* (the Reſiſtance of the external Air being taken away). But *in Vacuo* Water will riſe to the Height of 32 or 33 Feet (as will be ſhewn hereafter) by the Preſſure of the common Air; therefore ſince the remaining half Force of the internal Air is equal to the Preſſure of the common Air, it muſt cauſe the Spout to riſe to the Height of 32 or 33 Feet; and becauſe the Water keeps entering the Veſſel as faſt as it ſpouts out, it keeps the Air under the ſame Degree of Compreſſion, which therefore muſt conſtantly act with the ſame Tenor on the Water, and produce a continual Stream to the ſame Height.

4. When the Air-Veſſel is $\frac{2}{3}$ full of Water, the Space which the Air takes up is $\frac{1}{3}$ Part, whence its Spring will be 3 times as great as that of the common Air, therefore it will now project the Water with twice the Force of the Atmoſphere, and therefore to the Height of 64 or 66 Feet. When the Air-Veſſel is $\frac{3}{4}$ full of

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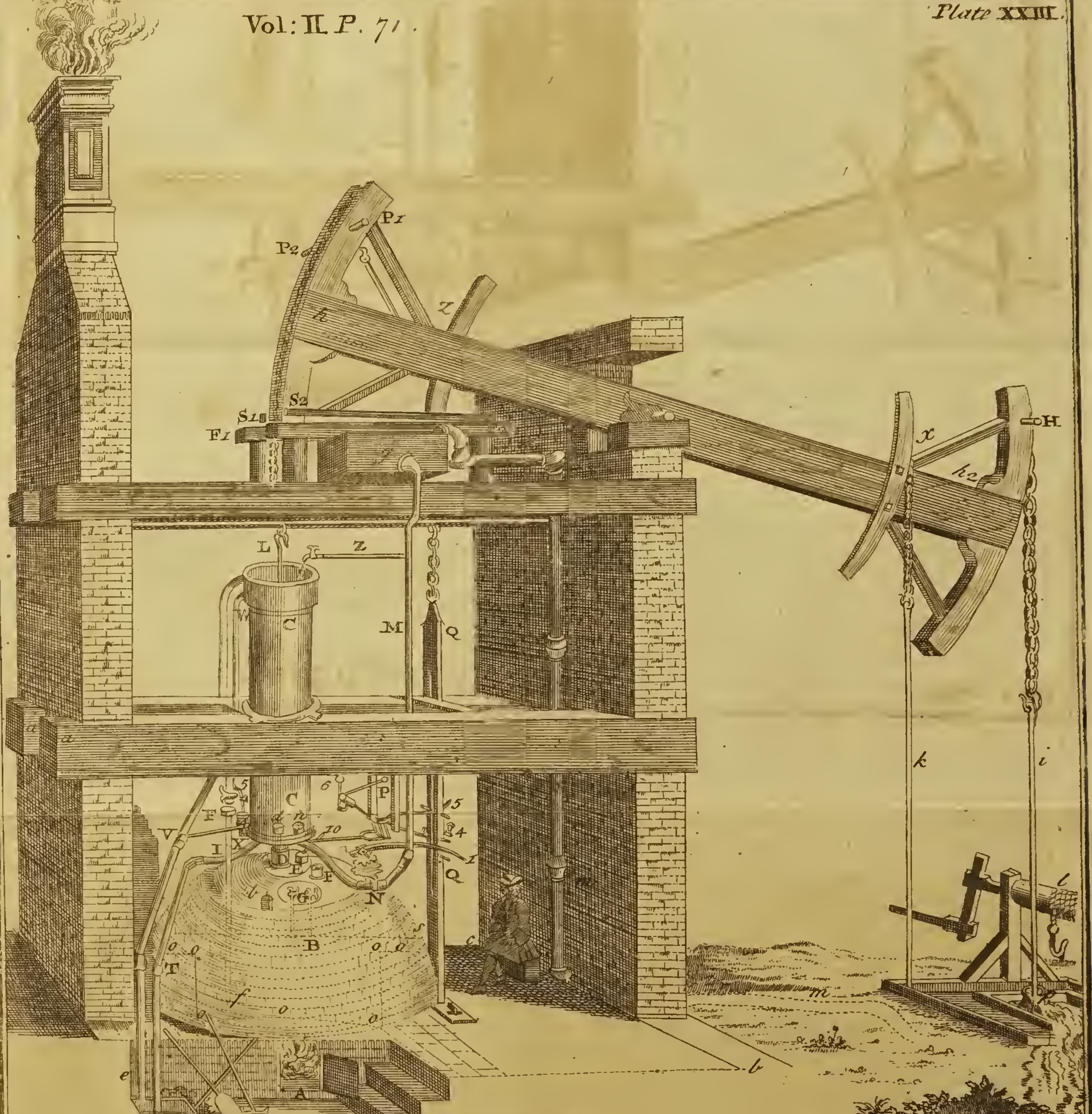
Plate XXIII.



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the following Manner. ABC is a Copper-Veſſel, partly fill'd with Water to D E, Pl.XXII. Fig. 2.

Water, the Air will be compreſſed into $\frac{1}{3}$ Part, and ſo will protrude the Water with a Force equal to 3 Atmoſpheres, and therefore to the Height of 96 or 99 Feet. And when the Veſſel is $\frac{4}{5}$ full of Water, the Air will take up $\frac{1}{5}$ Part, and will have its Spring 5 times as ſtrong as at firſt, and will therefore force the Water 132 Feet high.

5. Hence 'tis eaſy to obſerve the Law by which the Spring of the Air will act on the Surface of the Water below it, *viz.* according to the Numbers in the Table below.

Height of the Water.	*Of the Air compreſs'd.*	*Proportion of the Air's Spring.*	*Height of the Spout.*
$\frac{1}{2}$	$\frac{1}{2}$	2	33 Feet.
$\frac{2}{3}$	$\frac{1}{3}$	3	66
$\frac{3}{4}$	$\frac{1}{4}$	4	99
$\frac{4}{5}$	$\frac{1}{5}$	5	132
$\frac{5}{6}$	$\frac{1}{6}$	6	165
$\frac{6}{7}$	$\frac{1}{7}$	7	198
$\frac{7}{8}$	$\frac{1}{8}$	8	231
$\frac{8}{9}$	$\frac{1}{9}$	9	264
$\frac{9}{10}$	$\frac{1}{10}$	10	297

And thus for any greater Degree of Compreſſion.

6. As the Air-Veſſel is the Cauſe of the continual Stream, we may naturally infer, that if ſuch an Air-Veſſel were adapted to the *Common Houſe Pump*, it would then become a *Fire-Engine*, or very uſeful for extinguiſhing Fires that may happen in or near the Houſe. Now this may be effected in a Manner either the ſame with, or analogous to, that which follows. Pl. XXI. Fig. 6. A B C D is the Barrel of the Pump, P H the Rod and Piſton, C W the Pipe going down to the Water of the Well at W. Towards the lower Part of the Barrel is a ſhort Tube, by which the Air-Veſſel F E is fixed to, and communicates with, the Barrel of the

which being ſet over a Fire, and made to boil, will fill the upper Part D B E with a vaſtly elaſtic Vapour, the ſufficient Strength whereof is known by its forcing open a Valve at *e*: This heated elaſtic Steam is, by turning a Cock at F, let into the Barrel *a b c d*, where by its elaſtic Force it raiſes the Piſton G, which drives the Air above it through a proper Clack at the Top. After this, that the Piſton may by

Pump. A M N L is a Collar of Leathers ſo fixed on the Top of the Barrel, and adapted to the Rod, that it may move freely in the Leathers, without permitting the Air to paſs in or out between. The Nozzle or Spout D has a Stop-Cock S to ſhut off the Water, or let it paſs out as required. Q is a Piece ſcrew'd on to direct the Stream by a ſmall pliant Leathern Pipe at the End.

7. Now 'tis plain, when the Piſton is raiſed from the Bottom of the Pump Barrel, the Water above it will be forced into the Air Veſſel, and there compreſs the Air (the Stop-Cock being ſhut); it will alſo compreſs the Air on the Top of the Barrel, for the Water will not be higher than the Spout D at firſt, when the Stop-Cock is ſhut: But afterwards, as the Air is confin'd, it will be compreſs'd at Top, and the Water will riſe to l.

8. This compreſs'd Air, in each Place, will act upon the Water by its Spring, and, upon turning the Stop-Cock, will force it out in a continual Stream, through a Pipe at Q, and that with a greater or leſſer Degree of Force, as Occaſion requires, that being abſolutely in the Power of the Perſon working the Pump. This Invention we owe to Sir *James Creed*, one of the very few Votaries which the Philoſophic Muſes find among Perſons of Quality and Fortune.

its Weight deſcend, a little cold Water, from the Fountain *f g h i*, is let in at the Bottom by turning a Cock at *k*, which, in Form of a Jet, condenſes the hot Steam in the Barrel into 13000 times leſs Space than before it took up, which makes a ſufficient *Vacuum* for the Piſton to deſcend in. The Piſton G, and Lever H I, being thus put into Motion, do accordingly raiſe and depreſs the Piſton K in the Barrel of the forcing Pump L M on the other Side; which, by the Pipe N, draws the Water from the Depth W, and forces it to riſe and ſpout through the Tube O, continued to any Height at Pleaſure. This Engine is uſed to draw the Water from Coal-Pits and other Mines. Beſides theſe, there are various other Water-Engines of a more complex Structure; but as they all act on the ſame Principles with theſe two deſcribed, 'tis needleſs to ſay any thing of them here (LXXX, LXXXI, LXXXII.)

(LXXX.) 1. The FIRE-ENGINE is the moſt admirable, curious, and compounded Machine that we find among all thoſe whoſe Inventions have been owing to the Diſcoveries of Modern Philoſophy, and which affords the greateſt Advantages to Mankind. The Fire-Engine is of two Sorts, which I ſhall here give the Theory of, and Figures of each according to the lateſt Perfection in which they are made. The Account given above being only in relation to its firſt State and original Simplicity;

Simplicity: But it is now of a very different Form, though acting upon the ſame Principle, as will appear from the following hiſtorical Theory thereof.

2. The MARQUIS OF WORCESTER, in his *Century of Inventions*, (printed *A. D.* 1663.) is the firſt that I know of who propoſed raiſing great Quantities of Water by the Force of Fire, turning Water into Steam; and he mentions ſome Engine of this Kind, which he ſaw play'd a continual Stream in the Manner of a Fountain, forty Feet high. He alſo ſays, that a Perſon attending it turn'd two Cocks; that one Veſſel of Water being conſumed, another begins to force and re-fill with cold Water; and this alternately and ſucceſſively, the Fire being tended and kept conſtant by the ſame Perſon in the Interim between turning the Cocks.

3. Captain SAVERY, having read this Account, immediately attempted to raiſe Water by Fire, and was the firſt who erected an Engine for this Purpoſe, of the Form we have ſince had them. To ſecure the Invention to himſelf, he bought up all the Marquis's Books that he could find, and burnt them; and then gave out that he diſcover'd the Method by Accident. He made many Experiments to bring the Machine to Perfection, and erected ſeveral for Gentlemens Seats, but could not ſucceed for Mines, the Depth from which Water was to be raiſed from thence being ſo great, that it required the Steam too dangerouſly ſtrong to be attempted in his Way.

4. Thus the Progreſs of the Engine was ſtopped, till long afterwards Mr. *Newcomen* an Ironmonger, and Mr. *John Cowley* a Glazier, (both Diſſenters of the *Baptiſt* Denomination at *Dartmouth*) contrived another Way to raiſe Water by Fire, where the Steam to raiſe the Water from the greateſt Depths of Mines is not required greater than the Preſſure of the Atmoſphere; and this is the preſent Structure of the Engine, and which is now of about 35 or 40 Years ſtanding.

5. We have already ſhewn (*Annot.* LXXVIII. 67.)
Plate XXIII. that Water may be drawn from any Depth by a Compound Pump. Suppoſe the Depth = 150 Feet, and the Diameter of the Pump-Barrel at *p* be $7\frac{3}{4}$ Inches, then the Weight of a cylindric Column of Water 150 Feet high,

high, and $7\frac{3}{4}$ Inches Diameter, will be about 3000 *lb.* If now the Rod of this Pump were hung by a Chain to the End of a Lever *h h*, as at H, and at the other End another Power were applied, as at P, with a ſuperior Force, the Pump might be worked, and the Water raiſed by that Power.

6. Now this Power cannot be human, or the Strength of Men; for it will require 100 Men to pull down the Beam, each pulling with a Force of 30 *lb.*; and 100 more to relieve theſe when weary; for the Pump in a Mine muſt not ſtand ſtill: But 200 Men would be too expenſive. If we allow Horſes, (and one Horſe equal to five Men) there muſt be 20 Horſes working at a time, and 20 more to relieve theſe; which great Number of Horſes, though leſs expenſive than the Men, will be found too great an Expence for moſt Mines: And neither Men nor Horſes would be able to give more than 15 or 16 Strokes in a Minute.

7. Some other Power therefore muſt be applied; and the Philoſopher readily offers his Service with his Air-Pump, and Preſſure of Air. His Propoſal is to fix a large Barrel or Cylinder C C, truly bored, about 8 or 9 Feet long and 22 Inches wide, on the Top of his Air-Pump; and in this is a Piſton well leather'd, whoſe Rod L is faſtened to a Chain fixed to the End of the Lever at P. Now, ſays he, allowing 15 *lb.* of Preſſure upon every ſquare Inch, the whole Preſſure of Air upon ſuch a Piſton will be 5800 *lb.* which is near twice the Force to be overcome. But this muſt be effected by exhauſting ſo large a Barrel by the Air-Pump, and which he cannot do more than twice in an Hour; ſo that he can produce only two Strokes in the Time that 960 are neceſſary to keep the Water from overflowing the Mine.

8. But though the firſt Expedient fails, he makes a ſecond Propoſition, and that ſucceeds; which is as follows. B is a large Boiler, whoſe Water by the Fire under it is converted into elaſtic Steam. The great Cylinder C C is fixed upon it, and communicates with it by the Pipe D *d*; on the lower Orifice of which (within the Boiler) moves a broad Plate, by means of

the Steam-Cock or Regulator E 10, ſtopping or opening the Paſſage, to prevent or permit the Steam to paſs into the Cylinder, as Occaſion requires. The Diameter of the Pipe D is about 4 Inches.

9. The Steam in the Boiler ought always to be a little ſtronger than the Air, that, when let into the Barrel, it may be a little more than a Balance to the Preſſure of the external Air, which keeps down the Piſton at the Bottom *d n*. The Piſton being by this means at Liberty, the Pump-Rod will, by its great Weight, deſcend at the oppoſite End to fetch a Stroke: For the Weight of 70 or 80 Yards of Iron-Rod amounts to about 9 or 10 Hundred; whereas the Piſton, and other Weights at the other End, is not above 4 or $4\frac{1}{2}$ Hundred. The End of the Lever at the Pump therefore will always preponderate the other, and deſcend when the Piſton is at Liberty.

10. The Handle of the Steam-Cock E 10 being turned towards *n*, opens the Pipe D to let in the Steam; and being turned towards O, it ſhuts it off, that no more can enter. The Piſton is now raiſed towards the Top of the Barrel at C, and the Barrel is full of Steam. The Lever O 1 muſt then be lifted up, to turn by its Teeth the Injecting-Cock at N, which permits the Water brought from the Ciſtern *g*, by the Pipe *g* M N, to enter the Bottom of the Barrel at *n*, where it flies up in the Form of a Fountain, and ſtriking againſt the Bottom of the Piſton, the Drops being driven all over the Cylinder will by their Coldneſs condenſe the Steam into Water again, and precipitate it to the Bottom of the Barrel.

11. Mr. *Hen. Beighton* made an Experiment to determine the Rarity of the Steam, and found the Content of the Barrel of Steam was 113 Gallons; and ſince there were 16 Strokes in a Minute, therefore $113 \times 16 = 1808$ Gallons of Steam *per* Minute. He alſo obſerved, that the Boiler required to be ſupplied with water at the Rate of 5 Pints *per* Minute; and ſince 282 cubic Inches make a Gallon, $35\frac{1}{4}$ make a Pint, and $5 \times 35\frac{1}{4} = 176\frac{1}{4}$ in 5 Pints; alſo the cubic Inches of Steam are $1808 \times 282 = 509856$; if then we ſay, As $176\frac{1}{4}$:

509856

509856 :: 1 : 2893; or one cubic Inch of Water is expanded into 2893 Inches of Steam: Conſequently the Steam in the Barrel is reduced to $\frac{1}{2893}$ Part, when turned to Water by the Jet of cold Water; and therefore a ſufficient Vacuum is made in the Barrel, for the Piſton to deſcend unbalanced, by the Preſſure of the Air.

12. The Piſton being forced down raiſes the other End of the Lever, and conſequently the Piſton of the Pump, which brings up and diſcharges the Water at *p*. Now this whole Operation of opening and ſhutting the Steam-Regulator and Injection-Cock, will take up but little more than 3 Seconds, and will therefore produce very well 16 Strokes in a Minute.

13. That the Ciſtern *g* may always be ſupplied with Water, there is an Arch *x* fixed near the Arch H at the Pump-End, from whence another Pump-Rod *k* with its Piſton draws Water from a ſmall Ciſtern near the Mouth of the Pit, (ſupplied from the Water raiſed at *p*) and forces it up the Pipe *m m m* into the Ciſtern *g*, which therefore can never want Water.

14. That the Leathers of the Piſton C may be always ſupple, and ſwelled out ſo as to be always Air-tight, a ſmall Stream of Water is ſupplied from the Injecting-Pipe M by the Arm *z*. On the Top of the Cylinder is a larger Part or Cup L, to hold the Water that lies on the Piſton, leſt it ſhould overflow when the Piſton is got to its greateſt Height, as at W; at which Time if the Cup be too full, the Water will run down the Pipe V to the waſte Well at Y.

15. The Water in the Boiler, which waſtes in generating Steam, is ſupplied by a Pipe F *f* about 3 Feet long, going into the Boiler a Foot below the Surface of the Water. On the Top of this Pipe is a Cup or Funnel F, ſupplied by the Pipe W with Water from the Top of the Ciſtern, which has the Advantage of being always warm, and ſo not apt to check the Boiling of the Water in the Copper.

16. That the Boiler may not have the Surface of the Water too low, (which would endanger burſting) or too high, (which would not leave Room enough for Steam)

Steam) there are two Gage-Pipes at G; one going a little below the Surface of the Water when at a proper Height, and the other ſtanding a little above it. When every thing is right, the Stop-Cock of the ſhorter Pipe being open'd gives only Steam, and that of the long one, Water; but if otherwiſe, both Cocks will give Steam when the Surface is too low, and both give Water when it is too high; and hence the Cock which feeds the Boiler at F may be opened to ſuch a Degree, as always to keep the Surface of Water to its due Height.

17. The cold Water, conſtantly injected into the Cylinder to condenſe the Steam, is carried off by the Eduction-Pipe *d* T Y, going from the Bottom of the Cylinder to the Waſte Well Y, where going a little under Water it has its End turned up, with a Valve Y to keep out the Air from preſſing up into the Pipe, but permitting the injected Water coming the other Way to be diſcharged; whereby the Cylinder is kept empty.

18. Leſt the Steam ſhould grow too ſtrong for the Boiler, and burſt it, there is a Valve fixed at *b*, with a perpendicular Wire ſtanding up from the Middle of it, to put Weights of Lead upon, by which to examine the Strength of the Steam puſhing againſt it from within. Thus the Steam is known to be as ſtrong as the Air, if it will raiſe up ſo much Weight on the Valve as is at the Rate of 15 *lb.* to an Inch ſquare; becauſe that is the Weight of Air (nearly) on every ſquare Inch, as will be ſhewn in its Place. When the Steam becomes ſtronger than what is required, it will lift up the Valve, and go out. This Valve is called the *Puppet-Clack.*

19. The Steam has always a variable Strength, yet never $\frac{1}{10}$ ſtronger or weaker than common Air: For it has been found that the Engine will work well, when there is the Weight of 1 *lb.* on every ſquare Inch of the Valve *b*. This ſhews that the Steam is then $\frac{1}{15}$ Part ſtronger than the common Air. Now as the Height of the Feeding-Pipe from the Funnel F to the Surface of the Water S *s* is not above 3 Feet, and $3\frac{1}{2}$

Feet of Water is $\frac{1}{10}$ of the Preſſure of the Air; if the Steam were $\frac{1}{10}$ Part ſtronger than Air, it would puſh the Water out at E; which ſince it does not do, it cannot be ſtronger than Air, even in this Caſe, where, the Regulator being ſhut, it is moſt of all confined.

20. When the Regulator is open'd, the Steam gives the Piſton a puſh, which raiſes it up a little way; then falling a greater Space, it comes to be of the ſame Strength, and ſo a Balance to the Atmoſphere; thus the Piſton, being at Liberty, riſes to the Top W. The Steam, now expanded into the whole Capacity of the Cylinder, is weaker than the Air, and would not ſupport the Piſton were it not for the greater Weight at the other End of the Lever, which keeps it up. The Steam each Stroke drives the injected Water of the preceding Stroke out of the Eduction-Pipe *d*TY, and would itſelf follow, and blow out at the Valve Y, (which is not loaded) if it were ſtronger than the Air, which it never does. If it were exactly equal to the Strength of the Air, it would juſt drive all the Water out at Y, but could not follow itſelf, the Preſſure being on each Side the Valve equal by Suppoſition. If it be weaker than the Air, it will not force all the Water out of the Pipe *d*TY, but the Surface will ſtand ſuppoſe at T, where the Column of Water TY added to the Strength of the Steam is equal to the Preſſure of the Air. When the Steam is $\frac{1}{10}$ weaker than the Air, the Height $TY = 3\frac{1}{2}$ Feet. Now ſince the whole perpendicular Diſtance from *d* to Y is but 4 Feet, and the Steam always ſufficient to expel the Water; it is plain it can never be more than $\frac{1}{10}$ Part weaker than the Air, when weakeſt.

21. There is Air in all the Water injected; and tho' that Air cannot be taken out or condenſed with the Steam, yet will it precipitate and fall through the Steam to the Bottom of the Cylinder, as being much heavier. For it has been ſhewn, that Steam is to Water, as 1 to 2893 in its Denſity, but the Denſity of Air is to that of Water, as 1 to 864 (as will be ſhewn) therefore the Rarity of Steam is to that of Air, as 2893 to 864; the

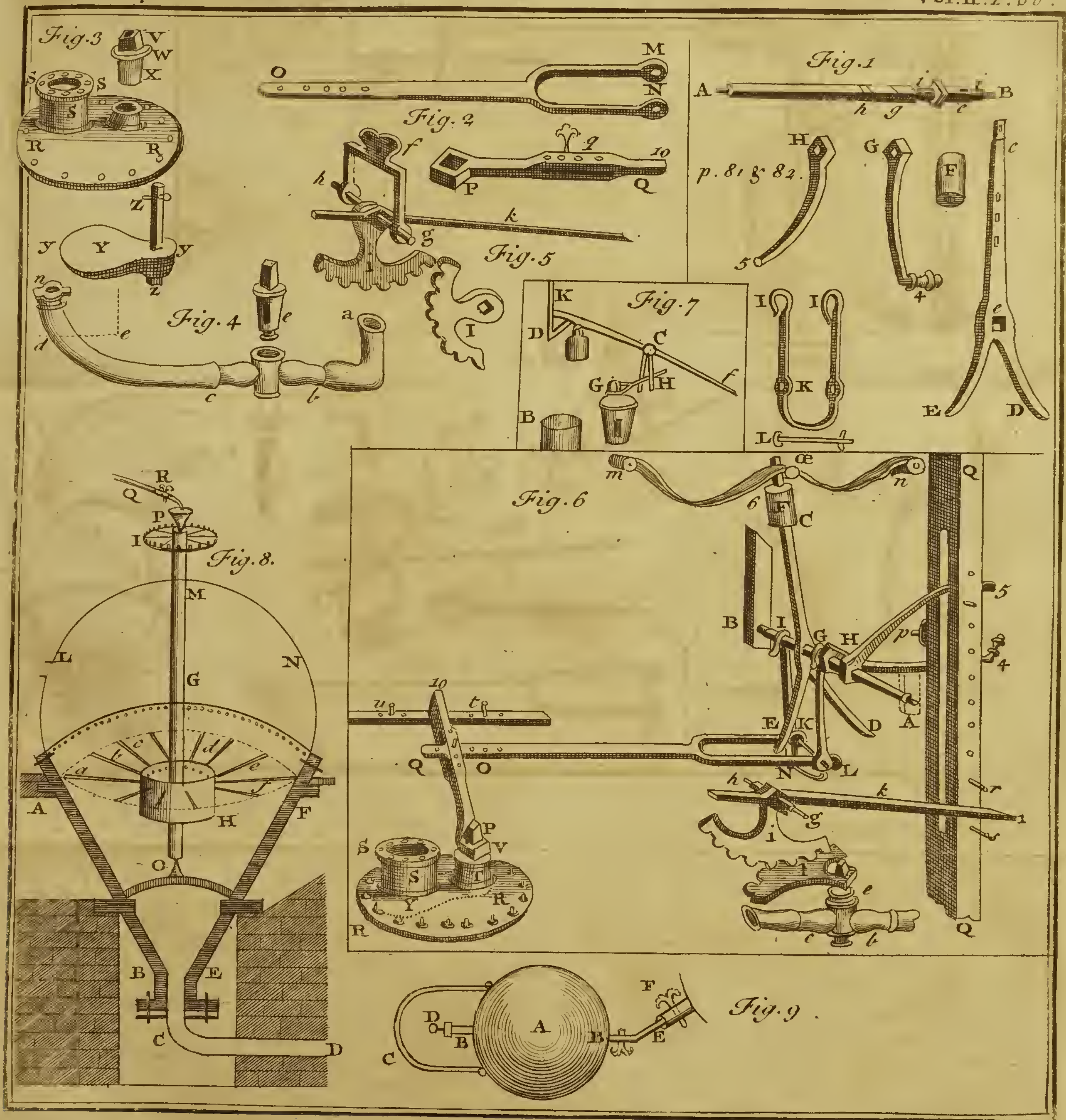
Air will therefore fall through the Steam to the Bottom, and from thence be driven out through a ſmall Pipe opening into the Cup at 4, on which is a Valve. Now when the Steam firſt ruſhes into the Cylinder, and is a little ſtronger than the outward Air, it will force the precipitated Air to open the Valve at 4, and make its Eſcape; but the Steam cannot follow, becauſe it is weaker than the outward Air, as the Piſton gives it room, by aſcending, to expand. This Valve (from the Noiſe it makes) is call'd the *Sniſting-Clack.*

22. Among the greateſt Improvements of this Engine, we may reckon that Contrivance, by which the Engine itſelf is made to open and ſhut the Regulator and Injection-Cock, and that more nicely than any Perſon attending could poſſibly do it. For this Purpoſe there is fixed to an Arch Z, at a proper Diſtance from the Arch P, a Chain, from which hangs a perpendicular Piece or working Beam Q Q, which comes down quite to the Floor, and goes through it in a Hole, which it very exactly fits. This Piece has a long Slit in it, and ſeveral Pin-Holes, and Pins, for the Movement of ſeveral ſmall Levers deſtined to the ſaid Office of opening and ſhutting the Cocks, after the following Manner.

23. Betwen two perpendicular Pieces of Wood on each Side P, there is a ſquare Iron Axis A B (in the following *Fig.* where this Part is repreſented by itſelf for Diſtinctneſs) which has upon it ſeveral Iron Pieces of the Lever Kind. The firſt is the Piece C E D, called the Y, from its repreſenting that Letter inverted by its two Shanks E and D; on the upper Part is a Weight F to be raiſed higher or lower, and fixed, as Occaſion requires. This Y is fixed very faſt upon the ſaid Iron Axle A B.

24. From the Axle hangs a Sort of an Iron Stirrup I K L I, by its two Hooks I I, and having on the lower Part two Holes K, L, through which paſſes a long Iron Pin L K, and key'd in the ſame. When this Pin is put in, it is alſo paſs'd through the two Holes in the Ends E, N, of the horizontal Fork or Spanner E Q N, join'd

Plate XXIV. Vol. II. P. 80.



at its End Q to the Handle of the Regulator V 10. From Q to O are ſeveral Holes, by which the ſaid Handle may be fixed to that Part of the End which is moſt convenient.

25. Upon the Axis A B is fixed at Right Angles to the Y, an Handle or Lever G 4, which goes on the Outſide of the Piece Q Q, and lies between the Pins. Another Handle alſo is faſtened upon the ſame Axle, *viz.* H 5, and placed at half a Right Angle to the former G 4; this paſſes through the Slit of the Piece Q Q lying on one of its Pins. Hence we ſee, that when the working Beam goes up, its Pin in the Slit lifts up the Spanner H 5, which turns about the Axle ſo faſt, as to throw the Y with its Weight F from C to 6, in which Direction it would continue to move, after it paſs'd the Perpendicular, were it not prevented by a Strap of Leather fixed to it at O E, and made faſt at the Ends *m* and *n*, in ſuch Manner as to allow the Y to vibrate backwards and forwards about a Quarter of a Circle, at equal Diſtances on this Side and that of the Perpendicular.

26. As Things are repreſented in the Figure, the Regulator is open, its Plate T Y being ſhown on one Side the Pipe S which joins the Cylinder and Boiler. The Piſton is now up, and alſo the working Beam near its greateſt Height; the Pin in the Slit has ſo far raiſed the Spanner H 5, that the Weight F on the Head of the Y is brought ſo far from *u*, as to be paſt the Perpendicular, and ready to fall over towards *m*, which when it does, it will by its Shank E, with a ſmart Blow, ſtrike the Iron Pin K L, and drawing the Fork O N horizontally towards the Beam Q, will draw the End 10 of the Regulator towards 6, and thereby ſhut it, by ſlipping the Plate Y under the Holes of the Throat-Pipe S.

27. Immediately after the Regulator is ſhut, the Beam riſing a little higher, with its Pin S on the Outſide upon the lower Part lifts up the End *k* 1 of the Handle of the Injecting-Cock, and opens it by the turning of the two Parts with Teeth. The Jet immediately making a *Vacuum*, the Beam again deſcends, and the Pin *r*

depreſſing

depreſſing the Handle *k* 1 ſhuts the Injection-Cock; and the Beam continuing to deſcend, the Pin *p* bears down the Handle G 4, and throwing back the Y, its Shank D throws forward the Fork N Q, and again opens the Regulator to admit freſh Steam. After this every thing returns as before; and thus is the Engine moſt wonderfully contrived to work itſelf.

28. After the Engine had been made, as above deſcribed, for many Years, it received another Improvement of very great Advantage; and that was, inſtead of feeding the Boiler with warm Water from the Top of the Cylinder (as related *Art.* 15.) by the Pipe W above, and F *f* below; they contrived to ſupply it with the ſcalding hot Water which came out of the Eduction-Pipe *d* T Y, which now, inſtead of going to the waſte Well at Y, was turned into the Boiler on the top Part; and as the Eduction-Pipe before went out at the Side of the Cylinder, it was now inſerted in the Bottom of the ſame; and though the Preſſure of the Steam in the Boiler be ſomewhat greater in the Cylinder, yet the Weight of Water in the Eduction-Pipe being added to the Force of Steam in the Cylinder, will carry the Water down continually, by overcoming the Reſiſtance in the Boiler.

29. To conclude this Theory of the Fire-Engine, I ſhall here add a moſt curious and uſeful Table of the Calculation of the Power of Fire-Engines for the various Diameters of the Cylinder, and Bore of the Pump, that are capable of raiſing Water from 48 to 440 Hogſheads *per* Hour, at any Depth, from 15 to 100 Yards. It was compoſed by Mr. *Henry Beighton*; and is founded on this Principle, that the Ale-Gallon of 282 cubic Inches of Water weighs 10 *lb.* 3 *oz.* *Averdupois*, and a ſuperficial ſquare Inch is preſſed with the Weight of 14 *lb.* 13 *oz.* of Air when of a mean Gravity. But allowing for ſeveral Frictions, and to give a conſiderable Velocity to the Engine, it is found by Experience, that no more than 8 *lb.* of Preſſure muſt be allowed to an Inch ſquare on the Piſton in the Cylinder, that it may make about 16 Strokes in a Minute, about 6 Feet each.

30. Theſe Calculations are but for common Practice; for with large Boilers the Piſton will make 20 or 25 Strokes *per* Minute, and each of them 7 or 8 Feet, and then a Pump of 9 Inches Bore will diſcharge more than 320 Hogſheads *per* Hour, and for other Sizes proportionably. The Uſe of the Table is eaſy; for ſuppoſe it required to draw 150 Hogſheads *per* Hour, at the Depth of 90 Yards; then in the 7th Column I find the neareſt Number 149, and againſt it, in the firſt Column, I find a 7 Inch Bore for the Pump; then under the Depth 90 on the Right-Hand, in the ſame Line, I find 27 Inches for the Diameter of the Cylinder fit for the Purpoſe, and ſo for any other.

Diameter of Bore.	Will hold in a Yard.	Will draw by 6 Feet Stroke.	Weight in one Yard.	At ſixteen Strokes a Minute.	63 Gallons to a Hogſhead.		In one Hour.		Diameter of the Cylinder in Inches.	The Depth in Yards.												
Inch.	Gallons	Gallons.	lb. Aver.	Gallons.	Hog.	Gal.	Hog.	Gal.		15	20	25	30	35	40	45	50	60	70	80	90	100
12	14.4	28,8	146	462	7	21	440			18½	21¾	24	28½	30½	32½	34½	37¼	40				
11	12,13	24	123,5	338	6	20	369	3		17	19¾	22	24¾	26¼	28	29¾	31¼	34¼	37	39½		
12	10,02	20,04	102	320	5	5	304	6		15½	18	20	22	23¾	25¼	27	28¾	31¼	33¾	36	38	40
9	8,12	16,2	82 7	259,8	4	7	247	7		4	16¼	18	20	21½	23	24½	25	28	30½	33	35	36½
8½	7,26	14,5	73,9	232,3	3	43	211	2		13½	15¼	17	19	20¼	21¾	23	24	26¼	28½	31	32½	35¼
8	6,41	12,8	65,3	205,2	3	16	195	2		12½	14¼	.6	18½	19	20½	21½	23	25	27	29	30½	32½
7½	5,65	11,3	57,6	181,1	2	55	172	3		11	13¾	15	16½	18	19	20	21¼	23¼	25	27	28½	30¼
7	4,91	9,8	50	157,1	2	31	149	4		10¾	13	14	15½	16¾	18¾	19	20½	22	24	25½	27	28½
6½	4,23	8,4	43	135,3	2	9	128	5		10	12	13	14	15½	16¼	18	19	20	22	23	24½	26½
6	3,61	7,2	36,7	115,	1	52	110	1		9½	11	12	13	14	15½	16	17	19	20½	22	23	24½
5½	3,13	6,2	31,8	99,2	1	36	94	3			10	11	12	13	14	15	15¾	17	19	20	21	22½
5	2,51	5,0	25,5	80,3	1	7	66	6				10	11	11¾	13	13¾	14	15½	16¾	18½	19¼	20½
4½	2,02	2,4	20,5	64,0	1	1	60	6					10	11	11¾	12	13½	14	15	.6	17	18¼
4	1,6	3,2	16,2	51,2	0	51	48	51						9	10	11	11½	12	13½	14	15	16

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Plate XXV.

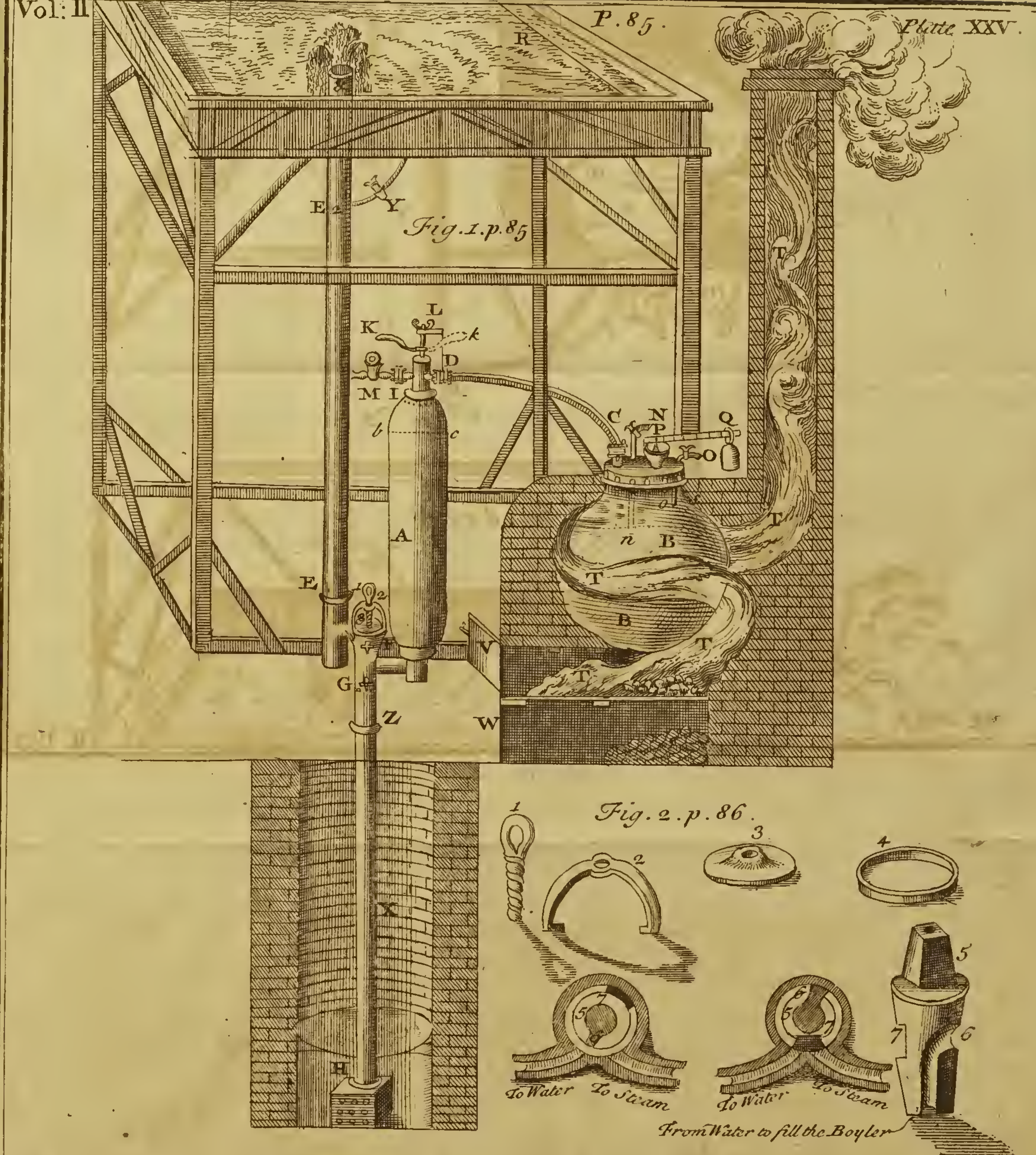
Fig. 1. p. 85

Fig. 2. p. 86.

(LXXXI.) The Method of conſtructing a Fire Engine according to the original Inſtitution of the Marquis of *Worceſter* and Capt. *Savery*, wherein the Water was to be raiſed ſolely by the Preſſure of elaſtic Vapour or Steam, is very uſeful, and cheap in reſpect of the other Sort ; and when the Height, to which the Water is to be raiſed, does not exceed 100 or 150 Feet, then this Engine is applicable with great Advantage, it requiring but a ſmall Fire, (not bigger than what is generally uſed in a Parlour Chimney) is of a very ſimple and eaſy Structure, and admirably adapted for playing of Fountains to very great Heights.

2. Dr. *Deſaguliers* has given us the Structure of this Engine on a Copper Plate with his laſt Improvements, and becauſe I think this better than that which Profeſſor *Muſſchenbroek* has exhibited in his *Eſſai de Phyſique*, eſpecially for raiſing Water, I ſhall here give the ſaid Figure and its Explication.

Pl. XXV. Fig. 1.

3. The Boiler B B is a large Copper Body of a globular Form, which will beſt of all withſtand the very great Force of Steam, that is in this Caſe neceſſary. Round the Body of this Boiler the Fire and Flame are conducted, as ſhewn at T T T. It has a Copper Cover ſcrew'd on, which contains the Steam-Pipe C D, and two Gage-Pipes N, O, which, by turning their Cocks, ſhew the Height of the Surface of the Water within, as in the other Engine.

4. On the ſame Cover at P is a Valve, over which lies a Steel-yard with its Weight Q to keep it down, the Strength of the Vapour being this Way moſt exactly eſtimated : For being in the Nature of a Lever, of the third Sort, 'tis plain, if the Beam of the Lever be divided into 10 equal Parts, and the firſt of them be upon the Middle of the Valve, and the Weight Q hangs at the 2d, 3d, 4th, &c. Diviſions, that then the Force of the Steam which can raiſe up the Valve will be 2, 3, 4, &c. times as great as the Weight. If the Area of the Valve be a ſquare Inch, and Q = 15 *lb.* hanging at the 2d Diviſion is raiſed by a Steam puſhing up the Valve, it will ſhew the Steam will then preſs with the Force of 2 Atmoſpheres, and ſo on to 10 Atmoſpheres ;

but great Care muſt then be taken about it, leſt the Steam, ſo very ſtrong, burſt the Boiler to Pieces.

5. The Steam is carried from the Boiler to a Copper Veſſel A by means of the Pipe CD, and is let into the ſame by turning the Handle K of the Steam-Cock DI. The Key of this Cock is kept down by the Screw L, held up by the Gibbet DL. The Handle turn'd from K to *k* admits a Paſſage to the Steam into the Copper Receiver A.

6. The Receiver A communicates at Bottom with the Sucking-Pipe ZH going down to the Water H in the Well X; and above with the Forcing-Pipe EE, which goes up a little above the Water of the Reſervoir R; between theſe two Pipes are two Valves F and G, both opening upwards. The Steam being let in upon the Water of the Receiver A, forces it up through the Valve F, and the Pipe EE to the Reſervoir, and then the ſaid Receiver is full of hot Steam.

7. This Steam in the Receiver is condenſed by a Jet of cold Water coming from the Forcing-Pipe by the ſmall Pipe MI, being let in, and ſhut off by the Cock at M. The Steam being condenſed by this Jet, will be reduced within a very ſmall Space, and ſo make a *Vacuum*, upon which the Water in the Well will ruſh up the Forcing-Pipe to reſtore the Equilibrium, and thus again fill the Receiver A; the little Air being compreſs'd within a ſmall Compaſs at the Top above *a b*.

8. That there may always be Water in the Force-Pipe for the Jet, there is a little Pipe which brings the Water to it from the Reſervoir, with the ſmall Stop-Cock Y to ſhut it off upon Occaſion. The Valves at F and G are examined at any time, by unſcrewing the Pin 1 to looſen the Strap 2, and let down the Flanch 3, all which Parts are ſhewn larger in the Figures below.

9. By the peculiar Contrivance of the Cock at DI, and its Key, the Water is made to paſs from the Force-Pipe to the Boiler, to ſupply the Waſte in Steam. This is plainly ſhewn in the Sections of the Cock and Key, where 5 is the Top of the Key; 6 is a Hole on one Side, which goes down to the Bottom, to convey the Steam or Jet of Water alternately to the Receiver; Pl.XXV. Fig. 2.

7 is a Notch on the other Side to take in the Water from the Force-Pipe, and conduct it to the Boiler B. How this is done, is eaſy to conceive from a View of the two Sections of the Cock and Key in two Poſitions within it.

10. The Boiler may hold about 5 or 6 Hogſheads, and the Receiver 1 Hogſhead: It will work 4 or 5 Hours, without recruiting; about 4 Strokes *per* Minute will produce upwards of 200 Hogſheads *per* Hour. This Steam makes a *Vacuum* ſo effectually, as to raiſe Water from the Well to the Height of 29 or 30 Feet; and ſuppoſe the Steam able to lift up the Steel-yard, with its Weight hanging at the 6th Diviſion, it will then be able to raiſe a Column of Water about 50 Yards high, as being then 6 Times ſtronger than the Preſſure of the Air, as is eaſy to underſtand from what has been ſaid on this Subject of the Common Water-Engine, (See *Annotat.* LXXX. 5.) the Water being raiſed in a ſimilar Manner in both Machines; there by the Preſſure of condenſed elaſtic Air; and here by the Preſſure of rarified elaſtic Steam.

11. The Elaſticity of the Steam increaſing with its Heat, and the Space into which it endeavours to expand itſelf being as its Elaſticity or Strength, and ſince we know (by what precedes) that Water expanded into Vapour, of the ſame Strength of the Air, occupies near 3000 times the Space as before, it follows, that when the Steam has twice that Strength, it will be expanded into 6000; if it be 5 times as ſtrong as the Air's Spring, it will become expanded into 15000 times that Space, and ſo on, beyond the Reach of Experiments. From all which it appears how near a perfect Vacuum we may approach by this Engine, or any other, by means of rarified Vapour.

(LXXXII.) In the *Philoſophical Tranſactions*, N° 461. there is an Account of an Improvement made in the Fire-Engine by Mr. *Payne*, as follows. He has contrived two Iron Pots or Veſſels of a conical Form inverted, as repreſented by ABEF, on the upper Part of which is fix'd a round globular Copper Head (of about $5\frac{1}{2}$ Feet Diameter) as L M N. There is then placed on the Inſide a ſmall Machine H, call'd the *Diſperſer*, with

Plate XXVI. Fig. 8.

with Spouts *a*, *b*, *c*, *d*, *e*, &c. round the Sides fix'd to it, and the Bottom thereof reſts on a Centre-Pin O. In this Machine is fix'd an upright Tube G, with Holes at the Bottom, and a Funnel P on the Top to receive a Spout of Water from a Conduit-Pipe Q, by the Stop-Cock R.

2. This Veſſel being compleated, two or more of them are placed in a reverberatory Arch for conveying the intenſe Heat of a ſtrong Fire, the Flame whereof encompaſſes the Iron Veſſels, and keep them in a red Heat, during the Time of their Uſe; at which Time the Cog-Wheel I, being turn'd by proper Machinery, whirls the Diſperſer about with great Velocity, and cauſes the Water in it (by a centrifugal Force) to fly thro' the Spouts againſt the Sides of the red-hot Pots, which immediately converts the greateſt Part thereof into Vapour or elaſtic Steam, which is conveyed by a common Pipe or Cock to the Barrel of the Engine to put the Piſton in Motion; and the waſte Water is convey'd away at Bottom, by means of a Pipe C D, with a Valve at D to keep out the Air.

3. By many Experiments made on this Engine, Mr. *Payne* has proved the following Particulars, *viz.* That ſuch an ignified Pot or Veſſel will rarify or expand 50 Gallons of Water, Wine-Meaſure, *per* Hour.

4. That a Cubic Inch of Water will make in Practice 4000 Inches of Steam; and therefore 50 Gallons will produce 46000000 *per* Hour, or 770000 *per* Minute; hence two ſuch Pots will produce 1540000 cubic Inches of elaſtic Steam *per* Minute.

5. That by an Experiment made at a Fire-Engine, 40 Gallons of Water *per* Hour, made into elaſtic Steam, will effectually give Motion to a 24 Inch Cylinder-Engine.

6. That 112 *lb.* of Coals is ſufficient, in this Method, to rarify 90 Gallons of Water *per* Hour into Steam, whereas in the common Fire-Engine, with Boilers, 'tis uſual to expend three times as great a Quantity.

7. That 95 Gallons of Water expanded into Steam will work a 36 Inch Cylinder-Engine; from all which

he concludes, that his new Invention will ſave at leaſt 60 *per Cent.* in Pit-Coals to work a Fire Engine.

8. The Method by which he found how much Steam could be produced from a given Quantity of Water was this. He procured a Copper Globe A 12 Inches in Diameter, with two Braſs Cocks B B oppoſite to each other; by one of theſe the Globe was faſten'd to a Pipe E, which was fix'd to the upper Part of one of the Steam Veſſels (as at N) and was fill'd with Steam by the Cock F; upon the other Cock was fix'd a ſmall Valve D, fitted to a Bore of one Inch Diameter; and C is a Handle or Bale to ſuſpend it by.

9. The whole thus fitted weighed 15 *lb.* 3 *oz.* *Troy*, or 12 *lb.* $9\frac{1}{2}$ *oz.* *Averdupois*; the Capacity or Content was about 925 cubic Inches. The Globe being fix'd to the Steam Veſſel, all the Cocks were turn'd, till the Steam by paſſing thro' had ſufficiently heated the Globe; then being ſtopp'd, the Steam condenſed, and the Air let in, when weigh'd in a very nice Balance, the Weight was 15 *lb.* 3 *oz.* 2 *dwt.* ſo that the whole Quantity of Steam was but 2 Penny-Weights, or $\frac{1}{10}$ of an Ounce *Troy*. Whence one Ounce of Water makes 9250 cubic Inches of Steam.

10. The Globe fill'd with Water weighed 45 *lb.* 7 *oz.* After this, it being fill'd with Steam, and one of the Cocks put under Water, the Water ruſh'd into the Globe, and the Cock being again turned, and the Globe of Water now weighed, the Weight was 44 *lb.* 9 *oz.* which deducted from the former Weight, gave 14 *oz*; which ſhew'd that all the Air was nearly excluded, or that a *Vacuum* was approached to in the Ratio of 713 to 727; or the Part poſſeſſed by Air was but the $\frac{14}{727}$ Part of the Whole.

11. The Air being again thus excluded out of the Globe by Steam, and both Cocks turn'd, the Globe was thus weigh'd, and the Weight was 12 *lb.* $10\frac{1}{2}$ *oz.* then one Cock was opened and the Air let in, upon which the Scale deſcended, and $\frac{1}{2}$ an Ounce *Averdupois*, being added to the other Scale, reſtored the Equilibrium, which was therefore the Weight of the Air, or 925 cubic Inches of Air weigh'd about 220 Grains.

12. But it was found above, that the Globe full of Steam weigh'd only 2 Penny-Weight *Troy*, or 40 Grains;

THE abovementioned *prodigious Rarefaction* of a ſmall Quantity of Water will be made very ſenſible by the *Æolipile*, whoſe Nature and Uſe in the Production of a *continual and moſt powerful Blaſt of Steam*, or *vapourous Wind*, is a *Phænomenon* equally curious and ſurpriſing, as will appear from the Experiment (LXXXIII.)

Grains; whence the ſpecific Gravity of elaſtic Steam is to that of Air as 40 to 220, or as 1 to 5 nearly; and ſince that of Air is to Water as 1 to 860, therefore the ſpecific Gravity of Steam is to that of the Water, as 1 to 860×5=4300.

12. Again, ſince $\frac{1}{10}$ of an Ounce of Water expelled near 925 cubic Inches of Air; then one cubic Inch of Water (which weighs $\frac{527}{1000}$ of an Ounce *Troy*) will expel about 4630 cubic Inches, but Mr. *Payne* takes 4000 for a Standard in Calculation.

(LXXXIII.) 1. The ÆOLIPILE is an Inſtrument contriv'd for Uſe, as well as Jucundity. It has its Name from the Fable of ÆOLUS, the God of Wind and Tempeſts, who was deſcribed, as having his Palace in the Caves, and locking up the Winds in the Hollows of the Mountains. The Word literally interpreted is, *the Gates of Æolus* ('Αιόλȣ ϖύλαι) becauſe of the Similitude of its Effect. For it conſiſts of a Copper globular Body, in which is inſerted a ſmall Neck or Pipe, with a very ſmall Orifice, from whence (when the Veſſel has Water in it, and heated) a Vapour iſſues with prodigious Violence, Impetuoſity and Force, and blows in the Manner of a terrible tempeſtuous Wind.

Pl. XXI. Fig. 7. 2. The Body of this Inſtrument conſiſts of two Parts, A and B, ſolder'd together with hard Solder, to endure a great Heat of Fire. On the upper Part is alſo ſolder'd the Pipe CD, whoſe Orifice or Hole at D is about the 40th or 50th Part of an Inch in Diameter. The Way to fill it with Water is, to ſet it firſt, while empty, on the Fire, which will heat it, and drive out

V. THE laſt Cauſe of the Motion of Fluids which I mention'd was that of Attraction. We have already ſeen how by this means any Fluid will aſcend above the common Surface in *Capillary Tubes*, &c. But the moſt notable and obvious Motion of Fluids ariſing from Attraction, is that of the Tides; the Waters of the immenſe Ocean, forgetful, as it were, of their natural *Quietus*, move, and roll in ſwelling Tides obſequious to the ſovereign Power of the Moon, and weaker Influence of the Sun.

To illuſtrate this, let N E S Q repreſent the Earth, covered over with Water ABDF; N S the Axis of the Earth, E Q the Equator, T R the Tropic of *Cancer*, *t r* the Tropic of *Capricorn*, M the Moon

Pl. XXII. Fig. 3.

all the Air; then with a Pair of Tongs you take it off the Fire, and hold the End of the Pipe D under Water, and the Water will run in to poſſeſs the Space deſerted by the Air, by the Preſſure of the Atmoſphere.

3. For an Experiment, it need not be more then half full of Water, which will be a conſiderable Time in being changed into Vapour. The ſtronger the Fire, the better will be the Steam, and the more forcibly will it fly out, and it will continue this ſurpriſing Blaſt, till it is wholly evacuated; then the Noiſe ceaſes, and a ſtill fine *Aura* follows the Steam. Great Care muſt be taken not to have the Hole at D ſtopp'd in putting it in the Fire, for if that ſhould happen, the Æolipile will burſt with a great Exploſion (equal to that of a Cannon) and may do very great Miſchief, as I have twice experienced, and once it was like to have been fatal; for

in her Orbit, S the Sun in his. Now ſince all Bodies are endued with an attracting Virtue, the Moon will attract all the Water in the neareſt Hemiſphere F A B, with Degrees of Force which are inverſely as the Squares of the Diſtances from all Parts; and therefore with the ſtrongeſt Force where the Diſtance is leaſt, *viz.* in the Point A directly under her; and this Attraction being in this Hemiſphere contrary to that of the Earth, the Water in all Parts from B and F towards A will have their Gravity gradually decreaſing, and be lighteſt of all at the Part A; and conſequently muſt there ſtand higher than at the Point F, where they are more attracted by the Earth, and ſo heavier, as is evident from the Laws of *Hydroſtatics.*

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then, when it burſt, the Power was ſo great, as to rend aſunder the Body of the Inſtrument, not in the Jointure, but in the ſolid Metal, which was at leaſt 8 Hundredths of an Inch thick. It happily ſtood ſo as to fly againſt the inner Part of the Mantle-Piece, which beat it both Sides together by the Violence of the Blow; which, had it flown into the Room, muſt have been fatal to any one or more who ſhould have been in its Way.

4. To give an Idea of the great Force neceſſary to burſt the Æolipile in the Manner above mentioned, let us proceed to Calculation. It has been found by Experiment, that a Copper Wire $\frac{1}{10}$ of a *Rhine-Land* Inch in Diameter was broke aſunder by the Weight of $299\frac{1}{4}$ *lb.* hung on to it. Now the *Rhine-Land* Foot is to the *London* Foot, as 139 to 135; the Diameter therefore

AGAIN: In the other Hemiſphere FDB, the Attraction of the Moon conſpires with that of the Earth; but decreaſing as the Squares of the Diſtances increaſe, the joint Force of Attraction will every where decreaſe from F and B towards D, the Point oppoſite to the Moon; where, again, the Waters will be lighteſt, and therefore ſtand higheſt to preſerve the *Equilibrium*. Whence it appears, that by this Sum and Difference of the Moon's and Earth's Attraction, there will neceſſarily enſue a Protuberance or Swelling of the Waters, which we call *Tides of Flood*, in the two Points A and D directly under the Moon. Alſo in the two Points F and B, as the Waters are there moſt attracted, ſo they will be heavieſt, and conſequently riſe to the leaſt Height from the Earth's Surface, whence

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of the Wire in *Engliſh* Meaſure was $\frac{136}{1350}$ of an Inch, whence the Area of its tranſverſe Section will be $\frac{212531}{25515000}$, or very nearly $\frac{21}{2551}$ of an Inch ſquare.

5. The Diameter of the Æolipile is 4 Inches, its Circumference therefore is 12,566; its Thickneſs about $\frac{8}{100}$, or 0,08 of an Inch, whence $12,566\times0,08=1,00528=$the Area of the rended Circumference of the Æolipile. Then ſay, by the Rule of Three, If $\frac{21}{2551}$ or $\frac{2}{355}$ require $299\frac{1}{4}$ or 300 *lb.* to break it, What will 1 ſquare Inch require? The Anſwer is, 38250 *lb.* for $\frac{2}{255}$: 300 *lb.* :: 1 : 38250 *lb.* So the Force of the Steam which burſt the Æolipile was *Thirty-eight Thouſand and two Hundred and fifty Pounds*, or one Thouſand nine Hundred and twelve Tons and an half.

6. This may ſerve as a notable Inſtance of the incre-

they are call'd *Tides of Ebb*, or the *Ebbing of the Water*.

IF to the Power of the Moon we add that of the Sun, we ſhall have the Tides conſiderably augmented at the *Conjunction* in S, or *Oppoſition* in H, that is, *at the New and Full Moons*, which are call'd *the Spring Tides*; as thoſe which happen when the Sun is at O or P are call'd *Neap Tides*, the Waters at A and D being then loweſt, becauſe the Attraction of the Moon is then counteracted by that of the Sun.

IT is farther to be obſerved, That of *the two Tides of Flood* at A and D, that at A is greateſt to any Place T in *Northern Latitude*, when the Moon is in *Northern Signs*, and above the Horizon; for the Point A is then nearer the *Zenith* of the Place G, than the oppoſite Point D is to the ſame

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dible Force of elaſtic Steam, it being three times as great as the Force of Gun-powder in the largeſt Cannon, as calculated in *Annotat.* XXV. Hence it appears of what prodigious Uſe this expanded Vapour would be, could we rarify Water into Vapour as ſoon, and with the ſame Eaſe, as we can ſet Fire to Powder. This Property of Air has already ſupplied us with *Air-Guns*; and with much greater Effect might we have *Steam-Guns*, and other Inſtruments, if that could be done.

7. To try this, one need only take a Vial with one Drop of Water in it, and ſet it over the Fire, till the Drop begins to riſe in Vapour, and drive out the Air, then putting the Cork faſt into the Mouth of the Vial, ſet it on the Fire, and incline it to the farther Side of the Room; after ſome little Time the Steam will diſ-

Place at R twelve Hours afterwards; and conſequently, the Height of the Tide S G is greater than that of the oppoſite Tide R *g*. The contrary happens when the Moon is in *Southern Signs.*

THAT there are *two Tides of Flood,* and *two of Ebb,* ſucceeding each other *alternately at the Interval of ſix Hours,* is obvious from the *Figure.* That they happen *later each Day near an Hour,* is owing to their exact Correſpondence to the *Motion of the Moon,* which daily *culminates* ſo much later. That they happen *not when the Moon is in the Meridian, but about three Hours after,* is owing to the Force of the Moon being then greater than when in the Meridian of any Place; as the Heat of the Day is greater at Three o' Clock than at Twelve; and the Heat of the Summer is greater in *Auguſt,*

charge the Cork with a conſiderable Exploſion, and throw it as far as a Grain of Gun-Powder would have done.

8. Or thus; let a few Drops of Water be put into a Piſtol or Gun-Barrel, cloſed on the Breach-End, and a leaden Bullet thruſt hard into the Barrel; let the End of the Barrel then be put over the Fire to heat, and expand the Water into Steam, which when it begins to come out of the Touch-hole, will ſhew the Air to be all excluded; then let the Hole be ſtopp'd with a ſmall Plug, and in ſome time the Vapour will diſcharge the Bullet, with as much Force as a common Charge of Powder, and with the ſame Report and Execution.

9. And though it may be difficult to contrive any Method of making Vapour of a ſufficient elaſtic Force

than at the 10th of *June*. Laſtly, *that the greateſt Spring Tides happen not at the* 10th *of* March, *and* 12th *of* September, *but in* February *and* October, is becauſe the Sun being neareſt the Earth in *December*, his Influence is then ſtrongeſt, and ſo muſt quicken the Time of the greateſt *Vernal Tides*; and being weakeſt in *June*, the Time of the *Autumnal Tides* will neceſſarily be retarded.

Such are the *Phænomena* of the *Tides*, according to a general Theory, and are found to be verified in thoſe Parts of the Earth's Surface which are *wholly cover'd with Water*, as the Theory requires; I mean, the vaſt *Atlantic and Weſtern Oceans* (LXXXIV.)

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in large Quantities, ſo as by Conduct-Pipes to charge large Guns therewith, at a Diſtance from the Steam-Veſſel; yet ſomething of this Kind may be effected, by expanding a ſmall Quantity of Water at the Bottom of the Cannon into Vapour, by firſt putting a red hot Iron Bullet down the Bore to the Water, which it ſoon diſſipates into Vapour, and when confin'd at the Touch-hole, and another with which the Cannon is charged, it will ſoon become ſtrong enough to drive it with Force enough for any propoſed Effect. So that the *Steam-Gun*, or even a *Steam-Cannon*, is perhaps a Matter not merely *in poſſe*, but may be hoped for in Reality, if all proper Expedients be tried, and Experiments made for that Purpoſe

(LXXXIV.) 1. The Theory of the Tides is a Point of great Curioſity and Importance, and has therefore excited ſeveral learned Gentlemen to endeavour an Explanation thereof, in ſuch Ways, as they think the various Phænomena will be beſt and moſt eaſily underſtood.

RPJCB

Plate XXVI

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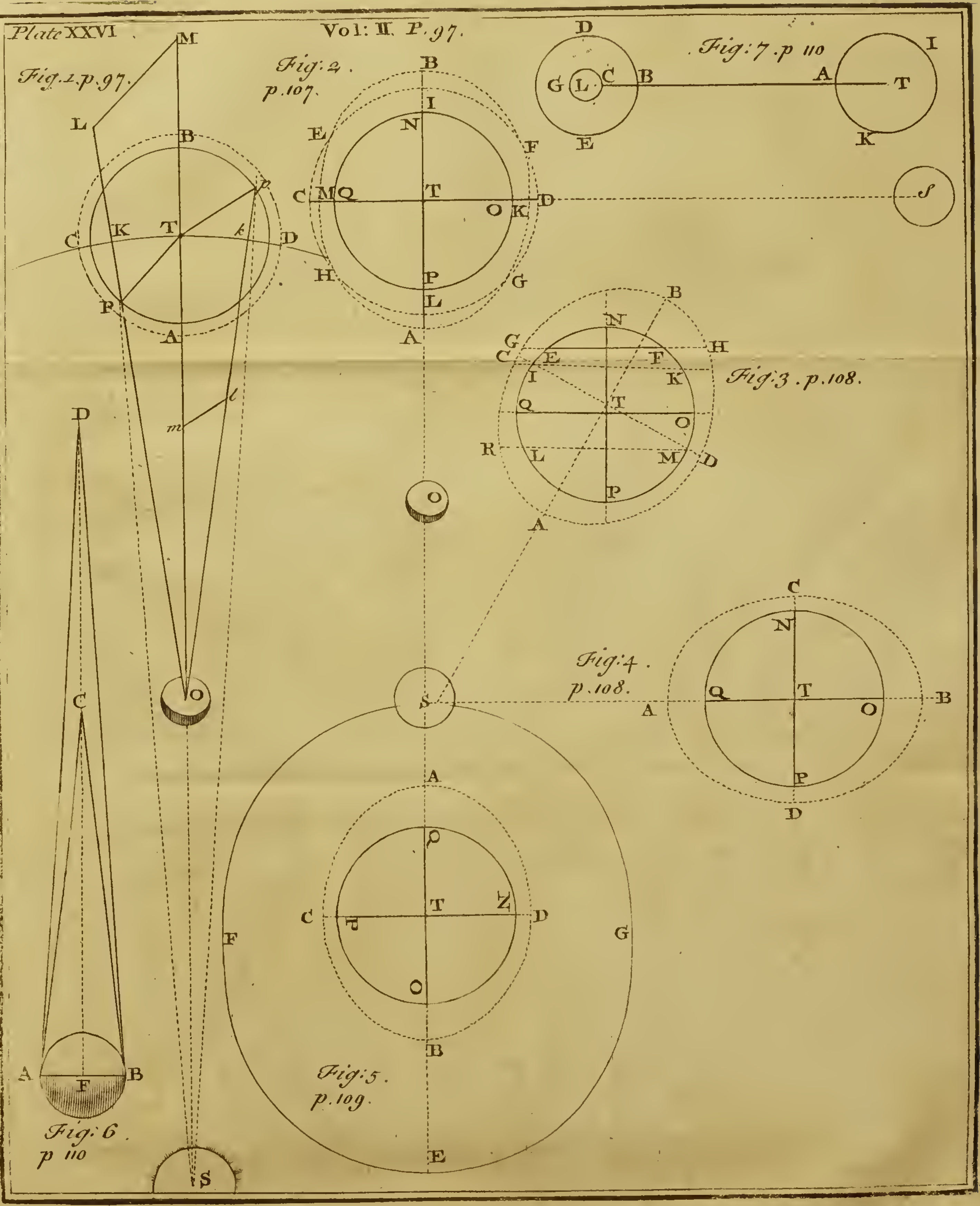

Fig. 1. p. 97.

Fig: 2. p. 107.

Fig: 3. p. 108.

Fig: 4. p. 108.

Fig: 5. p. 109.

Fig: 6. p 110

Fig: 7. p 110

ſtood. I have conſider'd them all, and can find none which I think ſo natural and obvious as that which I have here propoſed. Nature herſelf here points out all the Phænomena, and that too plainly to want any Aſſiſtance from the various Contrivances of Art; which I find by frequent Experience is more apt to puzzle than afford any *Eclairciſſement* in this particular Caſe.

2. The Mathematical Principles of Theory are as follow. Let T be the Centre of the Earth, O the Moon, and A C B D the Circumference of the Earth, which ſuppoſe all around environ'd with Water, whoſe outmoſt Surface is repreſented by the dotted Circle, and its Diſtance from the Earth's Surface is indefinitely ſmall in reſpect of the Earth's Semidiameter. Now 'tis plain were the Moon away, or her Attraction at the Earth nothing, the Waters would every where have an equal Share of the Earth's attractive Power (ſuppoſing the Earth quieſcent) and ſo would take their Place at an equal Diſtance from the Centre, and therefore conform themſelves to a ſpherical Figure, agreeable to what was ſhewn under the *Laws of Hydroſtatics*.

Plate XXVI. Fig. 1.

3. But ſince the Earth is not quieſcent, but moveable upon its Axis, the Centrifugal Force generated under he Equator (where it is greateſt) will there diminiſh he Power of Gravity upon the Waters, and render hem leſs heavy there than on any other Part, and as he Centrifugal Force decreaſes towards each Pole, the Weight of the Water will increaſe, and be greateſt under either Pole. And ſince Fluids, which communicate with each other, maintain an Equilibrium, and if hey are of different Weights, they have alſo different Denſities, and conſequently require different Altitudes o ſuſtain each other *in Equilibrio*; therefore it follows, hat the Waters under the Equator, which are lighteſt, muſt have a greater Altitude (or Diſtance from the Earth's Surface) to be *in Equilibrio* with the Waters under the Pole, which are heavier, and will therefore have a leſs Altitude. Hence the Figure of an univerſal Ocean will not be ſpherical, but a flatted Spheroid like that of the fluid Earth, and for the ſame Reaſon, n *Annotat.* XXXIV.) whoſe longeſt Diameter is that f the Equator continued.

4. But as this reſults from the diurnal Rotation of the Earth only, which is always uniform, and every where conſtantly the ſame, it can cauſe no Alteration in the Depth of the Water, in any particular Place, and therefore can have no Share in the Cauſe of the *Flux and Reflux* of the Sea, at leaſt ſo little as not to deſerve Notice, and cannot be diſcern'd by the Senſes. However, what that is will be ſhewn in the Sequel of this Note.

5. The alternate Riſing and Falling of the Waters is therefore owing to ſome other Cauſe or Cauſes than what can be found in or upon the Earth; that is, it muſt be owing to ſome of the heavenly Bodies, which we ſee at a Diſtance from us. This the Ancients made no Doubt of; and aſcribed it to the Influence of the Moon, with good Reaſon, as they ſaw ſo plain a Congruity between the Motion of the Waters, and that of the Moon. They could not alſo but obſerve, that at each New and Full Moon the Tides were greater than at other Times, and were therefore aſſured, the Sun was likewiſe concern'd in the Cauſe of the Tides. Yea, *Pliny* aſſures us, that they were ſo nice as to obſerve the annual *Phænomena* of the Tide; and knew that the Tides were greater in Winter than in Summer. But they had no Theory to account for the diurnal, menſtrual, and annual Differences of the Tide. This was reſerv'd for the Invention of the celebrated Author of our Philoſophy, whoſe Footſteps, therefore, we ſhall here inſiſt on as near as poſſibly we can, in explicating all the particular *Phænomena* of the Tides.

6. Since (as we have ſhewn) all Bodies mutually attract each other, the Moon at O will attract the Earth at T, and its Waters, as well as be attracted by it. But ſince the Law of Attraction is to decreaſe, as the Squares of the Diſtances increaſe, the Water at A, P, C, B, or at A, D, *p*, B, will be differently affected by the attractive Power of the Moon at O; the Attraction at A will be greateſt as being neareſt, that at B leaſt, and at C or D, it will be at a Mean, and the ſame nearly as at T the Centre of the Earth, becauſe of the very great Diſtance T O in reſpect of T C.

7. Let us conſider the Caſe of one aqueous Particle at P, and compare it with that of A in the neareſt Hemiſphere C A D; and of another at p compared with that of B in the fartheſt Hemiſphere C B D. Draw the Lines O P and O p; in O P continued out let O K repreſent the mean Attraction, or that of the Particle C, and let it be made as $OL : OK :: OK^2 : OP^2$; then will $OL : OK :: \frac{1}{OP^2} : \frac{1}{OK^2}$, or $OK : \frac{1}{OK^2} :: OL : \frac{1}{OP^2}$, wherefore O K and O L will repreſent the attractive Forces of the Moon on the Particles at C and P, as being inverſely as the Squares of the Diſtances.

8. In like Manner, if we take $Ol : Ok :: Ok^2 : Op^2$, then will Ol be the Force with which the Particle p is affected by the Moon. Continue out O T, and draw L M parallel to PT, and lm to p T. Then is the Force O L reſolvable into the two Forces L M, and M O, and the Force O l into lm, and mO. Now ſince L M and lm directly conſpire with P T, and p T, they will expreſs Forces to be ſuperadded to the Attraction of the Earth upon the Particles P and p; and therefore unleſs theſe additional Forces were every where the ſame, the Waters about the Earth's Surface cannot have an equal Power of Gravity towards the Earth's Centre; and ſo cannot exiſt in a ſperical Form.

9. In order to ſee this, we have (by ſimilar Triangles L O M and P O T) $LM : PT :: LO : PO$; whence $LM = \frac{PT \times LO}{PO}$; but ſince L O is as $\frac{1}{PO^2}$ (by *Art.* 7.) we have $LM = \frac{PT}{PO^3}$, or ſince PT is given $LM = \frac{1}{PO^3}$, and for the ſame Reaſon $lm = \frac{1}{pO^3}$, that is, $LM : lm :: pO^3 : PO^3$; but ſince P O is leſs than pO, L M will be much greater than lm, conſequently the Water at P and p will be affected with very different Forces, and therefore will not tend equally to the Earth's Centre T, or be at an equal Diſtance from it. And this Difference of Forces will be greateſt of all at the

Points A and B, for there theſe perturbating Forces will be as AO^3 and BO^3 inverſely, which Cubes have the greateſt Difference of all others.

10. Now this is only one Part of the Attraction of the Moon which diſturbs the Waters; the other Part M O makes this Perturbation ſtill greater; for if T O be the Force with which the Earth's Centre is attracted by the Moon O, 'tis plain the Waters can never accede directly towards the Moon with the ſame Velocity that the Earth does, unleſs when M O and *m* O are equal to T O, that is, at the Points C and D. From MO ſubduct TO, the Remainder TM will be the Exceſs of Force, by which the Waters in every Point P, of the Hemiſphere CAD will be accelerated towards the Moon more than the Earth itſelf; and T *m* will expreſs the Force by which the Earth tends towards the Moon more than the Water at any Point *p* in the remote Hemiſphere C B D.

11. The Proportion of the Part of the Force M O, in reſpect to OP, is thus found; as MO : OT :: LM : P T; wherefore $MO = \frac{OT \times LM}{PT}$; but ſince both Q T and P T are conſtant, we have M O as L M, *i. e.* M O is as $\frac{1}{PO^3}$. Therefore at A, M O will be as $\frac{1}{AT^3}$; and at B, *m* O will be as $\frac{1}{OB^3}$, that is, in reſpect to the Points A and B, we have M O : *m* O :: $OB^3 : AO^3$; and ſince theſe Cubes are the greateſt of all others, the Waters will be accelerated in A towards the Moon with the greateſt Difference of Attraction, and at B with the leaſt; conſequently the Water will there on this Account alſo recede moſt from a ſpherical Figure.

12. Therefore, ſince both Parts of the Moon's Force are proportional to the Cubes of the Diſtance of the Luminary inverſely, they will both conſpire to make the Waters at A and B recede from the Centre of the Earth T, and put on a ſperoidical Figure, whoſe longeſt Axis B A goes towards the Moon's Centre O. The Force $MO = MT + TO$, but ſince the Part T O is

always the ſame, the perturbating Part will be only TM, which will be as MO — TO, and ſo when MO $=$ TO, then TM $= o$; which happens when P is at C or D; and then alſo LM coincides with PT, and both with CT; whence ſince in that Caſe the Line OK becomes OC $=$ OT, the perturbating Forces will in Part vaniſh (*viz.* TM $= o$) and the other Part LM will make the Waters at C heavier than in any other Place; and ſince every thing happens inverſely, with regard to the Particle *p*, it follows, the Waters about the Points C and D will be at a leſs Diſtance from the Earth's Centre than any where elſe, whence CTD will be the ſhorteſt Diameter of the aqueous Spheroid ACBD.

13. The Sun at S is ſhewn to produce the ſame Sort of Effects upon the Water, about the Earth; only in a leſſer Degree. For tho' the *Momentum*, or abſolute Force of the Sun upon the Waters is much greater than that of the Moon, yet the Parts of the Force which diſturb the Figure of the aqueous Globe, *viz.* LM and *lm*, and TM, and T*m*, will in this Caſe be leſs than before, on account of the much greater Diſtance TS than TO. As the perturbating Forces of the Moon in the Points A and B are as BO^3 to AO^3, thoſe of the Sun will be as SB^3 to SA^3. But the Difference of theſe latter Cubes is much leſs than that of the former, becauſe of their vaſtly larger Roots SB and SA. For Example, Suppoſe OA $=10$, OB $=11$; and SA $=100$, and SB $=101$; then $OA^3 : OB^3 :: 1000 : 1331 :: 100 : 1033$ nearly. But $SA^3 : SB^3 :: 1000000 : 1030301 :: 100 : 103$ nearly.

14. Hence we ſee, that both the Luminaries produce a Tide in two oppoſite Parts of the Earth's Surface A and B, in a Space of a Day proper to each, *i. e.* in the Space of a *Lunar and Solar Day*, or the Time in which the Moon, or the Sun, after leaving the Meridian, comes to it again. The Lunar Day (becauſe of the Moon's quick Motion conſpiring with that of the Earth) is about 50 Minutes longer than the Solar Day of 24 Hours. And thoſe Tides will always be moving the ſame Way, ſometimes together, ſometimes ſeparately,

rately, that of the Moon always following after that of the Sun.

15. In order to compute the Forces of the Sun and Moon to raiſe the Waters, we will ſuppoſe (at preſent) that P is the Moon, and T the Earth, then L M will be the Addition of the Moon's Gravity to the Earth ariſing from the Action of the Sun S. When P is at C, then L M will be as C T, which let us call A. The Gravity of the Earth towards the Sun will be expounded by T S, which call F, that is, let C T : T S : A : F. Let P be the periodical Time of the Earth about the Sun, and p that of the Moon about the Earth; alſo let f be the Force of Gravity in the Moon towards the Earth; then will $F : f :: \frac{TS}{P^2} : \frac{TC}{p^2}$ (by *An.* XXXIV. 6.) therefore $F : f :: \frac{F}{P^2} : \frac{A}{p^2}$; conſequently, $\frac{FA}{p^2} = \frac{Ff}{P^2}$, that is, $\frac{A}{p^2} = \frac{f}{P^2}$, whence $A : f :: p^2 : P^2$; *or the additional Gravity to the Moon at* C, *is to the Gravity of the Moon towards the Earth, as the Square of the Moon's Period to the Square of the Earth's Period.* Now $p = 27$ Days 7 Hours 43′; and $P = 365$ Days 6 Hours 9′, whence $p^2 : P^2 :: 1000 : 178725 :: 1 : 178\frac{725}{1000} :: A : f$.

16. Now the Semidiameter of the Earth being put $= 1$, and $G =$ Power of Gravity on its Surface; ſince the Mean Diſtance of the Moon from the Earth is 60 Semidiameters of the Earth, we ſhall have $f : G :: 1^2 : 60^2 :: 1 : 3600$; therefore $3600 f = G$, and $f = \frac{G}{3600}$; which being put for f, in the Proportion above, we have $A : \frac{G}{3600} :: 1 : 178{,}725$; whence $178{,}725\ A = \frac{G}{3600}$. Therefore $A : G :: 1 : 178{,}725 \times 3600 = 643400$.

17. What we have hitherto ſaid is upon Suppoſition that the Moon revolves about the Centre of the Earth at Reſt; but this is not the Caſe, for both the Earth and Moon revolve about one common Centre of Gravity (as we have ſhewn *An.* XXXVI.) and allowing the Maſs of Matter in the Moon to that in the Earth as 1 to 39,788,

39,788, we find the true Mean-Diſtance of the Moon from the Earth revolving about the common Centre of Gravity, to be 60,5 Semidiameters of the Earth; but the Force by which it is retain'd in its Orbit at 60 Semidiameters, is to the Force retaining it at $60\frac{1}{2}$ (revolving in the ſame Time as before) as 60 to $60\frac{1}{2}$ (as was ſhewn *Annotat.* XXXIV. 8.) wherefore $60 : 60\frac{1}{2} :: f : \frac{60,5f}{60} =$ f truly equated. But here $f = \frac{60\,f}{60,5}$, and before we had $f = \frac{G}{60^2}$, therefore $\frac{60\,f}{60,5} = \frac{G}{60^2}$, and ſo $\frac{60,5G}{60 \times 60 \times 60} = f$; wherefore $A : G :: 1 : 643400 \times \frac{60}{60,5} = 638059,78 = b$, that is $A = \frac{G}{b}$.

18. After the ſame Manner it is ſhewn, if any Body P revolve about the Earth at its Surface, it would at C have an additional Gravity a, which would be as $\frac{G}{x}$; therefore $A : a :: 60\frac{1}{2} : 1 :: \frac{G}{b} : \frac{G}{x}$; whence $x = 60\frac{1}{2}\,a = 60\frac{1}{2} \times 638059,78 = 38602616,7$. Now becauſe the Waters will be affected juſt in the ſame Manner as ſuch a Body revolving at the Earth's Surface, by the Influence of the Sun, (becauſe both will yield to an impreſs'd Force) *therefore the Power of the Sun to increaſe the Gravity of the Waters at* C *and* D, *is to the Power of Gravity as* 1 *to* 38602616,7.

19. In order to eſtimate this Force at A and B, we muſt find the Value of L M there, by which the Particle at A is impell'd towards T; and of T M, by which it is impell'd in a contrary Direction towards S. Then will the Difference of theſe two Forces, *viz.* T M—L M, be the Force required. In order to this, it muſt be conſidered, that on account of the prodigious Diſtance of O in Compariſon of P, the Angle P O T will be exceeding ſmall, and the Lines O L, O M, may be looked upon as parallel, and therefore LM = P T, and P L = T M, very nearly.

20. Then when P coincides with A, and K with T, we have LM = AT = PK, and TM = PL = AT +

KL; wherefore $TM-LM=KL$, which therefore is the whole Force by which the Waters at A are drawn from the Earth's Centre T. In the ſame Manner it is ſhewn that $Tm-lm=kl=$ to the whole perturbating Force at B.

21. Again, ſince SP is nearly equal to SK, and it is $SL : SK :: SK^2 : SP^2$, and therefore $SL-SK : SK :: SK^2-SP^2 : SP^2$, that is, $KL : SK :: PK \times \overline{SK+SP} : SP^2$ (becauſe the Difference of the Squares of any two Numbers is equal to their Sum multiplied by their Difference; thus $\overline{a+b} \times \overline{a-b} = aa - bb$.) Therefore ſince $SK=SP$, we have $KL : SP :: PK \times 2SP : SP^2 :: 2PK : SP$; wherefore $KL=2PK$, and $PL=TM=3PK$.

22. Hence when P coincides with A, we have $PK=AT=PT=LM$, and therefore $TM=PK=3LM$; whence $TM-LM=KL=2LM=2AT=2CT$. But CT expreſſes the additional Gravity at C; therefore $KL=2TC$, is the Force by which the Waters are drawn from T towards S; and ſince $TC=a=\frac{G}{38602616,7}$, we have $KL=\frac{2G}{38602616,7}=\frac{G}{19301308,3}$; that is, the Diminution of the Gravity at A is double the Increaſe of it at C, and is the $\frac{1}{19301308,3}$ Part of the whole Gravity.

23. Hence the Sum of both the Forces $KL+LM$ $=\frac{2G}{38602616,7}+\frac{G}{38602616,7}=\frac{3G}{38602616,7}=\frac{G}{12867538,9}$; or the whole Force of the Sun to agitate the Waters of the Ocean will be the $\frac{1}{12867538,9}$ Part of the Gravity or Force of the Earth's Attraction. This is the Force in any given Place to which the Sun is vertical, and in his Mean-Diſtance from the Earth. In other Poſitions of the Sun, the Force will vary in the Proportion of the *verſed Sine of double the Sun's Altitude above the Horizon directly*, and *the Cube of its Diſtance from the Earth inverſely.*

24. It has been ſhewn that the centrifugal Force of the Parts of the Earth under the Equator is $\frac{1}{289}$ Part of Gravity, (*Annotat.* XXXIV. 20.) wherefore the centrifugal Force is to the Sun's Force to agitate the Sea, as 12867538,9 to 289, or as 44524,4 to 1. Alſo ſince the Diameter of the Equator is to that of the Poles as 230 to 229 (as was there ſhewn), if we ſay as 229 : 230 : : 4000 : 4017,5, the Difference 17,5 Miles, or 924000 Feet, is the Effect or Elevation at the Equator, occaſioned by the Centrifugal Force. Say then, As 44524,4 : 924000 : : 1 : 2,07=2 Feet $\frac{4}{5}$ of an Inch= 24 84 Inches, which therefore is the Height which the Sun can elevate the Waters to at A and B above that at the Points C and D.

25. In this Computation, *Profeſſor* EULER has thought fit to find great Fault with Sir ISAAC NEWTON, and his Principles, calling this Altitude of 25 Inches an *enormous and erroneous Effect*, becauſe his own Calculation produces but $\frac{1}{2}$ a Foot, or a little more, for the Height to which the Sun can elevate the Sea. But ſince Meſſrs. *Bernoulli* and *MacLaurin* have each of them ſeparately inſtituted the Calculus, and reſolv'd the Problem, with ſuch an exact Agreement with each other, and with Sir *Iſaac*, there is no Room to doubt of the Truth of it; and it appears to depend on as certain Principles, and more evident than thoſe which Mr. *Euler* has advanced in Contradiction to Sir *Iſaac Newton*.

26. The Power of the Sun to move the Waters has been computed by means of the periodical Times of the Earth and Moon; and after the ſame Manner might be computed the Force of the Moon to diſturb the Waters, were there any Satellite revolving about the Moon to compare its periodical Time with the Moon's periodical Time; but as there is none, we muſt have recourſe to other Methods, the eaſieſt of which is, to collect the Proportion of the Force of the Sun and Moon from the Proportion of their Effects on the Waters of the Sea; that is, by obſerving the Heights to which they riſe when they act together, and when they act contrarily; thus *Stirmius* obſerved at the Mouth of

the *Avon*, at the third Stone below *Briſtol*, that the Water in the Syzygies was about 45 Feet, and in the Quadratures about 25 Feet. Alſo at *Plymouth*, Captain *Colpreſs* obſerved thoſe Elevations to be as $20\frac{1}{2}$ to $11\frac{1}{2}$, or 41 to 23. At *Havre de Grace* the Altitudes have been obſerved as 17 to 11. The Mean of all theſe Proportions is 10 to 6. Let $S =$ the Sun's Force, and $L =$ the Moon's Force; then will $L + S : L - S :: 10 : 6 :: 5 : 3$; whence $L : S :: 4 : 1$, or the Force of the Moon will be about 4 times greater than that of the Sun.

27. And this would be a determination ſufficiently exact, were the Sun and Moon in the Equator, in their Mean Diſtances from the Earth; alſo were the Tides to be greateſt and leaſt in the Syzygies and Quadratures, and the Diſtance of the Moon the ſame in each; but ſince all thoſe Things are found otherwiſe in Fact, we ſhall have the above-mentioned Proportion varied by them in ſuch manner, that it will become at laſt $1{,}017522\,L + 0{,}7986355\,S : 0{,}9830427 \times 0{,}8570327\,L - 0{,}7986355\,S :: 5 : 3$; or, putting Letters for the Numbers, $a = 1{,}0175$, $b = 0{,}7986$, $c = 0{,}983$, $d = 0{,}857$, we have $aL + bS : cdL - bS :: 5 : 3$.

28. Hence $aL + bS = 5$, and $cdL - bS = 3$; whence $\frac{5 - aL}{b} = S$, which ſubſtituted in the other Equation gives $cdL - 5 + aL = 3$, or $cdL + aL = 8$, whence $L = \frac{8}{cd + a} = 4{,}301$; then $\frac{5 - aL}{b} = 0{,}7808 = S$. Wherefore $S : L :: 07808 : 4{,}301 :: 1 : 5{,}508$; but if we take the Mean-Proportion of Tides as 9 to 5, as Sir *Iſaac Newton* has done, then $S : L :: 1 : 4{,}4815$. Since the Sun elevates the Water 25 Inches, the Moon will elevate it $25 \times 5{,}508$ Inches $= 137{,}7 = 11\frac{1}{2}$ Feet. The Sum of both Forces ſhould elevate the Water $13\frac{1}{2}$ Feet at the New and Full Moon, and the Difference of the Forces ſhould raiſe the Water $9\frac{1}{2}$ Feet, as in the Quarters; and theſe Heights anſwer pretty well to Obſervations: For in the *Atlantic* and *Æthiopic* Oceans the Tides are found from 6 to 15 Feet Altitude, and generally higher in the great *Pacific* Ocean.

In ſhort, that the Phænomena of the Tides, in this reſpect, may anſwer to the Theory, the whole Globe ſhould be covered with very deep Water; at leaſt the Ocean ſhould not be leſs than 90 Degrees wide, or one Quarter of the Globe.

Plate XXVI. Fig. 2.

29. The Sum of all that has been ſaid is this: If NOPQ be the Surface of the Earth, T its Centre, IFKGLHCE a Circle repreſenting the ſpherical Surface of the Waters covering the Earth, and affected only with the Gravity of the Earth; upon placing an attracting Body at S at the Diſtance ST, the Waters will no longer continue their ſpherical Figure, but be immediately drawn into a ſpheroidical Figure ACBD, in ſuch manner, as to be depreſs'd at C and D to M and K, and elevated from L and I to A and B; and the Elevation AL or BI is double the Depreſſion CM or DK. That if S be the Sun, then $AP - OK = AL + KD = 25$ Inches; or $11\frac{1}{2}$ Feet, if S be the Moon. That at the Points E, F, G, H, (which are call'd the *Octants*) the Water is neither elevated nor depreſs'd. That if any other Body be placed at O (as the Moon) in the ſame Right-line TS; then by the joint Influence of both S and O, the Elevation at A and B will be increaſed, and the Depreſſion at C and D likewiſe. Laſtly, if S be in the Situation *S*, or vertical to the Point D, 'tis plain its Action to raiſe the Water D will be directly contrary to that of the Moon in depreſſing it there; wherefore the Depreſſion will not be ſo great as before; for the ſame Reaſon the Elevation at A and B will be diminiſhed, being now only as the Difference of the two Forces, whereas before they were as the Sum.

30. We ſhall now conſider the Phænomena of the Tides which remain; and firſt, it is evident, that if PN be the Axis of the Earth, and QO the Diameter of the Equator, then the Moon ſituated at O, over one of the Poles, would accumulate the Water over each Pole, and the Spheroid would be ſo poſited as to have its longeſt Axis AB coinciding with the Axis of the Earth PN. In this Poſition of the Spheroid, 'tis plain, there could be no ſuch Thing as a Tide in any Part of the Ocean over all the Earth, for every Section of the

Spheroid, parallel to the Equator, would be a Circle; conſequently in any Parallel of Latitude, the Water would be at an equal Diſtance from the Earth's Surface every Moment of the diurnal Revolution, or natural Day.

Plate XXVI. Fig. 3. 31. Suppoſe the Moon were removed from the Direction of the Earth's Axis, and poſited at S, then will the Axis of the aqueous Spheroid A B be turned towards S, and make an Angle with the Earth's Axis, as A T P or B T N. Then we obſerve, that ſince C, D, are the Places of loweſt Water, that Parallel I K which paſſeth through the Point I on one Side the Equator, and L M which paſſes through M on the other, will divide the Earth into three Zones, in two of which, *viz.* E N K and L P M, there will be but *one Tide each Day*, of the ſame Kind; for inſtance, in the Parallel E F, a Perſon at F will have high Water, and at E low Water 12 Hours after. Again, in all the Zone I K M L, there will be two Tides of the ſame Kind each Day, as is evident from the Figure. Theſe Limits, or the Arch Q I or O M is the Complement of the Moon's Declination from the Equator.

32. If the Moon at S be over the Equator, the longer Axis of the Spheroid A B will now coincide with the Plane of the Equator Q O, and the ſhorter Axis C D with the Axis of the Earth N P. Here 'tis obvious, that in this Situation of the Spheroid, the Waters in the Parts A, B, with reſpect to thoſe at C D, will give the greateſt Difference of high and low Water poſſible to all Parts of the Earth; and that there is no Place but thoſe two at the Poles N, P, but what has two Tides of Flood, and two of Ebb every 24 Hours. And this Difference of the Flux and Reflux will decreaſe from the Equator to the Poles. Fig. 4.

33. I have already obſerved that the greateſt Elevation of the Waters is not when the Luminary is in the Meridian, but about three Hours after, becauſe the Motion communicated to the Waters during the Arrival to the Meridian is not immediately deſtroyed, but remains ſome Time, and receives a farther Augmentation from that which is impreſs'd for about three Hours after. For the ſame Reaſon, we obſerve, the greateſt

and leaſt Tides happen not on the Day of the Syzygy, or Quadrature, but on the 3d or 4th after; the Sum or Difference of the Forces of the Luminaries not being till then at a *Maximum*.

34. Let S F E G be the Orbit of the Moon about the Earth Q N O P, which as it is not circular but elliptical, the Centre of the Earth T will not be always at an equal Diſtance from the Moon; but the Moon will be ſometimes neareſt the Earth, as when at S, and ſometimes fartheſt off, as at E. The Point S is called the *Perigæum*, or Perigee; and the Point E the *Apogæum*, or Apogee. The Power of the Moon in her Perigee is to that in the Apogee nearly as TE^3 to TS^3; and conſequently the greateſt Tides will be on the Day of the Perigee, or rather a few Days after, for the Reaſons above-mention'd.

Plate XXVI. Fig. 5.

35. This is very nearly agreeable to Experience. For in the Year 1713, *February* 26, the Elevation of the Water at *Briſtol* was 22 Feet 5 Inches; and on *March* 13th, it was 18 Feet 2 Inches. The Moon, in the firſt Caſe, was in her Perigee, and diſtant from the Earth 953 Parts, of which the Mean-Diſtance contains 1000. In the latter Caſe ſhe was in her Apogee, and her Diſtance was 1032. Now $1032^3 : 953^3 :: 22\frac{1}{2}$ Feet : 17 Feet 8 Inches, which is but 6 Inches leſs than the Altitude obſerved. All other Circumſtances were equal in both Caſes.

36. In like Manner, ſince the Earth moves about the Sun, in an Orbit in ſome ſmall Degree elliptical, the Sun will be at one Time neareſt to the Earth (and that is call'd the *Perihelion)* and at another Time fartheſt diſtant from it, (which is then call'd the *Aphelion)*. Now when the Earth is in her *Perihelion*, *(December)* or a few Days after, the Force of the Sun to move the Waters, will be greater than when in her *Aphelion* (in *June)*. Thus it was obſerved in the Year 1712, *December* 28, the Water aſcended 19 Feet 2 Inches, and on *June* 19 (of the preceding Year) it aſcended but to the Height of 18 Feet 4 Inches: The Diſtance of the Moon, &c. being the ſame in each Caſe, which will be found nearly to anſwer to the Ratio of the Cubes of

the Diſtances inverſely, or of the apparent Diameter of the Sun directly.

37. For ſince the apparent Diameter A B of any Body decreaſes with the Angle A C B under which it is ſeen, ſo that at C and D it will be as the Angles A C B and A D B; and theſe Angles decreaſe as their Radii (or the Diſtances C F, D F, increaſe; it follows that Plate XXVI. Fig. 6. the apparent Diameter A B is every where inverſely as the Diſtance, that is, at C or D we have A B as $\frac{1}{CF}$ or $\frac{1}{DF}$. Whence if the apparent Diameter of the Sun be $= D$, and T S the Diſtance of the Sun, then ſince $D = \frac{1}{TS}$, and the Sun's Force (F) to move the Sea as $\frac{1}{TS^3}$ (as ſhewn in *Art.* 9.) therefore $F = \frac{1}{TS^3} = D^3$; that is, *the Force of the Luminary to move the Sea is directly as the Cube of the apparent Diameter thereof.*

38. As the Force of the Moon to move the Waters is at a Medium about 5 times greater than that of the Sun, which is the $\frac{1}{12867539}$ Part of Gravity; that of the Moon will be the $\frac{1}{2573508}$ Part of Gravity; and therefore by far too ſmall to be ſenſible in the Motion of Pendulums, or any Kind of Static or Hydroſtatic Experiments. In the Motion of the Waters of the Sea only can it produce any ſenſible Effect.

39. To eſtimate the Elevation of Waters which the Earth will produce in the Moon; let us ſuppoſe the Quantity of Matter (Q) in the Earth to be to that in the Moon (*q*) as 40 to 1, that is, Q : *q* :: 40 : 1, and let us firſt ſuppoſe the Earth and Moon of equal Fig. 7. Bulk, and repreſented by A I K and B D E, and the Force (F) of the Earth, at the Surface of the Moon B, will be to the Force (f) of the Moon at the Surface of the Earth A, directly as the Maſſes of Matter in each (becauſe of the equal Diſtances T B and L A), that is, F : f :: Q : *q* :: 40 : 1. Again, let L B be to L C as the Diameter of the Earth to that of the Moon, which is as 365 to 100, then will the Force at B be to the Force at C as L B to L C, which let be as F to *f*,

then $F : f :: 365 : 100$, whence $F = \frac{365f}{100}$; and above we have $F = 40\,\mathrm{f}$, therefore $4000\,\mathrm{f} = 365 f$; and ſo $\mathrm{f} : f :: 365 : 4000 :: 1 : 11$ nearly; that is, *the Power of the Earth to raiſe the Water at the Moon* C, *is to that of the Moon at the Earth* A as 11 to 1, very nearly.

40. If then the Moon can raiſe the Water here 10 Feet, the Earth can raiſe the Water there to the Height of 110 Feet; but becauſe the Moon always turns about her Axis in the ſame Time as ſhe revolves about the Earth, and by this Means ſhews always the ſame Face to the Earth, the Waters (if any there be) will be raiſed on this and the oppoſite Side, and always continue over the ſame Part of the Moon's Surface, ſo that there can be no different Heights of Water there, and conſequently no Tides, except what ſmall ones may happen on Account of the Sun's Attraction, and her various Inequalities of Motion, and Diſtance from the Earth.

41. The Air, which is 860 times lighter than Water, muſt yield in Proportion more eaſily to the attracting Force of the Sun and Moon; and therefore the Tides will in this profound and vaſt *Aerial Ocean* be very conſiderable and importing. But of this the Reader may expect a further Account when we come to treat directly of the *Atmoſphere* in the next Lecture.

LECTURE VI.

Of PNEUMATICS, *or Doctrine of the* AIR, *or* ATMOSPHERE *in general. Of* Artificial *or* Factitious AIR; *the* great Quantity *thereof in* Natural Bodies; *various Experiments relating thereto. Of the* Weight *of the* AIR; *of the Nature of the* BAROMETER *for eſtimating the ſame; an Account of the ſeveral Kinds*, viz. *the* Perpendicular, Diagonal, Horizontal, Pendent, Wheel, *and* Water BAROMETERS. *The beſt Way of* making *the* Common Barometer. *The Nature and Uſe of the* NONIUS, *applied thereto, explain'd. The Uſe of the Barometer* in meaſuring *the* Heights of Mountains, *&c. The* SPRING *or* ELASTICITY *of the Air accounted for, and explain'd. The Nature of the* SEA-GAGE *explain'd. The* ALTITUDE *of the* ATMOSPHERE *determin'd. The Art of* SAILING *in the* AIR *proved impoſſible. The* ABSOLUTE WEIGHT *of the Air determin'd by Experiment. Its* variable Preſſure *on* HUMAN BODIES; *the* Quantity *thereof computed. An Account of fifty Experi-*

Experiments of the AIR-PUMP *relating to the Weight, Spring, and other Properties of the* Air. *A particular Deſcription of the* Air-Pump; *an* Air-Pump *of a* new Invention. *The* DIVING-BELL *explain'd. The* Nature *and* Uſe *of* THERMOMETERS *explain'd. The* Newtonian STANDARD THERMOMETER. Farenheit's *new* Mercurial Thermometer *explain'd.* HYGROMETERS *of ſeveral Sorts explain'd. The Common* AIR-GUN *explain'd. The* MAGAZINE AIR-GUN *particularly deſcribed.*

THAT Part of *Natural Philoſophy* which treats of the *Nature, Properties and Effects of the* ATMOSPHERE, or *Body of Air* encompaſſing the *Earth,* is called PNEUMATICS, from the Greek Word for *Wind* or *Breath.*

THE *Air* is generally eſteemed a *Fluid,* but yet differs from the general Nature of Fluids in three Particulars, *viz.* (1.) In that it is *compreſſible,* which Property no other Fluid has. (2.) It cannot be *congeal'd,* or any how *fixed,* as all other Fluids may. (3.) It is of a *different Denſity* in every Part, decreaſing from the *Earth*'s Surface upwards; whereas other Fluids are of an uniform Denſity throughout. The

Air is therefore a Fluid *ſui generis*, if it be properly any Fluid at all (LXXXV.)

THAT the Air was created at firſt with the Earth itſelf, is not to be doubted; and that ever ſince there has been a *conſtant Generation* of Particles of Air by the mutual Action of Bodies upon each other, as in *Fermentations*, and all Kinds of *natural and artificial Chemiſtry*, Sir *Iſaac Newton* thinks very reaſonable to ſuppoſe; and Mr. *Boyle* has given numerous Experiments relating to the Production of *artificial* or *factitious Air* (LXXXVI).

(LXXXV.) What is here ſaid of the incongealable Quality of the Air, relates to the Impoſſibility of changing it from a *Fluid to a Fixed State* by Cold, as Water is congeal'd or converted into Ice; and melted Metals are brought to their fixed State: And in this particular limited Senſe, the Air is *incongealable*, or uncapable of Fixation. But yet it is not abſolutely ſo; for we find by various Experiments, that Air has a fixed State in the Compoſition of natural Bodies, from which, when ſet at Liberty, it becomes a fluid elaſtic Air, like the common Air; and this again, from a *Fluid*, may be reduced to a *fixed State* in Compoſition with other Matter, though not *per ſe*; for we know, as yet, of no fixed Body conſiſting entirely of Air.

(LXXXVI.) 1. Since Air is abſolutely neceſſary for the Life of Man, and moſt Animals, yea, and Vegetables too, it was neceſſary at the firſt Formation of the Earth to render it a Habitation for Animals, and a proper Matrix for the Production of Plants. Now ſince there is a conſtant Generation of Air from all terreſtrial Subſtances (as we ſhall ſhew by and by) it follows, that the original Atmoſphere muſt be always increaſing

THAT the Air is a *heavy or ponderous Body*, muſt follow from the Nature of the

in Quantity and Bulk, unleſs we ſuppoſe all that is generated is again abſorb'd or refixed in the Subſtance of Bodies. And this alternate Tranſmutation of the State of Air is extremely manifeſt from numberleſs Experiments which have been made by Mr. *Boyle*, and Dr. *Hales*, of which I ſhall here give an Account of ſome of the principal of both Kinds, as follows.

2. The Production of artificial or factitious Air is cauſed either (1.) by ſlow Degrees from Putrefactions and Fermentations of all Kinds; or (2.) more expeditiouſly by ſome Sorts of chymical Diſſolutions of Bodies; or (3.) and laſtly, almoſt inſtantaneouſly by the Exploſion of Gunpowder, and the Mixture of ſome Kinds of Bodies. Thus, if Paſte or Dough with Leaven be placed in an exhauſted Receiver, it will, after ſome Time, by Fermentation, produce a conſiderable Quantity of Air, which will appear very plainly by the Sinking the Quickſilver in the Gage. Thus alſo any Animal or Vegetable Subſtance, putrifying *in Vacuo*, will produce the ſame Effect.

3. Gunpowder, fired *in Vacuo*, inſtantly generates a large Quantity of Air in the Receiver, which cauſes the Quickſilver to ſubſide. And in the remarkable Experiment of Dr. *Slare*, half a Drachm of Oil of Carraway-Seed, poured upon a Drachm of the Compound Spirit of Nitre, produced ſuch a prodigious Quantity of Air, as inſtantly blew up the Receiver, which was ſix Inches in Diameter, and eight Inches deep. The Preſſure, therefore, of the Atmoſphere on the exhauſted Receiver, which it overcame, was above 400 *lb.* reckoning 15 *lb.* to a ſquare Inch.

4. But Dr. *Hales*, in his *Vegetable Statics*, has greatly excelled in his Experiments of this Kind, and in the Methods of making them: One of which was by *Diſtillation*, the other by *Fermentation*. That by *Diſtillation* is as follows: the Matter to be diſtill'd is put into the Retort *r*, and then at *a* is cemented very faſt the

Plate XXVII. Fig. 1.

Matter of which it doth conſiſt; and ſince thoſe Particles ariſe from Bodies of every

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Glaſs Veſſel ab, which was very capacious at b, and had an Aperture cd, or Hole at the Bottom. The Bolt-Head ab being thus immerſed in Water, with one Leg of an inverted Syphon put up as far as z, the Water would riſe in the Bolt-head, and drive out the Air through the Syphon, which being taken out, the Water will remain in the Veſſel to the Part z; at the ſame Time, while the Bolt head is under Water, it is placed in the Veſſel xx, which with the Bolt-head and Retort is carried to the Chymical Furnace, where the Retort has the Heat and Fire gradually communicated to it, and the Bolt-head ab and Veſſel xx well ſcreen'd from the Heat of the Fire.

5. As the Matter diſtill'd, all except the Air, would go down into the Water of the Bolt-head and Veſſel; the Air that was generated or deſtroyed by the Proceſs would be ſhewn by cauſing the Surface of the Water in the Bolt-head to ſtand below or above the Point z, as at y, when all was ſet aſide till it became quite cold. Thus if the Body diſtilling generates Air of an elaſtic Quality, that added to the former will not permit the Water y to riſe ſo high as z, and the Space between z and y below will ſhew how much Air was produced from its fixed State.

6. But if, when all is cold, the Surface of the Water y be ſeen above the Point z, it then ſhews that the diſtill'd Body did deſtroy, that is, imbibe or abſorb, a Part of the natural Air above z; and the Space between z and y, fill'd with Water, will ſhew what Quantity was changed from a repellent elaſtic to a fixed State, by the ſtrong Attraction of the abſorbing Particles of the diſtill'd Body. This Quantity of generated or abſorbed Air it is eaſy to meaſure in Cubic Inches, by ſtopping the End of the Bolt-head with a Cork, and then from a Quantity of Water of a known Weight to fill it firſt to z, and afterwards to y; and the Difference of Weight in the two Bulks of Water gives the Number of Cubic

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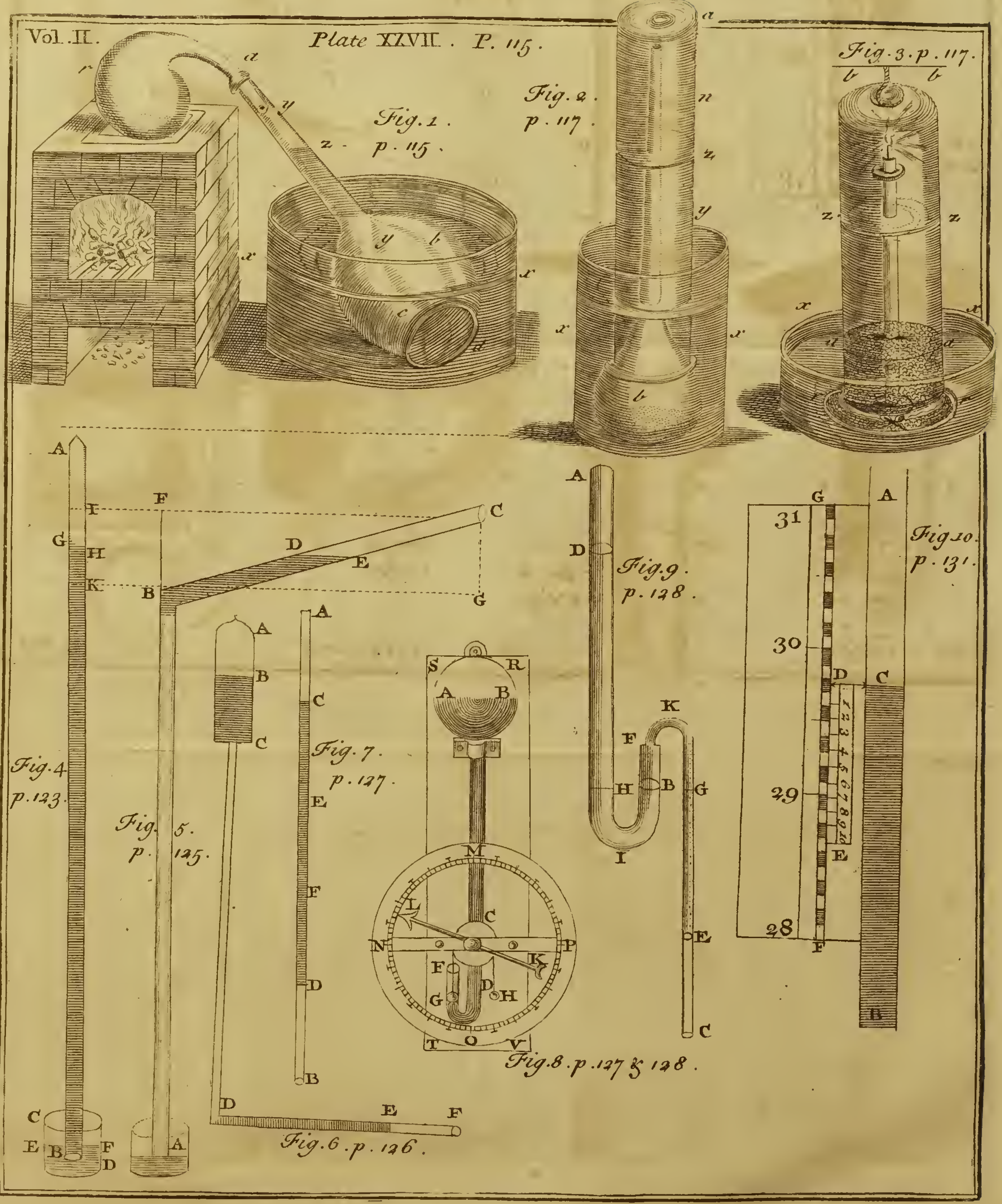

Fig. 1. p. 115.

Fig. 2. p. 117.

Fig. 3. p. 117.

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Fig. 5. p. 125.

Fig. 6. p. 126.

Fig. 7. p. 127.

Fig. 8. p. 127 & 128.

Fig. 9. p. 128.

Fig. 10. p. 131.

Kind in or upon the Earth, 'tis evident the conſtituent Parts of Air are of a moſt

Inches from a Table of ſpecific Gravities, in the Manner we have formerly ſhewn.

7. The other Method which the Doctor made uſe of for eſtimating the ſurprizing Effects of Fermentation ariſing from various Mixtures of ſolid and fluid Subſtances, in generating and abſorbing Air, was as follows: He put the Ingredients into the Bolt-head b, and then run the long Neck thereof into a tall cylindric Glaſs *a y*, and inclining both almoſt horizontally in a arge Veſſel of Water, the Water ran into the Veſſel *y*, and driving out Part of the Air would poſſeſs its Place upon turning them up and placing both in a Veſſel of Water *x x*, as you ſee in the Figure, where the Surace of the Water ſtands in the inverted Glaſs *a y* at he Point *z*.

Plate XXVII. Fig. 2.

8. If the Ingredients generated Air, then the Water vould fall from *z* to *y*, and the empty Space *z y* was qual to the Quantity of generated Air; but if on Fernentation they abſorbed or fixed the active Particles of Air, then the Surface of the Water would aſcend from to *n*; and the Cylinder *z n* would be the Bulk of Air bſorb'd, which is eaſily known in Cubic Inches.

9. When the Subjects for trying theſe Experiments ere a burning Candle, burning Brimſtone, Nitre, Gunowder fired, living Animals, &c. the Doctor uſed to nake uſe of a Pedeſtal, on the Top of which was a late whereon he laid the Matter to be fired; then inerting the tall cylindric Glaſs over it, and drawing the Vater up to *z z* with an inverted Syphon, he ſet fire to ne Matters lying on the Plate by means of a Burninglaſs concentering the Sun's Rays in its Focus upon the me. See the Figure.

Fig. 3.

10. But the Way that I make uſe of, and which is e moſt eaſy and expeditious poſſible, is inſtead of havg the cylindric Glaſs cloſe upon the Top at b b, to ve it open by a ſmall Neck, on which a braſs Cap is mented with a Female-Screw to receive a Stop-Cock,

heterogeneous Nature, and infinitely various in their ſpecific Gravities: Whence alſo it

to take off the Communication of the external Air when Occaſion requires. Thus the Uſe and Trouble of the Syphon is ſuperſeded; and in caſe of noxious Fumes, Vapours, *&c.* from *Aqua fortis*, burning Brimſtone, *&c.* a Syringe ſcrew'd on to the Stop-Cock will draw off the Air, and raiſe the Water to what Height you pleaſe, without the cumberſome Uſe of a large Pair of Bellows, as the Doctor made uſe of.

11. I ſhall here ſubjoin the Quantity of Air which various Subſtances produce by Diſtillation, which I have collected from the Doctor's Experiments, and reduced to Cubic Inches.

A Cubic Inch of Hog's Blood	33
Tallow	25
Deer's Horn	234
Oyſter-Shell	324
Heart-Oak	216
Peaſe	396
Amber	270
Oil of Anniſeed	22
Oil of Cloves	88
Honey	144
Yellow Bees Wax	54
Coarſeſt Sugar	126
Newcaſtle Coal	360
Freſh Earth	43
Antimony	28
Pyrites	83
Sea-Salt mixed with Bone-Calx	64
Nitre, with *ditto*	180
Sal Tartar ditto	224
Rheniſh Tartar	504
Calculus humanus	645
Stone in the Gall-Bladder	648

12. Theſe are the principal Experiments by Diſtillation. Others were made by Fermentation in various

will follow, that as the Matter which compoſes the Body of Air, or Atmoſphere, is

Mixtures, ſome of which generated Air, others abſorb'd it, and ſome did neither generate nor abſorb Air. The principal Subjects, which of themſelves abſorb Air, are the Fumes of burning Brimſtone or Matches, the Flame and Fumes of a burning Candle, the Breath of living and expiring Animals, as Rats, Mice, *&c.*

13. Thus the Doctor found that Linen Matches dipped in melted Brimſtone, and fired under a Glaſs in a Quantity of 594 Cubic Inches, abſorb'd 150, which was full one Fourth of the Whole. A Candle burning till it went out, its fumes afterwards conſumed $\frac{1}{11}$ Part of the whole Quantity of Air, which was 594 Cubic Inches. A half-grown Rat expired in the confined Air in ten Hours, and abſorb'd 45 Cubic Inches of Air, which was $\frac{1}{13}$ Part of the whole 594.

14. From what has been ſaid we ſee with how much Reaſon Sir *Iſaac Newton* philoſophized on this Subject in the following Words. " True permanent Air ariſes
" by Fermentation or Heat from thoſe Bodies the Chy-
" miſts call *fixed,* whoſe Particles adhere by a ſtrong
" Attraction, and are not therefore ſeparated and rari-
" fied without Fermentation; thoſe Particles receding
" from one another with the greateſt repulſive Force,
" and being moſt difficultly brought together, which
" upon Contact were moſt ſtrongly united." And again: " Denſe Bodies, by Fermentation, rarify into
" ſeveral Sorts of Air; and this Air by Fermenta-
" tion, and ſometimes without it, returns into denſe
" Bodies." See his *Optics,* Query 30, 31.

15. Now ſince Air is a heavy Body, a Cubic Inch whereof weighs very near $\frac{2}{7}$ of a Grain, it follows, that Air in its fixed State in Bodies makes a Part of their Subſtance, and in ſome of them a very great Part too, as is known from the Quantity and Weight of the Air diſcharged upon the Analyſis of the Bodies. This has eſcaped the Obſervation of Chymiſts, who have hitherto taught that all Bodies were ultimately reſolveable into

always variable, ſo will its Weight or Gravity be likewiſe; as we conſtantly experi-

what they call *Four Elements* only, viz. *Water*, *Oil*, *Salt*, and *Earth*. But by the following Table it will appear, that Air is an Element of Natural Bodies in as proper a Senſe as any of the other.

16. In the firſt Column of this Table you have the Bulk of the Body in Cubic Inches and Parts; in the ſecond, the Number of Cubic Inches of generated Air; in the third is the Weight of the Body in Grains; in the fourth is the Weight of the generated Air; and the fifth ſhews what Part of the Whole the Air makes.

	C. Inch.	*C. Inch.*	*Grs.*	*Grs.*	*Prop.*
Deer's Horn,	$\frac{1}{2}$	117	241	33	$\frac{1}{7}$
Oyſter-Shell,	$\frac{1}{2}$	162	266	46	$\frac{1}{6}$
Heart of Oak,	$\frac{1}{2}$	108	135	30	$\frac{1}{4}$
Indian Wheat,		270	388	77	$\frac{1}{5}$
Peaſe,	1	396	318	113	$\frac{1}{3}$
Muſtard-Seed,		270	437	77	$\frac{1}{6}$
Amber,	$\frac{1}{2}$	135	135	38	$\frac{10}{35}$
Dry Tobacco,		153	142	44	$\frac{1}{3}$
Honey with *Calx* of Bones,	1	144	359	41	$\frac{1}{9}$
Yellow Wax,	1	54	243	15	$\frac{1}{16}$
Coarſe Sugar,	1	126	373	36	$\frac{1}{10}$
Newcaſtle Coal,	$\frac{1}{2}$	180	158	51	$\frac{1}{3}$
Nitre with *Calx* of Bones,	$\frac{1}{2}$	90	211	26	$\frac{1}{8}$
Rheniſh Tartar,	1	504	443	144	$\frac{1}{3}$
Calculus humanus,	$\frac{1}{4}$	516	230	147	$\frac{1}{2}$

17. Thus we ſee that different Bodies contain different Quantities of fixed Air, from a Sixteenth to a Half of the whole Subſtance. From hence we may be fully ſatisfied of the Truth of Sir *Iſaac Newton*'s Reaſoning in the 31ſt *Query* of his *Optics*, in theſe Words: "The "Particles, when they are ſhaken off from Bodies by "Heat or Fermentation, ſo ſoon as they are beyond "the Reach of the Attraction of the Body, recede from

ence by the BAROMETER, of various Kinds and Structure. (LXXXVII).

“ it, and alſo from one another, with great Strength,
“ and keep at a Diſtance, ſo as ſometimes to take up
“ above a Million of Times more Space than they did
“ before in the Form of a denſe Body: Which vaſt
“ Contraction and Expanſion ſeems unintelligible by
“ feigning the Particles of Air to be ſpringy and ra-
“ mous, or roll'd up like Hoops, or by any other
“ Means than by a repulſive Power.”

18. That the Particles of Air cannot be thus coil'd up and detain'd in their elaſtic State in the Subſtance of Bodies, is eaſy to be ſhewn from Calculation. Thus, for Inſtance, one Cubic Inch of Oak yields 216 Cubic Inches of Air: Now ſuppoſe the Preſſure of the Atmoſphere be on every Square Inch about 15 *lb.* (as we ſhall ſhew) then in order to compreſs 216 Cubic Inches into one Cubic Inch, the Weight of 216 Times 15 *lb.* or 3240 *lb.* which would be the Force to confine it on each Side the Cube, which, as it has ſix Sides, will require 6 × 3240 = 19440 *lb.* or near twenty thouſand Weight, to confine this Air in its elaſtic State in one Cubic Inch, ſuppoſing it to be all Air; but, as it is not, the Force will be greater ſtill. This Force therefore of 19440 *lb.* muſt be exerted in every Cubic Inch of the Oaken Tree, which would rend it in pieces with a vaſt Exploſion. It is therefore not to be doubted but Air in Bodies does exiſt in a fixed and unelaſtic State; and that it is rouſed, and put into an active repellent State, by means of Fire and Fermentation.

19. They who would ſee the numberleſs Uſes that may be made of this important Doctrine of artificial Air, and the ſurprizing Scenes of Knowledge which it lays open in the moſt abſtruſe and difficult Parts of *Phyſics*, may conſult all the latter Part of the invaluable Book above mentioned, *viz.* Dr. *Hale*'s *Vegetable and Chymico-Statical Experiments*, ſome of which we ſhall take notice of alſo in the Sequel of theſe Notes.

(LXXXVII.) 1. The Weight of the Air is manifeſt from Reaſon and various Experiments. The Particles

SINCE the Particles of Air are ſuch as being ſeparated from Bodies beyond the

are affected by the Power of the Earth's Attraction, and muſt therefore all gravitate or tend towards its Centre, which is what conſtitutes Weight in them, and all other Bodies. The Experiments to ſhew the Weight of the Air are numerous which we ſhew on the Air-Pump, among which one is abſolute and very exact, by weighing it in a Balance, in the ſame manner as all other heavy Bodies are weigh'd.

2. The Method I take for this is, I believe, the moſt exact and nice that can poſſibly be thought of. For ſince (as we have ſhewn) the Friction of the Balance is in Proportion to the Weight with which it is charged, the leſs the Weight is the leſs will be the Friction, and conſequently the more nice and exquiſite will be the Experiment. In order to this I take a very thin large *Florence* Flaſk, whoſe Capacity is exactly known in Cubic Inches: This I exhauſt of all the Air as near as can be, and then hang it to the end of a very fine and exact Hydroſtatic Balance, which I counter-balance by Grain-Weights in a Scale hanging from the other End. When the Equilibrium is nicely obtain'd, I lift up the Valve, and let the Air ruſh into the Flaſk, which is ſenſibly heard, and ſeen to gravitate in the Glaſs, by cauſing it gradually to deſcend till it be fill'd with Air, and will then preponderate greatly. Then to reſtore the Equilibrium, I find by Experience 'tis neceſſary to add about 8 Grains for every Pint the Flaſk contains; which ſhews that a Gallon of Air weighs about a Dram, and a Buſhel an Ounce *Troy*; and becauſe one Pint = 28 Cubic Inches nearly, therefore one Cubic Inch of Air weighs $\frac{8}{28} = \frac{2}{7}$ of a Grain, at a Mean.

3. As the Air is a heterogeneous Fluid, it will vary in its Weight according to its different component Parts, and alſo according to its different Altitudes, which it muſt have as an elaſtic and fluctuating Fluid. Since few Bodies are lighter than Water, and that Water is moſt eaſily rarified into Vapour, it follows, that the Atmoſphere fill'd with aqueous Particles will be lighteſt,

Sphere of corpuſcular Attraction, are ſtrongly repell'd from thoſe Bodies; this Re-

as we generally find it is in moiſt rainy Weather; and alſo that it muſt oftener be in this light than in a heavier State: And that Inſtrument which ſhews the Variation of the Air's Gravity, or its different Weight at different Times, is call'd a BAROMETER, of which there are various Kinds, which are here deſcribed; but I ſhall firſt give an Account of the moſt ſimple Structure or Form of thoſe Inſtruments, which is as follows. A Glaſs Tube, hermetically ſeal'd at one End, is to be fill'd with Quickſilver, well defecated and purged of its Air: The Finger being then placed on the open End in immediate Contact with the Mercury, ſo as not the leaſt Particle of Air is admitted, the Tube is inverted, and carefully immerſed with the Finger on the open End in a Baſon of the ſame prepared Mercury; then upon removing the Finger, the Mercury in the Baſon will join that in the Tube; and the ſaid Column of Mercury in the Tube will be ſeen immediately to ſubſide, or ſink down to a certain Pitch or Altitude, if the Tube be above 31 Inches long, as it ought to be.

Plate XXVII. Fig. 4.

4. Let A B be ſuch a Tube of 34 Inches Length, and $\frac{1}{4}$ of an Inch in Diameter, (as it ought to be for this Purpoſe) hermetically ſeal'd at A, and open at B; let C D be the Baſon of Mercury, in which the Tube is immerſed inverted, the Surface of the Mercury in the Baſon E F, and in the Tube G H. Now, 'tis eaſy to underſtand, that if all this could be perform'd *in Vacuo*, as ſoon as the Tube was inverted, all the Mercury would deſcend into the Baſon, becauſe as a heavy Body it muſt tend towards the Centre of the Earth, till it meets with ſome Obſtacle, as the Baſon, to obſtruct its Motion, and ſupport it. I ſay, all this would happen *in Vacuo*, unleſs we can ſuppoſe any Power in the Tube ſufficient to ſuſtain the Column of Mercury; now there can be no ſuch Power but that of *Cohesion*, which indeed, in Tubes of a ſmall Bore, has been found able to ſuſtain it; but in ſo large a Bore, as we

pellency being mutual between the Particles, they do by this means conſtantly

ſuppoſe this Tube to have, that Power is by far too ſmall to ſupport ſo heavy a Column, which muſt therefore of Courſe ſink into the Baſon; and ſo ſtand upon the ſame Level in the Baſon, and in the Tube.

5. But ſince the Mercury does not totally ſubſide when this Experiment is made in the Air, the Column which remains in the Tube muſt owe its Suſpenſion to the Air as its Cauſe, ſince nothing within nor without the Tube can be ſuppoſed (with any Shew of Reaſon) to produce ſuch an Effect, beſides itſelf. Now allowing the Air to be a gravitating Fluid, it muſt neceſſarily cauſe ſuch an Effect, as the Suſpenſion of Mercury in the Tube; for by its Gravity a Force of Preſſure muſt be produced on all the Surface of Mercury in the Baſon, which is contiguous to it, and on every Part equally. Now ſince the Mercury in the Baſon is *in Equilibrio* with that in the Tube, it is plain the Weight of the Mercury in the Tube, and the Weight of the Air upon every circular Area of the Surface of the Mercury in the Baſon, equal to the Orifice of the Tube, muſt be equal, for elſe they could not balance each other, as we find they do; the Column of Mercury therefore is ſuſtain'd by the Counter-Preſſure of a Column of Air of the ſame Baſe, and whoſe Altitude is equal to that of the Atmoſphere.

6. That the Weight of the Columns of Air and Mercury, we have now been ſpeaking of, are preciſely equal to each other, will be farther evident, if we conſider, that upon Suppoſition the Quickſilver were thoroughly purg'd from Air, when it ſubſides in the Tube, it muſt leave a *Vacuum* in all that Part of the Tube above it, and ſo there is nothing to act upon its upper Surface to depreſs it; it will therefore always ſink or riſe to ſuch an Altitude, as the various Gravity of the Air requires, and of which it is therefore an adequate Expreſſion or Meaſure, as its Nature imports. This Invention was owing to that happy *Italian* Genius *Torricelli*, a Diſciple

repel each other, and therefore they muſt obſerve an equal Diſtance among them-

of the famous *Galileo.* And hence it is very often call'd the *Torricellian Tube,* and the *Torricellian Experiment,* &c.

7. Since, as we have ſhewn, this ſuſpended Column of Mercury exactly indicates the Gravity of the Air at all Times, it has employ'd the Attention of all Mankind, who very ſenſibly find themſelves affected with the different State of the Air; but more eſpecially has it merited the Conſideration of Philoſophers, who have taken all Opportunities to explore, by this Means, the two extremes of the Air's Gravity, *viz.* when it is leaſt and greateſt of all, by obſerving the leaſt and greateſt Altitude of this mercurial Column, which by long Experience we find to be very nearly between 28 and 31 Inches, it being very rarely leſs or more than thoſe Heights; whence 29 ½ Inches is fixed upon as the Mean-Altitude, expreſſive of the Mean-Gravity of the Air, which therefore let be repreſented by B H, and let the greateſt Altitude be B I, and the leaſt B K, then is I K call'd the *Scale of Variation.*

8. Now as there is a Space of 3 Inches in the perpendicular Tube for the Mercury to riſe and fall in, and ſince Fluids will always have the ſame Altitude in a Tube of any Form or Inclination; Artiſts have contrived Methods to increaſe the Motion of the Mercury, by making it move over a larger Space; and thus have invented ſeveral different Forms of Barometers, the principal whereof I ſhall here deſcribe. That which, next to the upright one, is in moſt Eſteem, is called the *Diagonal Barometer*, and is that in *Fig.* V. where A B C is a Tube ſeal'd at C, immerſed in Mercury at A; this Tube is perpendicular from A to B, where the Scale of Variation begins; from thence it is bent into B C, making an acute Angle F B C. This Part B C proceeds to the higheſt Limit in the Scale of Variation, *viz.* I C; and conſequently while the Mercury riſes from K to I in the common Barometer, it will move in

Plate XXVII. Fig. 5.

ſelves. When, therefore, thoſe aerial Particles are obliged by any impreſs'd Force

this from B to C (See *Annotat.* LXIX. 4.) and ſo the Scale will by this means be inlarged in the Proportion of B C to F B = I K, *i. e.* of the Diagonal to the leaſt Side of a Parallelogram.

9. The Fault which this Form is ſubject to is this, that as the Quickſilver is a very heavy Body, and ſupported on the Part B C, like a Body on an Inclined Plane, it muſt have a great Degree of Friction, which is the more ſo as the Part B C is more oblique, and conſequently the very ſmall and nice Variation of the Air's Preſſure cannot be ſo well ſhewn in this Form, where the Motion is not abſolutely free; and it very often happens, from the Inclination of the Part B C, that the Quickſilver divides into ſeveral Parts, and ſo requires the Trouble of filling the Tube anew too frequently. This Form is the Invention of Sir *Samuel Moreland.*

10. Another Form of the Barometer for enlarging the Scale of Variation, was contrived by Mr. *Caſſini*, and afterwards compleated by Mr. *John Bernouilli.* It conſiſts of a Tube A C D F, ſeal'd on the upper End A, and bent to a Right-Angle at D, whence it has the Name of the *horizontal rectangular Barometer.* The Mercury ſtands on both Legs from E to B. The Scale of Variation from A to C, is here made of a larger Part, and then it is evident in moving 3 Inches from A to C, it will move through ſo many Times 3 Inches in the ſmall Leg D F, as the Bore of D F is leſs than the Bore of A C, whence the Motion of the Mercury at E muſt be extremely ſenſible, but the Inconvenience here too is, that the Mercury is very apt to break off in the Leg E, as alſo to run out at the End F; beſides that, here alſo is a great Degree of Friction, and not only that, but the Part D F being of a very ſmall Bore, the Attraction of Coheſion will a little impede the free Motion of the Mercury therein. Plate XXVII. Fig. 6.

to approach nearer to each other, the *re-pulſive Power* will re-act or reſiſt the ſaid

11. The next Sort of Barometer that I ſhall deſcribe, is call'd the PENDENT BAROMETER, becauſe being a ſingle Tube only, it is hung up by a String at the End A. This Tube is a little tapering or conical, the End A being a little leſs than B; it is ſeal'd at A, and fill'd with Mercury from A to F = 31 Inches; then when the Tube is inverted, the Mercury, if the Gravity of the Air be not greateſt, will ſink down in the Tube and abide in that Part C D, which will admit its Length or Altitude C D to be the ſame with that in the Common Barometer. Thus by reaſon of the conical Bore of the Tube, as the Air grows lighter the Mercury will deſcend, till when it is lighteſt of all, the Mercury will ſtand from the lower Part of the Tube B to E, ſo that B E = 28 Inches. Conſequently the Mercury will, in ſuch a Tube, move from A to E, 32 Inches, if the Tube be 5 Feet or 60 Inches long, and ſo the Scale A E is here above 10 Times greater than in the Common Barometer. The Fault of this Barometer is, that the Tube being of a very ſmall Bore, the Attraction of Coheſion will be very conſiderable, and prove an Impediment to that Freedom of Motion as is neceſſary to ſhew a very ſmall Variation of the Air's Weight; for which Reaſon, when you ſhake the Tube a little, the Mercury will not ſettle again where it was, or have the ſame Altitude as before; beſides, that by its Weight it is very apt to break off in the lower Part, and fall out of the Tube. We owe this Invention to Mr. *Amontons*, who firſt publiſhed it in *A. D.* 1695.

Plate XXVII. Fig. 7.

12. The next Barometer which I ſhall mention is call'd the WHEEL BAROMETER, from its Structure, which is eaſily underſtood from the Figure. Where A B D G repreſents the Quickſilver in a Glaſs Tube, having a large round Head or Ball at Top, and turn'd up at Bottom to F; upon the Surface of the Mercury in the recurv'd Leg there is placed an Iron Ball G with

Force with an equal *Momentum*; and thus cauſe what we call the RENITENCY, ELAS-

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a String going over a Pulley C D, and is ballanced by another Weight H hanging freely in the Air. As the Surface of the Mercury at A B is very large, and that at G very ſmall, the Motion of the Quickſilver, and conſequently of the Ball G, will at Bottom be very conſiderable; but as the Weight G moves up and down, it turns the Pulley C D, and that a Hand or Index K L, by the Diviſions of a large graduated Circle M N O P; by which means the minuteſt Variations of the Air are plainly ſhewn, if the Inſtrument be ſo very accurately made that the Friction of the ſeveral Parts be inconſiderable. This is one of the many curious Inventions of Dr. *Hooke*.

13. Theſe are the principal Contrivances hitherto invented for enlarging the Scale of Variation in ſimple Mercurial Barometers. There are other Inventions of compound Barometers, *viz.* ſuch as are made of Mercury and Water, or other Liquors for that Purpoſe; but they are ſo difficult to make, ſo faulty when made, and ſo troubleſome to uſe, that it is not worth while to deſcribe them. However, as the Reader may have an Idea of one of the beſt Sort, I ſhall here give him that which owes its Invention to the Reverend Mr. *Rowning*, together with his Demonſtration of its Theory.

Plate XXVII. Fig. 9.

14. A B C is a compound Tube ſeal'd at A, and open at C, empty from A to D, fill'd with Mercury from thence to B, and from thence to E with Water; let G, B, H, be in an horizontal Line, then it is plain from the Nature of the Syphon, that all the compound Fluid contain'd in the Part between H and G, muſt ever be *in Equilibrio* with itſelf be the Weight of the Air what it will, becauſe the Preſſure at H and G, muſt always be equal. Whence 'tis evident, that the Column of Mercury D H is *in Equilibrio* with the Column of Water G E, and a Column of Air of the ſame Baſe conjointly, and will therefore vary with the Sum

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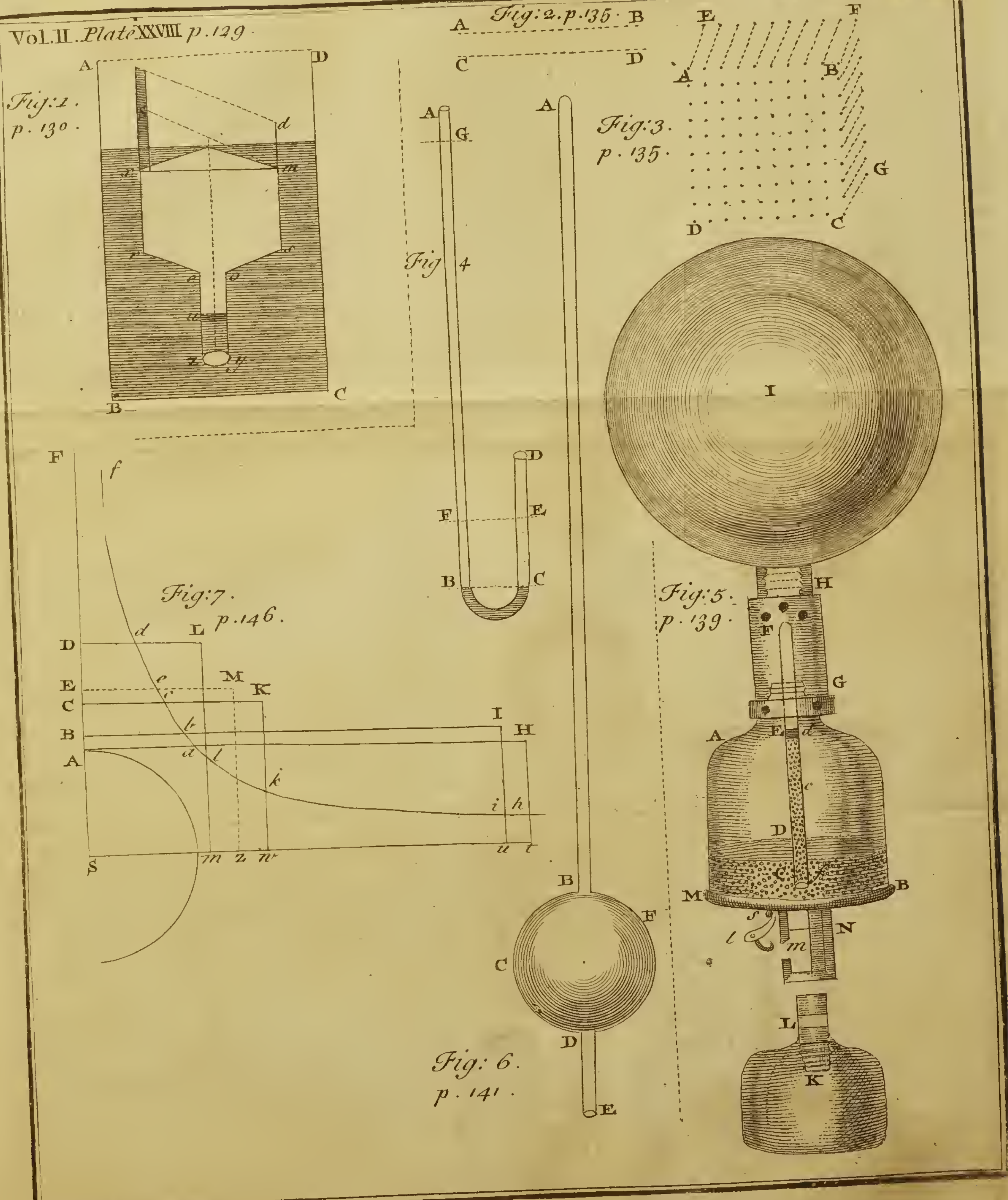

Fig: 1. p. 130.

Fig: 2. p. 135.

Fig: 3. p. 135.

Fig 4

Fig: 5. p. 139.

Fig: 6. p. 141.

Fig: 7. p. 146.

TICITY, or SPRING of the Air; which is ſo ſenſible by the common Experiment of

of the Variations of each of theſe; all which muſt now be computed.

15. The Variation of the Weight of the Air, which we will call V, is meaſured by the Space which the Mercury moves in the common Barometer in a given Time. Let x be the Space which the Water at E moves through in the ſame Time, and let the Diameter of the Tube A F be to that of the Tube F C as D to 1, hen will the Space moved through at B be as $\frac{x}{DD}$, and therefore G E the Difference of the Legs E K and K B, will vary in its Weight by $x + \frac{x}{D^2}$. Alſo ſince he Space moved through by the Mercury at B and D is is $\frac{x}{D^2}$, the Difference D H will vary its Weight by $\frac{2x}{D^2}$. But this *Variation* of Weight is equal to both the ormer, and ſince $x + \frac{x}{D^2}$ is an Altitude of Water, if we put m to 1 as the Specific Gravity of Mercury to Water, we ſhall have $m : 1 :: x + \frac{x}{D^2} : \frac{x + \frac{x}{D^2}}{m} =$ Altitude of Mercury of the ſame Weight, conſequently, $\frac{2x}{D^2} = V + \frac{x + \frac{x}{D^2}}{m}$; which Equation, when reduced, gives $x = \frac{VD^2m}{2m - D^2 - 1}$, which gives this Analogy; as $x : V :: mD^2 : 2m - D^2 - 1$, ſo is the Scale of Variation in this, to that in the common Barometer.

16. Hence if $m = 14$, and $D = 1$; we have $x : V :: 14 . 26 :: 7 : 13$; which ſhews that when the Tubes A F and F C are of an equal Bore, the Variation in

this is leſs than that of the common Barometer in the Ratio of 7 to 13. If $2m - D^2 - 1 = 0$, or $2m - 1 = D^2$, then $D = \sqrt{2m - 1} = 5,2$; whence it appears that when the Diameter of A F is to that of F C as 5,2 to 1, the Variation x will be infinite in reſpect of that in the common Barometer. If $D = 5$, then $x : V :: 175 : 1$; which ſhews how very large the Scale of Variation in this Barometer may be made in Compariſon of the common one. But I believe ſuch a Structure as this will afford more Pleaſure in Speculation than in Practice; and when all is done, the Barometer of the *common Form*, as it is moſt ſimple, ſo it will be found the moſt eaſy and accurate of all others.

17. Before I conclude this Article, I ſhall juſt mention the Barometer invented by the Rev. Mr. *Caſwell* of *Oxford.* Suppoſe A B C D be a Bucket of Water, in which is placed the Baroſcope *x r e z y o s m*, which conſiſts of a hollow Body *x r s m*, and Tube *e z y o*, made of Braſs, Tin, Glaſs, *&c.* The Bottom of the Tube *z y* has a Lead-Weight to ſink it, ſo that the Top of the Body may juſt ſwim even with the Surface of Water, by the Addition of ſome Grain-Weights. As the Inſtrument is put into the Water, with the Mouth downwards, the Water aſcends into the Tube to the Height of *y u*; there is added on the Top a ſmall concave Cylinder, or Pipe, to ſuſtain the Inſtrument from ſinking to the Bottom when the Air becomes heavier; *m d* is a Wire, and *m s*, *d e*, are two Threads, oblique to the Surface of the Water; of theſe Threads there may be ſeveral; and as the Water juſt touches the Top or Crown of the Inſtrument, when the Altitude of the Mercury is leaſt in the Common Barometer, ſo as the Air increaſes in Weight, the Inſtrument ſinks in the Water, and a ſmall Bubble is formed on the Thread, which continually aſcends and deſcends through all the Length of the Thread. From a Calculation on the Theory, it appears, that this Barometer is above 1200 Times more exact than the Common Barometer. See the whole Calculation in the Profeſſor's own Words in the *Phil. Tranſactions.*

Plate XXVIII. Fig. 1.

18. Though I have made and tried the Barometer above deſcribed, and find it to anſwer the Theory very

well, yet is it not fit for common Uſe, becauſe it can only ſhew the extreme minute Variations of the Air's Gravity for the *preſent Time*, by reaſon it is affected by the Heat as well as Weight of the Air. While the Degree of Heat remains the ſame, nothing can exceed this Inſtrument as a Barometer; but as the Heat of the Air varies, ſo will the Elaſticity of the included Air, which therefore will cauſe the Inſtrument to vary its Gravity, while that of the Air remains the ſame, and ſo cannot be of conſtant Uſe.

19. I have already hinted that the Common Barometer, after all, is the beſt Inſtrument to meaſure the Air's Gravity; which that it may do to the greateſt Perfection, the following Things are neceſſary. (1.) That the Tube be at leaſt of $\frac{1}{4}$ of an Inch Bore; $\frac{1}{3}$ of n Inch is a good Size. (2.) The Tube ought to be new, clean, and dry within when fill'd; in order to his, the Tube ſhould be hermetically ſealed at both Ends at the Glaſs-Houſe when made; one End of which may be cut off with a File when you intend to uſe it. (3.) The Diameter of the Ciſtern that holds he Mercury, in which the Tube is immerſed, ſhould be as large as conveniently may be, that the Mercury herein may have nearly at all times the ſame Altitude; otherwiſe the Index will not be true. (4.) The Mercury muſt be very pure, and free from any Mixture of Tin, Lead, or other Metal. (5.) It ought to be purged from Air entirely, as it may be by boiling it, and filling the Tube with it while boiling hot nearly. (6.) The Tube muſt be heated hot when fill'd, to avoid breaking by the boiling Mercury. (7.) It ſhould be rubb'd very hard, to excite the Electric Virtue, which will expel the Particles of Air from the Surface within. (8.) There ought to be a *Nonius* (as it is call'd) applied to the Index of the graduated Plate, to meaſure more accurately the Riſe and Fall of the Mercury.

20. This Artifice is of ſingular Uſe in this and many other Caſes. It bears the Inventor *Nonius*'s Name, and its Nature and Manner of applying it is as follows. A B is the upper Part of the Barometer, in which the Surface of the Mercury is at C. F G is the uſual Plate of 3 Inches Extent, from 28 to 31; and D E is the

Plate XXVII. Fig. 10.

the ſmall Plate call'd the *Nonius*, ſo contrived as to ſlide by the other in ſuch manner that its Index D may be always ſet on one Part to the Surface of the Mercury, and on the other End pointing to the Diviſion in the Scale of Inches correſponding thereto. Again, the *Nonius* is divided into 10 equal Parts, which together are equal to 11 of the Diviſions of the Scale; that is, D E = 11 Tenths of an Inch; and conſequently each ſmall Diviſion of the *Nonius* is equal to 1,1; two of them to 2,2; three of them to 3,3; and ſo on. Whence 'tis eaſy to obſerve, that if the Index D points between any two Diviſions of the Scale, as here between 29,7 and 29,8, we need only look back to ſee what Diviſion of the *Nonius* coincides with a Diviſion of the Scale, and that will ſhew how many Tenths of a Tenth, that is, how many Tenths beyond 29,7 in the preſent Caſe: But you obſerve the *Nonius* coincides with a Diviſion of the Scale at the fifth Diviſion; conſequently, the Mercury ſtands at 29,75 Inches in the Scale; and ſo you proceed with the greateſt Eaſe to the hundredth Part of an Inch, which is a great Degree of Exactneſs.

21. From what has been ſaid we may eaſily ſee the excellent Uſe of the Barometer in meaſuring the Heights of Places, as Mountains, Towers, &c. For ſince (as we ſhall ſhew) the ſpecific Gravity of Air (ſuch as is near the Earth's Surface) is to that of Mercury as 1 to 12040, 'tis plain 12040 Inches of Air in Height will balance one Inch Height of Mercury; conſequently, 1204 Inches, or 100 Feet, anſwers to $\frac{1}{10}$ of an Inch of Mercury. Therefore if a good Barometer be carried to the Top of a Mountain, or other high Place, the Mercury will ſubſide near one Tenth of an Inch for every 100 Feet of perpendicular Aſcent, and ſo will be a proper Index of the whole Height aſcended.

22. But ſince Mercury is not quite 14 times heavier than Water, the Number 12040 is ſomewhat too large and therefore a leſs Height than 100 Feet will anſwer to $\frac{1}{10}$ of an Inch Deſcent of Mercury in the Barometer; and what that is will be ſhewn from the Experiments made by Dr. *Nettleton* very exactly, as in the Table below.

Altitud

	Height.	Altitude of ☿.		Difference.	for $\frac{1}{10}$
		Bottom.	Top.		
Tower of *Halifax*	102 —	29,78 —	29,66 —	0,12 —	85
Coal Mine	140 —	29,48 —	29,32 —	0,16 —	87
Another, *ditto*	236 —	29,50 —	29,23 —	0,27 —	87
A ſmall Hill	312 —	29,81 —	29,45 —	0.36 —	86
Halifax Hill	507 —	30,00 —	29,45 —	0,55 —	91

23. Having the Height, to which the Mercury will tand at any one Elevation, it is eaſy to find at what Height it will ſtand at any other propoſed. For ſince he Denſity of the Air decreaſes in a Geometrical Ratio, as the Altitudes increaſe in an Arithmetical one, the atter will be as the Logarithms of the former reciproally: But the Weight of the Air is as the Denſity, nd the Height of the Mercury in the Barometer is as he Weight, therefore the Elevations are as the Logarithms of the Height of the Mercury reciprocally; and onſequently, if we take 30 Inches for the Standard Altitude, and 85 Feet for the Altitude requiſite to make it fall $\frac{1}{10}$ of an Inch; then by ſaying, As the Logarithm of $\frac{1}{29,9}$ is to 85, ſo is the Logarithm of $\frac{1}{29,5}$ to the Elevation which will make it fall $\frac{1}{2}$ an Inch; and ſo for any other.

24. After this Manner, the Doctor has computed the following Tables.

A TABLE ſhewing the Number of Feet aſcending, required to make the Mercury fall to any given Height in the Tube, from 30 to 26 Inches. As alſo the Number of Feet deſcending, requir'd to make the Mercury riſe, from 30 to 31 Inches.

Inch.	*Dec.*	*Feet.*	*Dec.*	*Inch*	*Dec.*	*Feet*	*Dec.*
31	0	834	79	28	4	1395	32
30	9	752	53	28	3	1485	13
30	8	670	01	28	2	1575	26
30	7	587	21	28	1	1665	70
30	6	504	15	28	0	1756	47
30	5	420	82	27	9	1847	5
30	4	337	21	27	8	1938	97
30	3	253	32	27	7	2030	72
30	2	169	10	27	6	2122	80
30	1	84	72	27	5	2215	21
30	0	00	00	27	4	2307	95
29	9	85	00	27	3	2401	02
29	8	170	29	27	2	2494	44
29	7	255	87	27	1	2588	20
29	6	341	73	27	0	2 82	33
29	5	427	89	26	9	2776	8
29	4	514	34	26	8	2871	62
29	3	601	08	26	7	2966	79
29	2	688	11	26	6	3062	32
29	1	775	44	26	5	3158	21
29	0	863	08	26	4	3254	46
28	9	951	01	26	3	3351	0
28	8	1039	25	26	2	3448	05
28	7	1127	80	26	1	3545	41
28	6	1216	66	26	0	3643	14
28	5	1305	83				

A TABLE ſhewing the Number of Feet requir'd to make the Mercury fall one Tenth of an Inch from any given Height in the Tube, from 31 to 26 Inches.

Inch.	*Dec.*	*Feet.*	*Dec.*	*Inch.*	*Dec.*	*Feet.*	*Dec.*
31	0	82	26	28	4	89	81
30	9	82	53	28	3	90	13
30	8	82	79	28	2	90	45
30	7	83	06	28	1	90	76
30	6	83	33	28	0	91	09
30	5	83	61	27	9	91	42
30	4	83	89	27	8	91	75
30	3	84	16	27	7	92	08
30	2	84	44	27	6	92	41
30	1	8	72	27	5	92	74
30	0	85	00	27	4	93	07
29	9	85	29	27	3	93	41
29	8	85	58	27	2	93	76
29	7	85	86	27	1	94	12
29	6	86	16	27	0	94	47
29	5	86	45	26	9	94	82
29	4	86	74	26	8	95	17
29	3	87	03	26	7	95	53
29	2	87	33	26	6	95	89
29	1	87	63	26	5	96	25
29	0	87	93	26	4	96	61
28	9	88	24	26	3	96	98
28	8	88	55	26	2	97	3
28	7	88	86	26	1	97	73
28	6	89	17	26	0	98	10
28	5	89	49				

a *blown Bladder*, and many others on the *Air-Pump*. (LXXXVIII.)

(LXXXVIII.) 1. That we may here exhibit a plain and clear Idea of the Force with which the Particles of Air repel one another, 'twill be neceſſary to proceed in the following Manner. If in any Diſtance A B, there are placed any Number of Particles at equal Intervals from one another; and in any other equal Diſtance C D, there are placed twice as many Particles at equal Intervals alſo; 'tis plain the Intervals between the Particles in C D will be but half ſo great as thoſe between the Particles in the Line A B. Hence the Number of Particles in any equal Parts of A B, C D, will be inverſely as their Diſtances from each other. Or, if we put N = Number of Particles, and I = to the Interval between each; then will N be always as $\frac{1}{I}$, for *Lines*, or linear Dimenſions. Plate XXVIII. Fig. 2.

2. But for *Superficies*, ſince they are as the Square of their like Sides, we ſhall have N^2 as $\frac{1}{I^2}$; and in like Manner, ſince *Solids* are as the Cubes of their like Sides, we ſhall have N^3 as $\frac{1}{I^3}$. But N^2 is as the Denſity of the Superficies; and N^3 as the Denſity of the Solid; conſequently the Denſity D, of a Superficies of this Sort, is as $\frac{1}{I^2}$; and of a Solid as $\frac{1}{I^3}$. And to facilitate the Idea, let A B C be a ſuperficies of ſuch Particles, equal to a ſquare Inch; and D F a Solid of a cubic Inch. Fig. 3.

3. Next, let it be ſuppoſed that each of theſe Particles repels thoſe, and thoſe only, which are next to it; and let this repulſive Force (F) be inverſely as the *n* Power of the Interval I, between the Centres of two adjacent Particles; that is, let F be as $\frac{1}{I^n}$. Hence 'tis manifeſt ſuch an Aſſemblage of Particles muſt conſti-

By reaſon of the *Spring of the Air*, its DENSITY muſt be always different in different Altitudes from the Earth's Surface; for the lower Parts of the Air, being preſſed by the Weight of the ſuperior Parts, will be made to accede nearer to each

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tute an *elaſtic Fluid*, or ſuch an one as, when compreſſed, or acted upon by any external Agent, will, by Virtue of its innate repellent Power, re-act or make Reſiſtance with an equal Degree of Force.

4. Now the Force of the ſuperficial Parts is as the Denſity D, and the repellent Force F between two Particles, conjointly, or as D×F; but D is as $\frac{1}{I^2}$, and F is as $\frac{1}{I^n}$; wherefore D × F is as $\frac{1}{I^2} \times \frac{1}{I^n} = \frac{1}{I^{n+2}}$, which therefore will expreſs the elaſtic Force of the Fluid. Now the Denſity D of the Fluid in the cubic Inch is as $\frac{1}{I^3}$, whence $I^3 D$ is as 1, I^3 as $\frac{1}{D}$, and I as $\frac{1}{\sqrt[3]{D}}$; which ſubſtituted for I in the Expreſſion of the elaſtic Force $\frac{1}{I^{n+2}}$, gives $D^{\frac{n+2}{3}}$; that is, the elaſtic or compreſſive Force is as the Cube Root of that Power of the Denſity, whoſe Index is $n+2$.

5. Hence if E the elaſtic Force be as the Denſity D, in any Fluid; then the general Expreſſion $D^{\frac{n+2}{3}}$, becomes $E^{\frac{n+2}{2}}$, whence in that Caſe $\frac{n+2}{3}=1$, and ſo $n+2=3$ and $n=1$. Conſequently, in ſuch a Fluid F is as $\frac{1}{I}$, or the Particles repel each other with Forces that are reciprocally proportional to the Diſtance of their Centres. Such then is the Property of the Air,

other, and the more ſo as the Weight of the incumbent Air is greater; and hence we ſee the *Denſity* of the Air is greateſt at the Earth's Surface, and decreaſes upwards in *Geometrical Proportion* to the Altitudes taken in *arithmetical Progreſſion.* Now it is found that the Air is four Times more rare

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whoſe Denſity is always proportional to the Forces which compreſſes it, which is alſo proved by the following Experiment.

Plate XXIX, Fig. 4.
P.28

6. Let Mercury be poured into an inflected Tube ABCD, open at both Ends, to a ſmall Height as B C. Then ſtopping the Orifice D very cloſe with a Cork or otherwiſe, meaſure the Length of confined Air DC very nicely, and pour Mercury into the other Leg A B, till its Height above the Surface of that in C D be equal to the Height at which it ſtands in the Barometer. Then it is plain the Air in the ſhorter Leg will be compreſſed with a Force twice as great as at firſt when it poſſeſſed the whole Space C D ; for then it was compreſſed only with the Weight of the Atmoſphere ; but now it is compreſs'd by that Weight, and the additional equal Weight of a Column of Quickſilver. Let E be now the Surface of the Mercury in the Leg C D, and upon meaſuring D E, the Space into which the Air is now compreſs'd, it will be found to be juſt half the former Space C D, that is, $DE = \frac{1}{2}DC$.

7. Hence it appears that the Spaces $S = DC$, and $s = DE$, which a given Quantity of Air poſſeſſes, under different Preſſures p and P, are as thoſe Preſſures reciprocally; that is, $S : s :: P : p$. And becauſe the Denſities d, D, where the Quantity of Matter is given (*An* LVI. 9.) are reciprocally as the Magnitudes of Bodies, *viz.* $d : D :: s : S$; therefore the Denſities of the Air are as the compreſſing Forces directly, *viz.* $d : D :: p : P$. This Property of the Air is the Principle to which we owe the Invention and Contrivance of ſeveral very uſe-

at the Height of ſeven Miles than at the Earth's Surface; and therefore at the Altitudes of 7. 14. 21. 28. 35. 42. 49. *&c.* the Rarity of the Air will be 4. 16. 64. 256. 1204. 4096. 16384. *&c.*

IF the Air were of an equal Denſity throughout, the Height of the Atmoſphere

ful Inſtruments and Machines, ſome of which I will exhibit here, and others in the Sequel of this Work.

8. We have ſhewn in the laſt *Annotation* that the Preſſure of the Air, in its State of Mean Gravity, will ſupport a Column of Quickſilver to the Altitude of $29\frac{1}{2}$ Inches; and (in *Annot.* LXIII.) it was ſhewn that the ſpecific Gravity of Mercury was to that of Water, as 14 to 1 nearly; therefore the ſaid Mean Preſſure of Air will ſuſtain a Column of Water to the Height of $14 \times 29{,}5 = 413$ Inches $= 34$ Feet 5 Inches. But ſince Mercury is not quite 14 Times as heavy as Water, we may take 400 Inches for the Meaſure of the Mean Gravity of the Air on Water, and 29,5 for Mercury; and then we ſhall have $DC : DE :: P : 29{,}5$ in Mercury; or $DC : DE :: P : 400$, in Water; conſequently $400\, DC = DE \times P$.

9. Again, let the Standard Altitude of Mercury or Water be $H = 29{,}5$ or 400. and let the Altitude $FG = h$; then will $P = H + h$, and then the above Equation will give this Analogy: As $S : s :: H + h : H$, whence $S : S - s :: H : h$, or $DE : EC :: H : h$; conſequently, by having DE or CE given, you know the Altitude $h = FG$. Thus for Example: Let $DC = 10$ Inches, *it is required to find what Altitude of Water* FG *will by its Preſſure raiſe the Surface at* C *one Inch?* Here $CE = 1$, $DE = 9$, and $H = 400$: Then $DE : CE :: H : h$, that is, $9 : 1 :: 400 : 44{,}4$; or $FG = 44\frac{1}{2}$ Inches nearly, or 3 Feet $8\frac{1}{2}$ Inches. Thus again, Query the Altitude FG that ſhall raiſe the Surface C 9 Inches, or $\frac{9}{10}$ of the Whole? Say, As $1 : 9 :: 400 : 3600 = FG$, or 300 Feet. Thus the Altitudes are found for every tenth Part of the whole Space DC, as in the following Table.

might be determined; for by Experiment we find the Length of a Column of Air 72 Feet high is equal in Weight to *one Inch* of Water of the ſame Baſe: Hence the Denſity of Air is to that of Water as 1 to 864. It is alſo found by Experiment, that the Weight of a Column of Air the Height of the Atmoſphere will be equal to the Weight

	Feet.	*Inches.*		*Feet.*	*Inches.*
1 ——	3	8	6 ——	50	0
2 ——	8	4	7 ——	77	9
3 ——	14	2	8 ——	133	4
4 ——	22	1	9 ——	300	0
5 ——	33	4	9½ ——	633	4

10. Hence is deduced the Nature and Structure of the Sea-Gage, invented by Dr. *Hales*, and Dr. *Deſaguliers*; whoſe Deſcription thereof I ſhall here give. AB is the Gage-Bottle, in which is cemented the Gage-Tube F f in the Braſs Cap at G. The upper End of the Tube F is hermetically ſeal'd or cloſed; the open lower End f is immerſed in Mercury C, on which ſwims a ſmall Thickneſs or Surface of Treacle. On the Top of the Bottle is ſcrew'd on a Tube of Braſs HG, pierc'd with ſeveral Holes to admit the Water into the Bottle A B. The Body K is a Weight hanging by its Shank L, in a Socket N, with a Notch on one Side at *m*, in which is forced the Catch *l* of the Spring S, and paſſing thro' the Hole L in the Shank of the Weight K, prevents its falling out when once hung on. On the Top, in the upper Part of the Braſs Tube at H, is fix'd a large empty Ball or full-blown Bladder F, which muſt not be ſo large, but that the Weight K may be able to ſink the Whole under Water.

Plate XXIX.

11. The Inſtrument thus conſtructed, is uſed in the following Manner. The Weight K being hung on,

of a Column of Water of the ſame Baſe, and 32 Feet, or 384 Inches high: Wherefore 864 multiplied by 384 will produce 331776 Inches, or a little above 5 Miles, for the Height of the Atmoſphere, were the Denſity every where the ſame as at the Earth.

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the Gage is let fall into deep Water, and ſinks to the Bottom; the Socket N is ſomewhat longer than the Shank L, and therefore, after the Weight K comes to the Bottom, the Gage will continue to deſcend, till the lower Part of the Socket ſtrikes againſt the Weight; this gives Liberty to the Catch to fly out of the Hole L, and let go the Weight K; when this is done, the Ball or Bladder I, inſtantly buoys up the Gage to the Top of the Water.

12. While the Gage is under Water, the Water having free Acceſs to the Treacle and Mercury in the Bottle, will by its Preſſure force it up into the Tube F f, and the Height to which it has been forced by the greateſt Preſſure, *viz.* that at the Bottom, will be ſhewn by the Mark in the Tube which the Treacle leaves behind it, and which is the only Uſe of the Treacle. This ſhews into what Space the whole Air in the Tube F f is compreſs'd; and conſequently, by the Rule (in *Art.* 9.) the Height or Depth of the Water, which by its Weight produced that Compreſſion, which is the Thing required.

13. If the Gage-Tube F f be of Glaſs, a Scale might be drawn on it with a Point of a Diamond, ſhewing, by Inſpection, what Height the Water ſtands above the Bottom, which Scale is made from the Numbers in the foregoing Table, where the Diviſion may be made for Hundredth Parts, as well as Tenths. But the Length of 10 Inches is not ſufficient for fathoming Depths at Sea, ſince it appears by the Table, that when all the Air in ſuch a Length of Tube is compreſ-

BUT ſince the Denſity of the Air decreaſes with the Preſſure, it will be more rarefied and expanded the higher we go ; and by this means the Altitude of the At-

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ſed into half an Inch, the Depth of Water is not more than 634 Feet, which is not half a Quarter of a Mile.

14. If to remedy this we make Uſe of a Tube 50 Inches long, which for Strength may be a Muſket-Barrel, and ſuppoſe the Air compreſſed into an 100dth Part, or $\frac{1}{2}$ an Inch ; then by ſaying, As 1 : 99 :: 400 : 39600 Inches, or 3300 Feet ; even this is but little more than half a Mile, or 2640 Feet. But ſince 'tis reaſonable to ſuppoſe the Cavities of the Sea bear ſome Proportion to the mountainous Parts of Land, ſome of which are more than three Miles above the Earth's Surface ; therefore to explore ſuch great Depths, the Doctor contrived a new Form for his Sea-Gage, or rather for the Gage-Tube in it, as follows. B C D F is a hollow metalline Globe, communicating on the Top with a long Tube AB, whoſe Capacity is $\frac{1}{9}$ Part of that of the Globe. On the lower Part, at D, it has alſo a ſhort Tube DE, to ſtand in the Mercury and Treacle. The Air contain'd in this compound Gage-Tube is compreſs'd by the Water, as before ; but the Degree of Compreſſion, or Height to which the Treacle has been forced, cannot here be ſeen through the Tube : Therefore to anſwer that End, a ſlender Rod of Metal or Wood, with a Knob on the Top, muſt be thruſt up to the Top of the Tube A B, which will receive the Mark of the Treacle, and ſhew it when taken out.

Plate XXVIII. Fig. 6.

15. If the Tube AB be 50 Inches long, and of ſuch a Bore as that every Inch in Length ſhould be a cubic Inch of Air, and the Contents of the Globe and Tube together 500 Cubic Inches ; then, when the Air is compreſs'd within a 100dth Part of the Whole, it is evident the Treacle will not approach nearer than 5 Inches of the Top of the Tube, which will agree to the Depth of 3300 Feet of Water, as above. Twice this Depth will compreſs the Air into half that Space

moſphere becomes indefinite, and terminates in pure *Æther*. But though we cannot aſſign the real Altitude of the Atmoſphere, it is certain from Obſervations and

nearly, *viz.* 2 ½ Inches, which correſponds to 6600 Feet, which is a Mile and a Quarter. Again, half that Space, or 1 ¼ Inch, will ſhew double the former Depth, *viz.* 13200 Feet, or two Miles and a half; which is probably very nearly the greateſt Depth of the Sea.

16. A Gage of this Kind may be of very great Uſe in many other Caſes. Thus the prodigious Force of Compreſſion ariſing from Freezing may be accurately tried: Let a Bomb of caſt Iron ſix or eight Inches Diameter, and about one Inch thick, be fill'd with Water; then if a ſmall Gage of this Sort be made and fix'd to a Stick, which is to be ſet upright in the Middle of the Bomb, ſo that the Gage-Bottle may be in the central Part; and if then the Hole of the Bomb be faſt ſcrew'd up, the Bomb cover'd over with a *freezing Mixture* (which is made of equal Quantities of Salt and Snow, or pounded Ice) in a little Time the Water will begin to freeze all round the Inſide of the Bomb, and by its Expanſion will produce a greater Force upon the Water, and a greater Degree of Compreſſion of the Air of courſe, than by any other Means yet known: And this may be continued till it ſhall burſt the Bomb, when the Gage taken out of the globular Shell of Ice (for the Water will be frozen only on the Outſide) will ſhew the exact Quantity of this Force of Compreſſion.

17. Dr. *Hales* (the Author of this Contrivance) actually made the Experiment, but not having well ſecured the Gage, it was broken to Pieces; but from computing the Force neceſſary to burſt an Iron Bomb an Inch thick, it appeared that this Force was about equal to 1340 Atmoſpheres, or the Preſſure of 1340 times the Weight of 33 Feet of Water. But this Computation was made upon Suppoſition that the Coheſion of caſt Iron is the ſame with that of Iron-Wire;

Experiment, that 45 or 50 Miles is the utmoſt Height where the *Denſity is ſufficient to refract a Ray of Light*; and therefore that may be eſteemed the Altitude of the Air to the leaſt ſenſible Degree of Denſity. (LXXXIX.)

but as it muſt be conſiderably leſs, ſo the Number 1340 may be diminiſhed to 1000; and then the Air muſt be compreſs'd into a 1000 times leſs Space than it had in its natural State, and muſt in that Caſe have been more denſe than Water: For its Denſity then to that of common Air was as 1000 to 1; whereas the Denſity of Water and Air are but as 860 to 1.

18. After the ſame Manner may be tried the Force with which dried Peas, Beans, &c. expand with Moiſture, when confin'd in a Bomb; for it muſt be a very ſtrong Veſſel indeed, ſince it has been found by Experiment they will burſt a Gun-Barrel in ſwelling. In like manner alſo the elaſtic Force of factitious Air generated from Bodies by Fermentation may be eſtimated in a very nice and entertaining Manner: With many other Things of this Sort, which the ingenious Reader will readily excogitate of himſelf.

(LXXXIX.) 1. The Denſity of the Air on one Hand, and the Rarity on the other, are both limited: No Condenſation can reach ſo far as to cauſe a Penetration of Parts; the utmoſt Limit, therefore, of Denſity, muſt be a perfect *Plenum*, or a given Quantity of Air reduced into a Space abſolutely full, or without any Pore; which is a Degree of Denſity that has not been, and probably never will be, in the Power of Art to effect.

2. On the other Hand, the Rarity of the Air cannot proceed *ad infinitum*, but has its Limit from its Gravity: For though the Rarefaction of the Air be ſtill greater as the Diſtance from the Surface of the Earth increaſeth, its Spring at length will be ſo weaken'd, that the Force by which the Particles tend upwards from thoſe next

SINCE the Gravity of the Air is ſo various, that at one time it will ſuſtain a Pillar of Mercury 31 Inches high, when at another it will raiſe it but to the Height of

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below them, will be leſs than the Force of Gravity by which they tend downwards. The Rarefaction of the Air muſt therefore be bounded, where theſe two oppoſite Forces come to balance each other.

3. Now though we cannot poſſibly define the Limits of the Atmoſphere, yet we may ſtill inveſtigate how much the Air is rarefied at any propoſed Altitude above the Earth's Surface: For doing which, ſeveral Methods have been propoſed; ſome of which are very tedious, and difficult to be underſtood. I ſhall here illuſtrate Sir *Iſaac*'s Theorem for that Purpoſe, which is very conciſe and plain. It requires only two different Denſities of the Air, at two given Altitudes above the Earth's Surface, to be known, and which we eaſily obtain by Experiment as follows.

Fig. 9.

4. Take a Vial A E F B, fill'd two Thirds full of Water to CD; in which let a long Tube I G (open at both Ends) be immerſed, and cloſely cemented to the Vial at A B, ſo that none of the included Air may eſcape. This done, blow a little Air through the Tube into the Vial, which increaſing the Spring of the contained Air, will cauſe it to raiſe and ſupport a Column of Water in the Tube, to ſuch a Height H, that its Weight, together with that of a Column of Air preſſing on its Surface H, is equivalent to the increaſed Spring of the confined Air.

5. The Vial and Tube thus prepared are to be carried up to the Top of a Tower, Mountain, or ſome high Place; and in the Aſcent, ſince the Column of Air preſſing on the Water at H is conſtantly ſhortened, ſo its Force of Preſſure will be diminiſhed. The Spring of the Air in the Vial will therefore keep the Column of Water conſtantly riſing in the Tube; ſo that when you have aſcended the Height of about 72 Feet, the Water in the Tube will have riſen from H to L, thro'

28 Inches, in the *Barometer*; it follows, that we may take 29 ½ Inches of Mercury for the *mean Altitude*, and conſequently its

the Space of one Inch; and ſo in Proportion for any other Altitude, as I have ſeveral Times found by trying the Experiment.

6. From hence it appears, that the Altitude of one Inch of Water is equivalent to the Altitude of 72 Feet, or 864 Inches of Air; and therefore the ſpecific Gravity of Air is to that of Water as 1 to 864, or, as Sir *Iſaac* has ſtated it, 860. Now ſince the ſpecific Gravity of Water is to that of Mercury as 1 to 14, therefore the ſpecific Gravity of Air to that of Mercury will be as 1 to $860\times14=12040$; and ſince the Height of Mercury ſupported by the Air in the Barometer is 2,5 Feet; if we ſay, As $1 : 12040 :: 2,5 : 2,5\times12040 = 30100$ Feet, which would be the Height of the Air were it every where as denſe as at the Earth, or about 5¾ Miles.

7. But ſince the Air is not uniformly denſe, we muſt ſeek its Height by another Method to be taught by and by. In the mean time, as the Air's Denſity conſtantly decreaſes, we ſhall ſhew how to find the Ratio of its Denſity at any Altitude to that at the Earth's Surface. Thus, ſince the Denſities are as the compreſſing Force, which is as the Altitude of the incumbent Column of Air, and ſince the Weight of Mercury is to Water as 1 to 14, it is plain that the Air which ſupports a Column of Mercury 2,33 Feet, will ſuſtain a Column of Water to the Height of 33 Feet. The Denſity on the Earth's Surface then is as 33.

8. Again; it is evident, ſince 860 Feet Altitude of Air is equal in Weight to 1 of Water, therefore at the Height of 860 Feet above the Earth, the Air (continuing in the ſame State) would ſuſtain only 32 Feet of Water. At the Height therefore of 860 Feet, the Denſity of the Air is as 32.

9. Hence the Denſity at any other Altitude is eaſily found by the Hyperbola *f a i h*, and its Aſymptotes SF

Weight for the *mean Weight* of a Pillar of Air of the ſame Baſe. But a Column of Mercury whoſe Baſe is *one Square Inch*, and Altitude $29\frac{1}{2}$, weighs about 15 *lb.* which is

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and St, S being the Centre of the Earth, and A its Surface: Then the Earth's Semidiameter S A = 4000 Miles nearly, or 21120000 Feet. Take A B = 860 Feet, and let the Denſity of the Air be required for any other Height, as A C = 7 Miles, or 36960 Feet. In the Points A, B, C, erect the Perpendiculars A H, B I, C K, which let be made proportional to the Denſities of the Air in the Points A, B, C; that is, let A H : B I :: 33 : 32, and A H : C K :: 33 : x; and from the Points H, I, let fall the Perpendiculars Ht, Iu. Plate XXVIII. Fig. 7.

10. Then putting SA = Aa = r, SB = a, SC = b, and SB : SA : : Aa : Bb = $\frac{rr}{a}$; thus Cc = $\frac{rr}{b}$; then Aa = Bb = $\frac{ar-rr}{a}$, and Aa—Cc = $\frac{br-rr}{b}$. And put $m : n$ :: 33 : 32 :: A H : B I. Then by the Nature of the Hyperbola, we have the Area $thiu$ as the Logarithm of $\frac{St}{Su}$, and the Area $thkw$ as the Logarithm of $\frac{St}{Sw}$; or the Area $thiu : thkw$ :: L.$\frac{m}{n}$: L.$\frac{m}{x}$. But (by *Coroll.* to *Prop.* XXII. *Lib.* 2. of the *Principia*) it is, $thiu : thkw$ :: A a — B b : A a — C c : : $\frac{ar-rr}{a} : \frac{br-rr}{b} :: \frac{a-r}{a} : \frac{b-r}{b}$.

11. Now, becauſe $a-r$ = A B = 860, and $b-r$ = A C = 36960, we have $\frac{860}{21120000} : \frac{36960}{21156960}$: : L.$\frac{m}{n}$: L.$\frac{m}{x}$; whence we have L.$\frac{m}{x}$ = 0,573190; therefore L.m — L.x = 0,573190; whence L.m — 0,573190 = L.x = 0,945324, the Number anſwering to which is 8,817 = x = C K, the Denſity required.

equal to the Preſſure of Air on every *Square Inch*; and therefore upon every *Square Foot* it will be 2160 *lb.*; and allowing 14½ *Square Feet* for the Sur-

Or, the Denſity at A is to the Denſity at C as A H to CK, or as 33 to 8,817, which is nearly as 4 to 1.

12. From the Nature of the Hyperbola it is plain, he Denſities *m*, *n*, *x*, muſt be in a *geometric Pro-reſſion*. But to ſhew theſe Things more generally: Let S C $= x$ be a variable Diſtance, and its Fluxion CE $= \dot{x}$; let the Denſity C K $= y$, the compreſſing orce in the Altitude C as v, and the Power of Gra-ity as g. Then will the ſpecific Gravity of the Air e there as $g y$; for it will be as the Denſity y when he Gravity g is given, and as the Gravity when the Denſity is given: and when neither is given, it will be onjointly as both.

13. Since the whole Weight or Preſſure of a Column f any homogeneous Fluid, of a uniform Denſity, is as s ſpecific Gravity multiplied by its Magnitude, (by *Annot.* LVI. 10.) and if the Baſe be the ſame as the id Gravity multiplied by the Altitude, and therefore s Fluxion as the ſpecific Gravity multiplied by the luxion of the Altitude; therefore we have $g y \dot{x} = -$, becauſe the Denſity of the Air thro' the very ſmall pace C E may be look'd upon as uniform; and ſince he Preſſure decreaſes as the Altitude x increaſes, there-re it is that we make the Fluxion of it negative, *viz.* $-\dot{v}$.

14. If the Gravity g be as $\frac{1}{x^m}$, and the Denſity y as ny Power n of the compreſſing Force v, *viz.* if $y : v^n$, nd therefore $v : y^{\frac{1}{n}}$, by taking the Fluxions we have $\frac{1}{n} y^{\frac{1-n}{n}} \dot{y} = \dot{v}$. In the Place of g and $\dot{v}$ in the Equa-on $g y \dot{x} = -\dot{v}$, let their Values be ſubſtituted, and we ave $\frac{1}{n} y^{\frac{1-n}{n}} \dot{x} = \frac{-\dot{x}}{x^m}$.

face of the Body of a middle-ſized Man, i
muſt ſuſtain a Preſſure of 31320 Pounds

15. If we put $n=1$, that is, if the Denſity be as the compreſſing Force, we have $\frac{\dot{y}}{y}=\frac{-\dot{x}}{x^m}$. Now, ſinc any Quantities x, $x+a$, $x+2a$, $x+3a$, in arithmetical Progreſſion, have all their Fluxions equal and th ſame, viz. $\dot{x}$; therefore if any Quantities $\frac{1}{x^{m-1}}$ be taken in ſuch a Progreſſion, their Fluxions $\frac{-m-1 \times}{x^m}$ or $\frac{-\dot{x}}{xm}$ will be all the ſame or conſtant; therefor $\frac{\dot{y}}{y}=-\frac{\dot{x}}{x^m}=1$; conſequently $\dot{y}:y$, that is, the Fluxions of the Denſities are as the Denſities themſelves which therefore are in Geometrical Progreſſion, as i manifeſt from the Doctrine of Fluxions.

16. If in the ſame Hypotheſis you put $m=1$, or ſuppoſe the Gravity to be every where uniform, or given then $\frac{\dot{y}}{y}=\frac{-\dot{x}}{x}$. If now we take $\frac{-\dot{x}}{x}$ conſtant, o make $\dot{x}:x$, then will the Diſtances x be in Geometrical Progreſſion; and in that Caſe alſo we have $\frac{\dot{y}}{y}=1$ or $\dot{y}:y$; whence alſo the Denſities y are in Geometrical Progreſſion.

17. The Fluent of the above Equation $-\frac{1}{n}y^{\frac{1-2a}{n}}\dot{y}=\frac{-\dot{x}}{xm}$ is $\frac{1}{1-n}y^{\frac{1-n}{n}}=\frac{-1}{m-1}x^{1-m}+Q$, a conſtant Quantity. Here, 'tis plain, it cannot be $n=0$, for ther $y^{\frac{1}{0}}=$Infinite; nor can $m=1$, becauſe then it woul be $x^{1-m}x^0=1$, and ſo the Denſity y would be ever

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or 14 *Tons*, when the Air is of a mean Gravity. This prodigious Force would

where the ſame, or conſtant; neither can $n = 1$, for hen $y^{\frac{1-n}{n}} = y^{\frac{0}{n}} = 0$. To determine the Value of Q, ve muſt firſt define the Altitude SF, where the Denſity vaniſhes, or $y=0$, and call it $a=$SF; then we have Q$=$ $\frac{-1}{n-1}a^{1-m}$; and hence $\frac{1}{1-n}y^{\frac{1-n}{n}} = \frac{a^{1-m}-x^{1-m}}{m-1}$; vhere 'tis plain $\frac{1-n}{n}$ ought to be a poſitive Number, nd leſs than Unity, that while the Diſtances x increaſe, he Denſities y may decreaſe.

18. If the Altitude at which the Denſity y vaniſhes e ſuppoſed infinite, then Q $= 0$, and the Equation is $\frac{1}{-n}y^{\frac{1-n}{n}} = \frac{1}{m-1}x^{1-m}$. For if in the Equation (*Art.* 7.) $y=0$, and $x=$Infinite, then $\frac{a^{1-m}}{m-1} - \frac{x^{1-m}}{m--1} = 0$; herefore $\frac{a^{1-m}}{m-1} = \frac{x^{1-m}}{m-1}$, and ſo $a=x=$Infinite, conrary to Suppoſition.

19. If now the Gravity be reciprocally as the Squares of the Diſtance, or $m = 2$, we have the Equation $\frac{1}{1-n}\,y^{\frac{1-n}{n}} = \frac{1}{m-1}x^{1-m}$ become $\frac{1}{1-n}y^{\frac{1-n}{n}} = \frac{1}{x}$; whence y will be reciprocally as $x^{\frac{n}{n-1}}$, which is a general *Formula* for any Hypotheſis of the Ratio of the compreſſing Power and Denſity. Thus, if you ſuppoſe the compreſſing Force in the duplicate Ratio of the Denity, that is, $y^2 : v$; then $y=v^{\frac{1}{2}}$, and $n=\frac{1}{2}$, and thereore $\frac{n}{1-n} = 1$, whence y will be reciprocally as x. Hence all thoſe Caſes of *Scholium* to *Prop.* XXII. *Lib.*

cruſh us into a very ſmall Compaſs, were it not that it is equal on every Part, and coun-

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I. of the *Principia* are derived, and any others at Pleaſure.

20. The Denſity of the Air decreaſing indefinitely, it is evident there is no certain Limit or Boundary of the Atmoſphere, which gradually rarefies into pure *Æther*, or *Aura*, as it is often call'd. But ſince one principal Effect of the Air is the Refraction of Light, and ſince the Particles of Light are the ſmalleſt Bodies we know of in Nature, 'tis reaſonable there to fix the Boundary of what we may properly call *Air*, in the Altitude where it begins to have the Power of producing this leaſt Effect in Nature, *viz.* the refracting a Ray of Light.

Plate XXIX. Fig. 2. 21. To diſcover this Altitude of the Air, we have the following Method. Let ADF be the Surface of the Earth, S the Sun below the Horizon, S B a Ray of Light touching the Earth; which is reflected by a Particle of Air, in the higheſt Part at B, in the horizontal Line BA to a Spectator at A. The Angle S B N is the Depreſſion of the Sun below the Horizon in this Caſe, which, becauſe it is at the Moment Twilight ends, is known from Obſervation to be about 18 Degrees. But becauſe B A is alſo a Tangent, the Angle A C D = S B N = 18 Degrees; and the Angle A C B $= \frac{1}{2}$ A C D = 9 Degrees; which would be true, did the Ray SB paſs through the Atmoſphere without Refraction, but becauſe it does not, but is refracted or bent towards H, the Angle A C B muſt be diminiſhed by the horizontal Refraction, which is about half a Degree; whence the Angle ACB = 8° 30′.

22. Therefore in the right-angled Triangle A C B we have all the Angles given, and one Side, (*viz.* A C = 4000 Miles, or the Semidiameter of the Earth,) to find the Side BC, thus,

As the Sine A B C = 81° 30′ = 9.995203
Is to the Side A C = 4000 = 3.602060
So is Radius 90° = 10.000000

To the Side B C = $4044\frac{1}{2}$ = 3.606857 terbalanced by the equal Re-action of the Spring of the Air within us. (XC.)

wherefore BC—HC=HB=44½ Miles, the Height of the Atmoſphere required.

(XC.) 1. Since a cubic Inch of Mercury weighs very nicely 8,1 *oz.* *Averdupois*, a Pillar of Mercury, whoſe Baſe is one ſquare Inch, and

$$\text{Altitude}\left\{\begin{array}{c}28\\29\frac{1}{2}\\31\end{array}\right\}\text{Inches, will weigh}\left\{\begin{array}{cc}\textit{lb.}&\textit{oz.}\\14&3\\14&15\\15&11\end{array}\right\}\textit{fere.}$$

2. So that the Air, at a mean Gravity, is equivalent to the Preſſure of 15 *lb.* upon every *ſquare Inch*; and therefore upon every *ſquare Foot* it will be equal to 2160 *lb.* and 2160×14,5=31320 *lb.* or 14 Tons nearly, the Weight or Preſſure ſuſtain'd by a middle-ſized Man. When the Air is lighteſt, this Preſſure is 13⅕ Tons; and when heavieſt, it is 14$\frac{3}{10}$ Tons, the Difference is 1,1 Ton, = 2464 *lb.* the Weight with which we are compreſs'd more at one Time than another.

3. This great Difference of Preſſure muſt greatly affect us in regard to the animal Functions, and conſequently in reſpect to our Health. If a Perſon, for Inſtance, be aſthmatical, he will find his Diſorder increaſe with the Levity of the Air; for ſince a pure, denſe, elaſtic Air, which is very heavy, is only capable to diſtend his Lungs in Reſpiration, when the Air is leſs compreſs'd by its diminiſh'd Weight, it will have leſs Elaſticity, and ſo be leſs capable of expanding the Lungs; the Valetudinarian will therefore find his Difficulty of Breathing increaſe in Proportion.

4. Again, the Reaſon why we think the Air lighteſt in fine Weather, when it is really heavieſt, is becauſe the greater Preſſure conſtringes and braces the Fibres and Nerves, and brings them to a due Tone, by which Means all the Blood-Veſſels act with their full Power and natural Vigour; hence a proper Velocity is given to the Fluids, and a greater *Momentum* to overcome Obſtructions in the Capillaries; thus by a briſk Circu-

THE Weight of the Air is proved by a great Variety of curious Experiments, the principal of which here follow.

lation of the Fluids, and a due Compreſſion of the Solids, we find ourſelves firm and well, alert and light, and therefore fancy the Air is ſo.

5. Whereas, on the contrary, when this Preſſure is leſſen'd by near 2500 *lb.* the Fibres are relaxed, the contractile Force of the Veſſels diminiſhed, a languid Circulation enſues, Obſtructions, Viſcidities, *&c.* happen, and produce Agues, Fevers, Aches, *&c.* in ſome; and in all, a Sort of Indolence or gloomy Inactivity, and Heavineſs; and therefore we imagine that it reſults from the Heavineſs of the Air, when it is juſt the contrary.

6. If it be required to find the Weight of the whole Atmoſphere on the Earth's Surface, we may proceed thus: Suppoſe the Earth's Diameter in round Numbers 8000 Miles, the Area of a great Circle will be 8000×8000 ×0,7854=50266400 ſquare Miles, which multiplied by 4, gives 201065600 ſquare Miles for the Surface of the Earth; but becauſe we took the Diameter a little too large, we may take 200,000,000 for the Number of ſquare Miles in the Earth's Surface; in one ſquare Mile are (5280×5280=) 27878400 ſquare Feet, therefore on the Earth's Surface we have 5575680000000000 ſquare Feet, which multiplied by 2160 (the Preſſure on each ſquare Foot, *Art.* 2.) gives 12043468800000000000 *lbs.* for the whole Preſſure. *N. B.* Since 2240 *lb.* make a Ton, the Preſſure 2160 *lb.* upon a ſquare Foot, is very near a Ton Weight.

Plate XXIX. Fig. 3. 7. I ſhall now preſent the Reader with a Solution of a very curious Problem, viz. *To find the Thickneſs* F H *of an hollow Ball or Globe* F D M E, *made of any given Metal, &c. whoſe ſpecific Gravity is known, ſuch that it ſhall ſwim immerſed in Part or wholly in any homogeneous Fluid, whoſe ſpecific Gravity is alſo known.* Let A B be the Surface of the Fluid, and let the Globe F D E ſwim therein, immerſed to the Depth LM, and let the ſpecific Gravity of the Metal be to that of the Liquor as 1 to 12.

(1.) By actually *weighing it in a nice Balance*; where we ſhall ſee that *one Gallon of Air* will weigh a Dram very nearly. (2.) By filling a Glaſs Tube with Mercury, and inverting it in a Baſon of the ſame Fluid, where it will appear that a Column will be ſupported in the Tube by the ſole Weight or Preſſure of the Air, to upwards the Height of 28 Inches. (3.) By taking the Air off from the Surface of the Quickſilver in the Gage of the Air-Pump, which then immediately riſes by the Preſſure of the external Air. (4.) By exhauſting a Receiver placed over the Hole of the Braſs Plate on the Pump, which will then be kept faſt on by the Preſſure of the incumbent Air. Or, (5.) More demonſtratively by exhauſting a ſmall Receiver under one larger, and letting in the Air at once upon it; which will then be faſten'd to the Plate as before, though not placed over the

8. Then putting the Diameter FM$=$D, HN$=d$, LM$=x$; we have the ſpherical Shell equal to the Sphere FDE—Sphere HIK, that is, $\frac{p\,D^2}{6}-\frac{p\,d^2}{6}$: alſo the Segment of the Liquor DME, is $\frac{pDx^2}{2D}-\frac{px^3}{3D}$; and, in Caſe of an Equilibrium between theſe Quantities, we have $\frac{pD^2-p\,d^2}{6}:\frac{p\,D\,x^2}{2\,D}-\frac{p\,x^3}{3\,D}::n:1$. Whence we ſhall get $D^3-d^2D=3\,Dx^2n-2nx^3$; and thence D^3-3Dx^2n

the Hole. (6.) By placing the Hand on the open Receiver, and exhauſting, the Weight of the Air on the Hand will be extremely ſenſible. (7.) By placing a plain Piece of Glaſs on the ſaid open Receiver, which, when the Air is a little exhauſted, will be broke into Pieces by the Weight of the Air. (8.) A Bladder tied over the ſame Glaſs will be broke in the ſame Manner. (9.) The Air exhauſted from a thin Bottle under a Receiver, and then ſuddenly let in, will, by its Weight, inſtantly reduce it to very ſmall Pieces. (10.) A Bottle broke by the ſame means another Way. (11.) By putting a Piece of Wood under Quickſilver in the Receiver, and then exhauſting the Air, and letting it in again, it will by its Weight force the Quickſilver into the Pores of the Wood, and very ſenſibly increaſe its Weight. (12.) The exhauſted Braſs Hemiſpheres prove

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$+2nx^3 = d^2D$; or $D^2 - 3x^2n + \frac{2nx^3}{D} = d^2$; whence at laſt $d = \sqrt{D^2 - 3x^2n - \frac{2nx^3}{D}}$. Therefore $\frac{D-d}{2} =$ the Thickneſs of the Shell required.

9. If we ſuppoſe the Body to ſwim in the Fluid wholly immerſed, then $x = D$, and $d = \sqrt{D^2 \times \overline{1-n}} = D\sqrt{1-n}$. Now admit FDE be a Sphere of Copper 10 Feet in Diameter, and that the Fluid Medium be Air, whoſe ſpecific Weight to that of Copper is as 1 to

not only the prodigious Weight of the Air, but alſo the Quantity thereof very exactly. (13.) By exhauſting Glaſs Bubbles ſwimming in Water, and letting the Air in again, it will force the Water into the Bubbles, and make them ſink. (14.) The Syringe with its Weight deſcending *in Vacuo*, and aſcending again upon the Admiſſion of Air, does very prettily prove the *Preſſure of the Air*, and the *Rationale* of Syringes in general. (XCI.)

$860 \times 9 = 7640$; hence $n = \frac{1}{7640}$, and therefore $1 - n = 1 - \frac{1}{7640} = \frac{7639}{7640}$; and becauſe $D = 10$ Feet, or 120 Inches, therefore $120\sqrt{\frac{7039}{7640}} = d = 119{,}992$; and ſo $\frac{D - d}{2} = 0{,}004 = F\,H$, the Thickneſs of the Metal requiſite for the Globe to ſwim in Air.

10. But in order to this, one Thing more is neceſſary, *viz.* that *the Concavity of the Globe be a pure Void or* Vacuum: for if it be fill'd with Air only, the Globe will ſink in the Air, be it ever ſo thin; becauſe in that Caſe it muſt be heavier than an equal Bulk or Globe of Air. Hence we ſee how impoſſible a Thing is that *Aerial Navigation*, which *Franciſcus de Lanis* and other Miracle-Mongers have amuſed us with, before true Philoſophy appear'd to deliver us from thoſe vain Speculations, and fruitleſs Attempts that may be grounded thereon.

(XCI.) 1. I ſhall here give the *Rationale* of the ſeveral *Phænomena* of the Experiments on the Air-Pump, as they are ſhewn in the Order of my Lectures on this Subject. The firſt of which is, *to ſhew the abſolute*

THAT Water riſes in *Pumps, Syphons,* and all Kinds of *Water Engines,* by the

Weight of the Air by weighing it in a Balance; of which we have already given an Account in *Annotation* LXXXVII. 2.

2. The *Second* is, *fixing a ſmall Receiver on the Plate of the Air-pump, by exhauſting the Air out of it.* The Reaſon of which is, that the Preſſure of the Air acts now alone on the Outſide of the Glaſs, and perpendicularly on its Top, and preſſes it down with a Force equal to ſo many Times 15 *lb.* as there are ſquare Inches in the Top of the Glaſs, or in the largeſt horizontal Section of it. The Spring of the Air, (which is always equipollent to the Preſſure) being now taken away from within the ſaid Receiver, will leave it to ſuſtain the entire Force of Preſſure, which will therefore fix it faſt down to the Plate.

3. The *Third* Experiment *fixes the Glaſs firmly on the Plate, not, as before, over the Hole, but on one Side of it.* This is to undeceive People in regard to the common erroneous Notion of a *Suction*, which they ſuppoſe is ſomething within-ſide of the Glaſs that draws it down as the Air paſſes out through the Hole. But when they ſee the Glaſs placed on one Side the Hole under a Receiver, and that as the Air is drawn out of the Receiver it will by its Spring all eſcape from under the Glaſs at the ſame time, and then when the Air is let into the Receiver all at once, it falls on the little Glaſs, and fixes it down in ſuch Manner that it is plainly ſeen to ſink into the Leather upon the Plate; I ſay, when all this is ſeen and conſider'd, it entirely eradicates that vulgar Error, and ſets the Truth in a clear Light.

4. The *Fourth* Experiment *fixes a Perſon's Hand on the Top of an open Receiver.* This is done by the Preſſure of the Air on the Top or Back of the Hand, when the Spring of the Air is wanting within the Receiver to counter-act it. This great Preſſure is very ſenſible to the Hand, though not hurtful; and the Skin and Fleſh is viſibly preſs'd down between the Metacarpal Bones.

Preſſure of the Air only, is made evident by taking off the ſaid Preſſure (in the ex-

The Spring of the Air in the Hand at the ſame time exerts itſelf, by extending the Skin and Fleſh of the Part of the Hand on the Glaſs as far down as poſſible, by which means the Blood flows thither in great Quantity, as in Cupping, and makes the Part look very red. If the Area of the Top of the Receiver be 4 Square Inches, the Hand will be preſs'd or kept on by a Weight equal to 60 *lb.*

5. The *Fifth* Experiment is *fixing the Braſs Hemiſpheres together by the Preſſure of the external Air, in ſuch Manner as to require two ſtrong Men to pull them aſunder.* This is done by exhauſting the Air from their Cavity, and thereby taking away the Spring, leaving the Preſſure to act alone. If the Diameter of the Hemiſpheres be 4 Inches, the Area will be 12,556 Square Inches, which multiplied by 15 gives 188,3 *lb.* by which they are compreſs'd together.

6. The *Sixth* Experiment *ſhews the Spring of the Air throwing the Air out of a Glaſs-Bubble through the Water in which it is placed, in Form of large round Bubbles of Air.* This is done by taking the Preſſure of the Air off the Surface of the Water in the Jar under the Receiver; and by that means the Spring of the Air having nothing to counter-act or confine it, will exert itſelf, and cauſe the Air to eſcape out of the Bubble, and from all Parts of the Water, in very ſmall Globules riſing up to the Top; whence, by the way, it will appear, that Water is a very porous Body, and all its Interſtices poſſeſs'd by Air, which is now expanded into viſible Volumes or Globules, and ſeen to make its Eſcape.

7. The *Seventh* Experiment is but a Part of the former, and ſhews, *that upon letting the Air again into the Receiver, it falls on the Surface of the Water, and by that means compreſſes the whole Body of Water, and drives Part of it into the evacuated Glaſs-Bubble, which then becomes heavier than Water, and ſinks to the Bottom.* As

hauſted Receiver) from a Baſon of Mercury, which then will not riſe in the Pipe

there is but very little Air left in the Bubble, its Spring will be very weak, and ſo will yield to the Force of the external Air compreſſing the Water, and therefore will give the Water Admittance till it becomes ſo far compreſs'd as to have a Spring equal to that of the outward Air, or to that which was in the Bubble at firſt. Its Denſity will then be the ſame alſo; and its Bulk, compared with the whole Bulk of the Bubble, will ſhew what Part of the whole Quantity of Air remained after Exhauſtion.

8. The *Eighth* Experiment *ſhews, that the ſame Bubble, placed with its Neck upon a hollow Glaſs over a ſmall Baſon, under the Receiver, upon exhauſting the Air the ſmall Quantity of Air in the Bubble will again expand itſelf, and drive out all the Water.* The Preſſure of the Air, which before kept the Water in the Bubble, being now taken away, the Spring of the Air in the Crown of the Bubble gradually exerts itſelf, and at laſt expels all the Water. From this Experiment it plainly appears, that the Spring of the Air is equal to the Preſſure, becauſe the Spring drives out all the Water which the Preſſure forced into the Bubble.

9. The *Ninth* Experiment *is the Expulſion of the Contents of an Egg through a ſmall Hole in the little End by the Spring of the Air contained in the great End of the Egg.* While the Egg is new and good, there is always a ſmall Quantity of Air contained in the great End between the Shell and the Skin or *Putamen*, which, upon taking off the Preſſure of the Air from the Hole, will expand itſelf, and drive out the *White and Yolk* through the ſaid Hole in the little End.

10. The *Tenth* Experiment *ſhews, that when half the Shell of the Egg thus emptied is taken off, the ſaid Bubble of Air will, upon Exhauſtion, ſo expand itſelf by its Spring as to raiſe up the Skin of the Egg, and throw it ſo far out as to make the Reſemblance of the entire Egg.* This will happen only when the Egg is quite new; for as

of the Syringe on drawing up the Piſton, as it will in the open Air.

the Egg grows ſtale, the Air will loſe its Spring by degrees, and the Egg will become putrid or addle. It is obſerved by Naturaliſts, that this included Bubble of Air is abſolutely neceſſary for the Production and Maturation of the Chick, which is effected by the Warmth and Fermentation occaſioned by the conſtant *Incubation of the Hen*.

11. The *Eleventh* Experiment *is to ſhew the great Quantity of Air contained in all ſolid Bodies.* For when a Piece of Braſs, Iron, Stone, *&c.* is put into the Water of a Jar under the Receiver, and the Air drawn out, the Spring of the Air contained in the Pores of thoſe ſolid Bodies, will, by expanding the Particles, cauſe them to appear on the Surface in numberleſs Globules, and exhibit a curious Spectacle to the Eye, like the pearly Drops of Dew on the Piles of Graſs; all which ſuddenly diſappear by letting the Air in again.

12. The *Twelfth* Experiment *ſhews, that a Piece of Cork with a Weight added to it, to make it juſt ſink in the Water, will be raiſed to the Top, or made to ſwim, by exhauſting the Air.* For the Bubbles of Air which are expanded from its Pores, and adhering to its Surface, render it lighter than Water, in which Caſe it muſt neceſſarily riſe to the Top, or ſwim.

13. The *Thirteenth* Experiment *ſhews, that Glaſs-Images and Bubbles, which ſink in Water, will, on exhauſting the Receiver, riſe to the Top and ſwim.* For the Bodies of theſe Images, *&c.* being hollow, are fill'd ſo far with Water as to make them juſt ſink; and the reſt of the Cavity being poſſeſſed of Air, this Air will, upon taking off the Preſſure of the external Air, exert its Spring, and drive out the Water from the Images and Bubbles; they then become lighter than the Water, and riſe to the Top. When the Air is let in again, the Water re-enters their Bodies, and they ſink down again.

THE SPRING *of the Air* is demonſtrable by various Experiments: As, (1.) By the

14. The *Fourteenth* Experiment *ſhews, that a Bladder nearly emptied of Air, and ſunk with a Weight to the Bottom of a Jar of Water, will, upon Exhauſtion, riſe to the Top and ſwim.* The Reaſon of which is, that when the Preſſure of the external Air is taken off, the Spring of the little incloſed Air will dilate and expand the Bladder to its full Bulk; and then the Quantity of Water equal to its Bulk will be heavier than the Weight and Bladder, and ſo will buoy them up to the Top, according to the Laws of *Hydroſtatics*, which ſee.

15. The *Fifteenth* Experiment *raiſes Beer or Ale into a large white Head or Froth to the Top of the Jar.* This happens on account of the great Tenacity of the Fluid; for when the Preſſure of the Air is taken off, the Air in the Beer expands itſelf into large Globules, to which the Particles of Beer adhering on every Side render them too heavy to riſe from the Surface, and fly away in the Air. The Bubbles of Air being thus raiſed are, as it were, conglutinated or ſtuck together by the adheſive Quality of the Liquor; and thus riſe in great Quantities, the upper Part being raiſed and ſuſtain'd by the Expanſion of that below. When the Air is let in, the Air-Bubbles contract, ſubſide, and retire within the Pores of the Fluid. In the ſame Manner Soap-Water, Yeaſt, &c. will riſe in a Head.

16. The *Sixteenth* Experiment *is exhibiting the Phænomena of Boiling Water in the exhauſted Receiver.* To this End the Water muſt be as hot as the Finger can well bear when put under the Receiver. Upon exhauſting, the Air-Bubbles will be ſeen to riſe very ſoon, and at firſt very ſmall; they ſoon appear bigger, and at laſt are ſo large, and riſe with ſuch Rapidity, as greatly to agitate the Water, and cauſe it to appear in all the Circumſtances of Boiling; which Agitation of the Water will continue till the Air be let in again, and then it will ceaſe, and all will be quiet and ſtill as at firſt. Some People imagine the Water grows hotter by boil-

great Expanſion of a ſmall Quantity of Air in an emptied Bladder, when the Air

ing under the Receiver, as it does over the Fire; not conſidering that Water boils only by the great Expanſion and Rarefaction of the Air it contains, from whatever Cauſe it proceeds, as from the Heat of Fire, from taking off the Preſſure of the Atmoſphere by the Air-Pump, &c.

17. The *Seventeenth* Experiment *ſhews, that a ſhrivel'd Apple will be plump'd out, and made to look fair, under the exhauſted Receiver.* The Reaſon of which is the Expanſion of the Air in the Subſtance of the Apple, when the Preſſure is taken off from its Surface; for though ſome Parts of the Skin be pervious to the included Air, (as appears by the little Streams riſing from the Pores of the Apple placed in Water) yet the greateſt Part of the Surface is not, and will not therefore ſuffer the Air to paſs out, but will yield and expand to its utmoſt Dimenſions, (and ſometimes burſt) on which all the Wrinkles diſappear, and the Apple puts on a youthful Face, till the Air be again let in, when it inſtantly returns to its former State of Decay and ſhrivel'd Countenance.

18. The *Eighteenth* Experiment *exhibits the beautiful Appearance of Air riſing from all Parts of a vegetable Subſtance very copiouſly through the Water in Vacuo.* For when the Preſſure of the incumbent Air is taken off, the Spring of the Air contain'd in the Air-Veſſels of Plants, will, by expanding the Particles, cauſe them to riſe from the Orifices of all the Veſſels, and that for a long time together, by which is ſhewn what a great Quantity of Air is contain'd in all vegetable Subſtances; and ſince it is ſeen to come out of the Sides or all over the Surface of a Piece of Stick, as well as from its tranſverſe Sections, it is a convincing Proof that the Texture of the Stems of Plants and Trees conſiſt of Veſſels in a longitudinal and alſo in an horizontal Poſition.

is taken off from the external Parts in the Receiver. (2.) By the Extruſion of a Fluid

19. The *Nineteenth* Experiment *ſhews the Method of injecting a vegetable Subſtance with Quickſilver.* Thus if a Piece of Stick be cut even at each End with a Penknife, and immerſed in Mercury, upon pumping out the Air from the Receiver it will at the ſame time come out of the Pores of the Wood through the Mercury, as will be viſible at each End. When the Air is let in again, it falls on the Surface of the Mercury, and forces it into the Pores of the Wood to poſſeſs the Place of Air. When the Wood is taken out and weigh'd, it will be found ſeveral times heavier than before; it will have changed its Colour, being now of a blueiſh Hue all over; and if ſplit or cut tranſverſely, the Quickſilver will be ſeen glittering in all its Pores, and through every Part.

20. The *Twentieth* Experiment *is the breaking of a Bladder by the Weight of the Air.* For if the Bladder be tied over one End of an open Receiver, as the Air is exhauſting the Spring will be weaken'd, and give way to the Preſſure of the Air on the Bladder, in which Caſe the Bladder will put on a concave Figure, which will be nicely ſpherical; and this will continue increaſing, till the Strength of the Bladder be overcome by the Preſſure, when it will break with a very great Report.

21. The *Twenty-firſt* Experiment *is the breaking a Glaſs-Bottle by the Preſſure of the Air.* For this Purpoſe the Bottle ought to be of a ſquare Form, and not cylindrical or globular; it ſhould alſo be not very thick, if ſmall. Then the Bottle is ſcrew'd on to the Hole in the Plate of the Pump, and the Air drawn out; by this means the Bottle ſuſtains the Preſſure from without, ſo long as its Strength will permit; then the Parts yield, and the Bottle is inſtantly reduced into very ſmall Pieces.

22. The *Twenty-ſecond* Experiment *breaks a Bottle by the Spring of the Air.* For the Mouth of the Bottle being ſecurely ſeal'd up, ſo that no Air from within can

out of a Glaſs Bubble, by the Expanſion of the Bubble of Air contain'd therein.

eſcape, it is put under the Receiver; and as the Air is drawn off from its Surface, the Spring of the Air included will take place, and act more and more forcibly againſt the Sides of the Glaſs, which having now nothing but its own Strength to defend it, as ſoon as that is overcome the Parts give way, and the Glaſs is burſt in Pieces.

23. The *Twenty-Third* Experiment *is to ſhew, that a Bladder being emptied of its Air, all to a very little, and then ſuſpended in the Receiver, the little Portion of Air will expand itſelf in ſuch manner upon Exhauſtion, that at laſt it will diſtend and fill out the Bladder to its utmoſt Bulk, and make it appear as one full-blown.* The Reaſon of which is apparent from what has been ſo often repeated above; as alſo, of its contracting again when the Air is let in.

24. The *Twenty-fourth* Experiment *ſhews, that the Syringe will deſcend from the ſuſpended Piſton* in Vacuo, *when the Hole at Bottom is ſtopped, and a ſmall Weight added to overcome the Friction.* If the Hole be ſtopped in the open Air, and the Piſton drawn up, it will be reſiſted by the Preſſure of the incumbent Column of Air; but *in Vacuo*, where this Air is taken away, the Piſton may freely riſe; or, which is all one, the Syringe may deſcend; as it will, if a ſmall Weight be added to overcome the Friction of the Piſton. When the Air is let in again, it will be ſeen to puſh up the Syringe upon the Piſton again.

25. The *Twenty-fifth* Experiment *ſhews, that Water riſes in Pumps, and Quickſilver in the Barometer, by the Preſſure of the Air only.* For a Glaſs Tube being ſcrew'd on to the above-mentioned Syringe, and immerſed in the Mercury in the open Air, if the Piſton be then lifted up, it will attenuate the Air contained in the Glaſs Tube, by giving it a greater Space to expand in, and by this means leſſen its Spring. The Preſſure then of the external Air will raiſe ſo much Mercury into the

(3.) By the Expulſion of the White and Yolk of an Egg through a ſmall Hole in

Tube, as its Weight added to that of the Spring of the included Air is an equipollent Force, and then an Equilibrium will enſue: But if the Mercury be placed under the exhauſted Receiver, and the Piſton lifted up, no Mercury will then be ſeen to riſe; which plainly ſhews the Cauſe, *viz.* the Air's Preſſure is in that Caſe taken away.

26. The *Twenty-ſixth* Experiment *ſhews, that the Spring of the Air has a Force equal to the Preſſure of the Air, by raiſing the Quickſilver to the ſame Height.* For if a Tube open at both Ends be cemented into a Glaſs Vial, nearly fill'd with Quickſilver, and placed under the exhauſted Receiver, as the Air is gradually exhauſted you will ſee the Mercury riſe from the Vial into the Tube above the Pump, by the Spring of the included Air, to the ſame Height as it is in the Gage-Tube below by the Preſſure, and that during the whole Time of the Exhauſtion. And this will always happen, let the Quantity of Air in the Vial be ever ſo ſmall, or what it will; the Phænomenon depending not upon the Quantity, but the Strength of the Spring.

27. The *Twenty-ſeventh* Experiment *ſhews the Method of making an artificial Fountain* in Vacuo, *by the Air's Preſſure.* For this Purpoſe a very tall Glaſs Tube is hermetically cloſed on the Top, and at Bottom by means of a Braſs Cap ſcrew'd on to a Stop-Cock, and that to the Plate of the Pump; then, when all the Air is exhauſted, the Cock is turned, and taken off the Plate, and immerſed in Mercury or Water: Then, upon turning the Cock again, the Fluid by the Preſſure of the Air will be ſeen very beautifully to play up in the Tube in the Form of a Fountain.

28. The *Twenty-Eighth* Experiment *ſhews, that the Magnetic Virtue from the Stone, or a touch'd Piece of Iron, affects the Needle* in Vacuo, *in the ſame manner as in open Air.*

the little End, by the Expanſion of the Air contained in the great End; and alſo,

29. The *Twenty-ninth* Experiment *ſhews, that the Attraction of Coheſion is the ſame* in Vacuo *as in the open Air.* For this Purpoſe a large Glaſs Tube, drawn out into a very fine Capillary at Top, when filled with Water will ſuſtain it to a certain Height in the Air: If the ſame be placed under the Receiver, and the Air drawn out, the Water will remain ſuſpended as before; which ſhews it to be wholly owing to the Force of Attraction.

30. The *Thirtieth* Experiment *ſhews, that Bodies, which equilibrate each other in the Air, loſe their Equilibrium* in Vacuo. Thus if a Piece of Lead at one End of a fine Balance, and a Piece of Cork at the other, are *in Equilibrio* in the Air, and thus placed under the Receiver, as ſoon as the Air begins to be exhauſted, ſo ſoon the Equilibrium will begin to be deſtroy'd, till at laſt, when all the Air is taken away, the Cork will deſcend, and ſhew itſelf really heavier than the Lead. The Reaſon of which is evident from Hydroſtatic Laws; for both Bodies being weigh'd in Air, each would loſe the Weight of an equal Bulk of Air, conſequently the Cork will loſe a greater Weight than the Lead in the Air; and therefore when the Air is taken away, the Weight that is reſtored to it being greater than what the Lead has retrieved, will cauſe it to preponderate, or weigh down the Lead *in Vacuo.* And hence we ſee that *a Pound of Feathers is really heavier than a Pound of Lead,* if weigh'd in the Air.

31. The *Thirty-Firſt* Experiment *ſhews the Air to be the Medium of Sounds.* For if a Bell be ſcrew'd on to the Air-Pump, it will ring in the Air, and be heard under a thin Receiver: But when the Air is exhauſted, the Sound is not heard, which plainly proves it to be propagated by means of the Air; and this is farther evinced by letting the Air gradually into the Receiver, becauſe if, in the mean time, you keep ſhaking the Bell,

(4.) By raiſing up the Skin of the Egg (after the Yolk is taken away, and one half

the Sound will increaſe in Proportion as the Glaſs is fill'd with Air.

32. The *Thirty-ſecond* Experiment *ſhews, that the Air is neceſſary for the Exiſtence of Fire and Flame.* Thus if Charcoal thoroughly lighted, and a Candle burning, be placed under the Receiver, as the Air is exhauſted the Coals will begin to decline and die away, and the Candle will go out by Degrees.

33. The *Thirty-third* Experiment *ſhews the Riſe of Vapours and Smoke to be owing to the Air*; becauſe when the Air is taken away, the Vapours, which at firſt riſe very plentifully from the wet Leathers of the Plate ſo as to obſcure the Receiver, begin to fall when the Air becomes greatly attenuated; and the Smoke, which at firſt roſe from the Candle extinct, now begins to deſcend; and when the Air is all exhauſted, the Receiver becomes quite clear, and free from all Appearance of Smoke or Vapour. Hence, by the way, we ſee the Reaſon why, when the Air grows lighter, it lets fall the Vapours, and the Weather becomes miſty, hazy, and wet or rainy.

34. The *Thirty-fourth* Experiment *ſhews the Exploſion of Gunpowder is owing to the Air.* For if it be kindled *in Vacuo*, the Air, that ſo ſuddenly expands itſelf from the Powder, and gives ſuch a Shock to the common Air, now finds none to encounter, and ſo makes no ſenſible Appearance, otherwiſe than by the ſinking of the Mercury a little in the Gage by its Spring.

35. The *Thirty-fifth* Experiment *ſhews how Halo's are produced by refracted Light.* Thus if a Candle be held on one Side of the Receiver, and the Eye placed at ſome Diſtance on the other, as ſoon as the Air begins to be exhauſted, and becomes attenuated and repleted with Vapours to a proper Degree, the Light of the Candle will be refracted through that Medium in Circles of various Colours, very much reſembling thoſe ſeen about the Moon in a hazy Air at Night.

of the Shell) by the Expanſion of the ſaid included Bubble of Air, ſo as almoſt to fill

36. The *Thirty-ſixth* Experiment *ſhews how the Lungs of an Animal are affected* in Vacuo; *in what Manner it dies, and is revived again.* For this Purpoſe a ſmall Bladder is tied to a Pipe, and ſcrew'd into a Bottle, which then repreſents the Lungs in the Thorax. This Pipe is perforated quite through to the Bladder, and is therefore analogous to the *Trachea* or Wind-pipe. The Air confin'd in the Bottle about the Bladder is in the ſame Circumſtances with that in the Breaſt about the Lungs. When this *Apparatus* is placed under the Receiver, one or two Exſuctions will attenuate the Air in the Receiver and Bladder, upon which the Spring of included Air in the Bottle will compreſs the Bladder, as that in the Thorax does the Lungs; and a few more Turns will cauſe the Bladder to be compreſs'd together. The Lungs being thus compreſs'd, the Animal is ſenſible of a prodigious Weight, the Circulation of the Blood through the Lungs is ſtopped, the Creature is all over convulſed, and at laſt expires in the greateſt Agonies of a moſt cruel Death. When the Air is let in again, the Bladder gradually expands, as do the Lungs of the Animal; and if it has not lain too long, the Blood will again paſs through them, and the Animal will recover its ſuſpended Life.

37. The *Thirty-ſeventh* Experiment *ſhews Air to be abſolutely neceſſary to moſt Sorts of Animals.* This we do by exhauſting the Air from a Cat, a Rat, a Mouſe, a Bird, *&c.* which ſoon die in the Manner above deſcribed. It is not always, indeed, that Gentlemen can thus ſuffer their Curioſity to get the Aſcendant ſo far over their Humanity, as to deſire ſo ſhocking a Spectacle. The Ladies (greatly to their Honour) ſhew more Conſideration, in generally voting againſt it.

38. The *Thirty-eighth* Experiment *ſhews Air is not abſolutely neceſſary to the Life of ſome Animals:* For it is well known that pumping the Air from a Toad, an Eel, a Viper, and all Sorts of Inſects, ſeems not immedi-

the Half-Shell. (5.) Glaſs Bubbles and Images fill'd with Water, ſo as to make

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ately to affect them. Indeed, the winged Inſects cannot fly, but they will crawl and run about very briſkly. Some ſay Fiſh will die for want of Air; I confeſs I never could kill any. They appear greatly diſturb'd, ſwoln, and ſickiſh at firſt; but Mr. *Hawkesby* ſays he has kept them a Week *in Vacuo*, and they recover'd their firſt Illneſs, and were at the Week's End as lively and alert as thoſe which had been kept as long in the Air.

39. The *Thirty-Ninth* Experiment *ſhews no Winged Animal can fly without Air.* For this Purpoſe a large Butter fly is a proper Subject, for as ſoon as it is put under the Glaſs it will fly and flutter about, but when the Air is taken away, nothing more of that kind is ſeen. If a fine Silk be tied about one of the Horns of this Animal, and it be thus ſuſpended in the Middle of the Receiver, it will at firſt fly towards every Side of the Glaſs, but when the Air is exhauſted, it cannot get out of the perpendicular Poſition into which it is brought by its Gravity, though it will be conſtantly endeavouring to do it.

40. The *Fortieth* Experiment *is that of Aduſt or Burnt Air*; this Air is brought into the Receiver through the Fire; and if a Candle be put down into it, it inſtantly goes out, and will do ſo for many times together; but every time the Candle burns longer than before; which ſeems to ſhew, that this Air is ſomewhat of the ſame Nature with that in Mines, commonly call'd *Damps*, and is, like that, purified again by Fire.

41. The *Forty Firſt* Experiment *ſhews that Aduſt Air is inſtant Death to moſt Sorts of Animals.* Thus a Sparrow put into this Air tumbles down with a kind of Vertigo, is convulſed, and dies directly; much after the ſame Manner as Men fall down dead in the contaminated Air of Mines, deep Wells, *&c.*

42. The *Forty-ſecond* Experiment *ſhews that all Bodies deſcend equally ſwift* in Vacuo. Thus a Guinea and Feather let fall from the Top of a tall exhauſted Re-

them juſt ſink in Water, will, upon exhauſting the Air from the Surface, riſe to

ceiver, come down to the Bottom in the ſame time, or both together. But when let fall from thence in the Air, the Feather will deſcend much ſlower than the Guinea, and with an oblique or indirect Motion.

43. The *Forty-third* Experiment *ſhews all Fermentation and Putrefaction depend on Air.* Thus Apples, Pears, Plums, Cherries, *&c.* which in the Air ſoon grow mellow, putrid, and rotten, will, if kept in an exhauſted Receiver, placed under Water, be preſerv'd a long time untainted, appear freſh and in their native Bloom. Thus Eggs alſo, which in the Air ſoon grow ſtale, putrid, and addle, will *in Vacuo* retain their Goodneſs, and be fit for uſe after a great while. Which is the Reaſon why many People keep them in Pots of Butter, Lard, *&c.* to preſerve them from the Air.

44. The *Forty-Fourth* Experiment *ſhews how neceſſary Air is for the Germination and Growth of Plants and Vegetables.* For if the ſame Seed be planted in two different Pots of Earth at the ſame time, and one of them be kept in an exhauſted Receiver, the Difference between the Appearance and Growth of each will be ſufficiently ſenſible to any that ſhall try the Experiment.

45. The *Forty-Fifth* Experiment *ſhews, that the Writing made with Phoſphorus upon Paper, laid on the Plate of the Pump, will* in Vacuo *appear luminous*, and not be extinguiſh'd like common Fire. It will alſo ſend up lucid Fumes or Clouds to the Top of the Receiver.

46. *If the Paper be wetted by Patches, on which the Lines have been drawn with Phoſphorus, inſtead of a Cloud it will give Flaſhes* in Vacuo. For theſe Experiments with Phoſphorus the Room ſhould be made very dark.

47. The *Forty-Seventh* Experiment *ſhews, that upon ſome Chemical Mixtures a ſtrong Effervescency, Ebulition, and Accenſion will happen* in Vacuo. Thus, if to an equal (but ſmall) Quantity of Vitriol, Oil of Tartar *per Deliquium*, and Oil of Cloves, you put two or three ſmall

the Top of the Veſſel. (6.) Alſo a Bladder fill'd with Air, and juſt made to

Pieces of Phoſphorus, the Mixture will take Fire in the open Air, and is put out by the Addition of a little Water. It will not only ſhine, but boil up into a Flame *in Vacuo*.

48. *Melted Lead, and other Metals, ſet to cool* in Vacuo, *have their Surfaces concave*; whereas they are convex in the open Air. The Reaſon of which is the ſame as of the Expanſion of Water when it congeals into Ice. Thus Ice becomes ſpecifically lighter than Water, and ſwims in it; as any ſolid Metal is ſpecifically lighter than when melted: Thus a leaden Bullet ſwims in melted Lead. What Agent Nature employs in the Affair of Congelation, is perhaps as yet unknown to Mortals; but whatever it be, 'tis certain that one Part of its Operation is to ſever the Particles, and fix them at a greater Diſtance from each other in the fixed, than they are in the fluid State.

49. *The Chemical Proceſs of Cryſtallization will not ſucceed* in Vacuo. If Salts be mixed with Water and evaporated to a Pellicle, and then placed under an exhauſted Receiver, and ſet in a cool Place as uſual, it will not ſhoot into Cryſtals, as in the open Air it readily will.

50. The *Fiftieth* Experiment ſhews, that if a Piece of Wood be cemented in the lower Part of the Neck of the open Receiver, and Mercury be poured upon it, after two or three Exhauſtions the Preſſure of the Air will be ſo great on the Mercury, as to cauſe it to deſcend through the Pores of the Wood in Form of a beautiful Shower; which will ſhine (if it be well cleanſed and the Weather dry) in a dark Room. The Air alſo will follow the Quickſilver through the Pores of the Wood, and cauſe the Gage to ſink.

51. To theſe Experiments of a *Vacuum*, I ſhall add the following Particulars relating to the CONDENSATION of Air: As, (1.) That the Veſſel ought to be very ſtrong to bear the Force of the Air's Spring thus in-

ſink with a Weight, will, upon Exhauſtion, ſoon riſe by the Expanſion of the contain'd

creaſed; for which Reaſon they are generally made of Braſs. (2.) If Glaſs be uſed for a Condenſer, it will not indeed ſuffer ſo great a Degree of Condenſation, but the Experiment will be pleaſanter, by viewing the Subject placed in the condenſed Air. (3.) The Spring of the Air will be greater in proportion to its Condenſation, and therefore (4.) The Sound of the Bell will be twice and thrice as loud as in the common Air, if the Air be made twice or thrice as denſe by Injection. (5.) A round Vial will be broke by condenſed Air, that could not be broke by the Preſſure of the common Air. (6.) Though Animals ſoon die by not having the natural requiſite Quantity of Air, yet they will not be eaſily kill'd by having that Quantity increaſed by Condenſation. (7.) If Air be condenſed upon Water in a Bottle, it will cauſe it to ſpout through the Tube of Communication to a very great Height, *viz.* to 30 Feet, if only one Atmoſphere be injected; to 60 Feet, if two; and ſo on. (8.) A Bladder, that will ſuſtain the Spring of common Air, will be broke by the Spring of condenſed Air. In ſhort, the Force of condenſed Air may be ſo far increaſed, as to countervail or antagoniſe the greateſt Power of Nature that we can apply. (9.) Water with Air condenſed upon it will conceive a much greater Degree of Heat than in the common Air, where it will boil much ſooner than in condenſed Air. (10.) So great may the Degree of Heat acquired in Water this Way be, as to melt ſoft Solder; and therefore Veſſels ſhould have their Parts put together with hard Solder, that are uſed about theſe Experiments.

52. From this vaſt Power of confined and elaſtic Air and Steam it is that we account for the prodigious Effects of *Papin's* DIGESTER in diſſolving Bones and reducing them to a Jelly, ſo as to become a wholſome and ſavoury Diet; for which Purpoſe they are put into a metalline Veſſel, with a Cover, which is faſt and

Air. (7.) The Spring of the Air will ſhew itſelf alſo by raiſing heavy Weights laid on a Bladder, half fill'd with Air, in a proper Veſſel under the Receiver. (8.) Beer, Cyder, Water, and porous Bodies, do emit great Quantities of Air under the exhauſted Receiver. (9.) Fiſhes are made ſo light or buoyant by increaſing the Spring of the Air in their Bladders, upon Exhauſtion, that they riſe to the Top of the Water, and cannot again deſcend to the Bottom. (10.) Shrivel'd Apples are made to look fair and ſmooth by the Spring of the contain'd Air filling out the Wrinkles. (11.) The Spring of the Air in a ſquare Bottle, cemented cloſe, will immediately burſt it in Pieces, upon exhauſting the incumbent Air. (12.) But that curious Experiment which ſhews the Force of the Spring of the Air to be equal to its Weight

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ſtrongly ſcrew'd down, and Air-tight. The Digeſter nearly fill'd with Water and Bones is ſet over a gentle Fire, which by Degrees rarefies the Water into Steam, which with the included Air in a ſhort Space of Time acts upon the Bones with ſo great an Energy, as to effect their utter Diſſolution, and cauſe them to mix and incorporate ſo intimately with the Water, or Broth, as to make a perfect *Coagulum* or Jelly when all is cold, which may be then ſliced out with a Knife. They who would ſee more of the wonderful Effects of this Inſtrument may conſult the Author's own Book upon the Subject.

or Preſſure, is by raiſing the Mercury, by the Expanſion of a ſmall Quantity of confined Air, to the ſame Height in an exhauſted Tube above the Pump, as that which it is raiſed to in the Mercurial Gage by the Preſſure of the Atmoſphere below it.

THE great Action of animal Life, *viz.* *Breathing* by *Inſpiration* and *Expiration* of Air, is owing to the *Preſſure* and *Spring* of the Air conjointly, as is evident by the *Contraction* and *Expanſion* of a Bladder in a ſmall Receiver, with a Bladder tied on at Bottom to repreſent the Diaphragm.

HENCE the Neceſſity of Air for *Reſpiration* and *animal Life* in moſt Sorts of Creatures, which die very ſoon in the *exhauſted Receiver:* Though ſome Animals will not be kill'd in this manner; as *Flies*, *Frogs*, *Toads*, ſome ſort of *Fiſhes*, &c.

THAT Air paſſing through the Fire, and heated Braſs Tube, is unfit for animal Reſpiration, is ſhewn by the ſudden Death of any Animal put into a Receiver fill'd therewith. Alſo Candles and living Coals, put into this aduſt Air, immediately go out. Hence the noxious and peſtilential Qualities of *Damps* and *ſuffocating Exhalations*, ſo frequent and fatally experienced in Mines, and other ſubterranean Places.

THAT Air in its natural State is neceſſary for *Fire* and *Flame*, is obvious from the *ſudden Extinction of a Candle, a live Coal*, &c. in the *exhauſted Receiver*. Alſo Gunpowder fired therein will not take Flame, or be *exploſive*, but *melt and die away*.

THAT the different Velocities with which heavy and light Bodies deſcend in the Air, is owing to the Air's Reſiſtance only, is manifeſt from the equal Velocity or Swiftneſs with which all Bodies deſcend in the *exhauſted Receiver*, as is ſhewn in the Experiment with a *Guinea* and a *Feather*.

AIR is likewiſe neceſſary for the Exiſtence and Propagation of Sounds; for a Bell placed under the Receiver, and rung, will not be heard when the Air is drawn out; but in condenſed Air, the Sound will be augmented in Proportion to the Condenſation.

THAT *Fermentation*, *Putrefaction*, &c. depend on the Air, and are promoted by it, is ſhewn by preſerving the Fruit in their natural Bloom and Perfection thro' the Winter in an *exhauſted Glaſs*.

THE Uſe of the *Diving-Bell* depends on the *Preſſure* and *Spring* of the Air: For ſince the Space which Air takes up is reciprocally as the Power compreſſing it, 'tis evident that at the Depth of 33 Feet of Water, where the Preſſure of the Atmoſphere is doubled, the Bell will be half fill'd with Water, at the Depth of 66 Feet it will be two Thirds fill'd; at the Depth of 99 Feet it will be three Fourths fill'd; and ſo on. Whence appears the Neceſſity of having the Veſſel in the Form of a *Bell*, that the perpendicular Height of the Water may be as little as poſſible. Hence alſo we ſee how neceſſary it is to have a very gentle Deſcent of the Bell, that the Divers may have Time to admit the Air, ſo greatly condenſed, by proper Degrees, leſt it ſhould burſt the fine Veſſels of their Bodies, and kill them: Together with ſeveral other Particulars relating to the Nature and Manner of uſing this Machine, which will be more fully explain'd in the Note below. (XCII).

(XCII.) 1. That the Reader may have a juſt Idea of the *Campana Urinatoria* or DIVING-BELL, according to the lateſt Improvements by Dr. *Halley* and Mr. *Triewald* of *Stockholm*, I have here exhibited two Figures of the ſame. The firſt is that of Dr. *Halley*'s Form, which was 3 Feet wide at Top, 5 Feet at Bottom, and 8 Feet high; and contain'd about 63 Cubic Feet, or near 8 Hogſheads, in its Concavity.

2. This was coated with Lead, ſo heavy that it would ſink empty; and the Weight was diſtributed about the Bottom I K, that it would go down in a perpendicular

Plate XXIX. Fig. 4.

The *Spring* of the Air is moſt evidently concerned in that Chirurgical Operation

Poſition and no other. In the Top was fixed a ſtrong but clear Glaſs D, to let in the Light from above; and likewiſe a Cock, as at B, to let out the hot Air that had been breath'd; and below, at L M, was fixed a circular Seat for the Divers to ſit on; and laſtly, from the Bottom was hung, by three Ropes, a Stage for the Divers to ſtand upon to do their Buſineſs. This Machine was ſuſpended from the Maſt of a Ship by a *Sprit*, which was ſufficiently ſecured by *Stays* to the Maſt-head, and was directed by *Braces* to carry it over-board clear of the Side of the Ship, and to bring it in again.

3. To ſupply the Bell with Air under Water, two Barrels, ſuch as C, of about 63 Gallons each, were made and caſed with Lead, ſo that they might ſink empty, each having a Hole in its loweſt Part to let in the Water, as the Air in them is condenſed in their Deſcent, and to let it out again when they were drawn up full from below. And to a Hole, in the Top of the Barrels, was fixed a Hoſe or hollow Pipe, well prepar'd with Bees-Wax and Oil, which was long enough to fall below the Hole at the Bottom, being ſunk with a Weight appended, ſo that the Air in the upper Part of the Barrels could not eſcape, unleſs the lower Ends of theſe Pipes were firſt lifted up.

4. Theſe Air-Barrels were fitted with Tackle, proper to make them riſe and fall alternately, like two Buckets in a Well; in their Deſcent, they were directed by Lines faſten'd to the under Edge of the Bell to the Man ſtanding on the Stage to receive them, who by taking up the Ends of the Pipes above the Surface of the Water in the Bell, gave Occaſion for the Water in the Barrels to force all the Air in the upper Parts into the Bell, while it enter'd below, and fill'd the Barrels. And as ſoon as one was diſcharged, by a Signal given, it was drawn up, and the other deſcended, to be ready for Uſe.

we call *Cupping*; for when a *Vacuum* is made by a Syringe in the Cupping-Glaſs

5. As the cold Air ruſh'd into the Bell from the Barrel below, it expell'd the hot Air (which was lighter) through the Cock B, at the Top of the Bell, which was then open'd for that Purpoſe. By this Method, Air is communicated ſo quick, and in ſuch Plenty, that the Doctor tells us, he himſelf was one of five who were together at the Bottom, in nine or ten Fathoms Water for above an Hour and an half at a Time, without any Sort of ill Conſequence; and he might have continued there, as long as he pleaſed, for any thing that appeared to the contrary.

6. In going down, 'tis neceſſary it ſhould be very gently at firſt, that the denſe Air may be inſpired to keep up, by its Spring, a Balance to the Preſſure of the Air in the Bell. Upon each 12 Feet Deſcent, the Bell is ſtopp'd, and the Water that enters is driven out by letting in three or four Barrels of freſh Air. By this Means, the Doctor ſays, he could (by taking off the Stage) lay the Bottom of the Sea, juſt within the Compaſs of the Bell, ſo far dry, as not to be over Shoes thereon.

7. By the Glaſs above ſo much Light was tranſmitted when the Sun ſhone, and the Sea was clear and even, that he could ſee perfectly well to write and read, and much more to take up any Thing under the Bell; and by the Return of the Air-Barrels, he could ſend up Orders, written with an Iron Pen, on ſmall Pieces of Lead, directing they were to be moved from Place to Place.

8. But in dark Weather, when the Sea was rough and troubled, it would be as dark as Night in the Bell; but then the Doctor found he could keep a Candle burning in the Bell, as long as he pleaſed; it being found by Experiment, that one Candle conſumes much about the ſame Quantity of confined Air as one Man does, *viz.* about a Gallon *per* Minute.

applied to any Part, the *Spring* of the Air
in the Fleſh under the Glaſs does ſtrongly

9. The only Inconvenience the Doctor complain'd of was, that upon firſt going down they felt a ſmall Pain in their Ears, as if the End of a Quill were forcibly thruſt into the Hole of the Ear. This may proceed from its being ſome Time before the Air can get from the Mouth, through the ſmall Canal of the *Euſtachian Tube*, which leads to the inner Cavity of the Ear; where, when it comes, it makes an *Equilibrium* with the outward Air, preſſing on the *Tympanum*, and thus the Pain, for a ſhort Time, ceaſes; then deſcending lower, the Pain of the Ears returns, and is again abated; and ſo on till you come down to the Bottom, where the Air is of the ſame Denſity continually.

10. One of thoſe Divers (who thought to out-wit Dame Nature for once) put a Piece of chew'd Paper in his Ears, which, as the Bell deſcended, was ſo forcibly preſſed into his Ears, that it was with great Difficulty the Surgeon could extract it. Thus a Bottle with only common Air in it, and cork'd down tight, if it be let down to a conſiderable Depth of Water, will be found, upon drawing it up again, to have had the Cork forced in by the Preſſure of the Water at that Depth.

11. This Bell was ſo far improved by the Doctor, that he could detach one of his Divers to the Diſtance of 80 or 100 Yards from it, by a Contrivance of a Cap or Head-piece, ſomewhat like an inverted Hand-Baſket, as at F, with a Glaſs in the fore Part, for him to ſee his way thro'. This Cap was of Lead, and made to fit quite cloſe about his Shoulders; in the Top of it was fixed a flexible Pipe communicating with the Bell, and by which he had Air when he wanted, by turning the Stop-Cock near his Head-piece. There was alſo another Cock at the End in the Bell to prevent any Accident happening from the Perſon without.

12. This Perſon was always well clothed with thick Flannels, which were warm'd upon him before he left

act, and by that means cauſes the Fleſh to diſtend and ſwell into the Glaſs, while the

the Bell, and would not ſuffer the cold Water to penetrate to hurt him. His Cap contain'd Air enough to ſerve him a Minute or two; then by raiſing himſelf above the Bell, and turning the Cock F, he could replenish it with freſh Air. This Pipe he coil'd round his Arm, which ſerved him as a Clue to find his Way to the Bell again.

13. This Diving-Bell receiv'd its laſt Improvement from Mr. *Martin Triewald*, F. R. S. Captain of *Mechanics* and *Military Architecture*, to his *Swediſh* Majeſty; the Manner and Form whereof is ſhewn in a Figure of his own drawing. A, B, is the Bell, which, as appears by the Scale of Feet under it, is much leſs than Dr. *Halley*'s, and therefore will come cheaper. It is ſunk with leaden Weights D, D, appended at the Bottom; the Subſtance of the Bell is Copper, and tinn'd within all over; and as in the Rivers and Coaſts of the *Baltic* Sea, the Water is very clear, ſo he has illuminated the Bell with three ſtrong convex Lenſes G, G, G, with Copper-Lids H, H, H, to defend them.

Plate XXIX. Fig. 5.

14. The Iron Ring, or Plate E, ſerves the Diver to ſtand upon when he is at Work; and it is ſuſpended at ſuch a Diſtance from the Bottom of the Bell, that when the Diver ſtands upright, his Head is juſt above the Water in the Bell, and it is much better there than higher up in the Bell, becauſe the Air is colder, and conſequently more freſh and fit for Reſpiration near the Surface of the Water, than towards the Top of the Bell.

15. But when there is Occaſion for the Diver to be wholly in the Bell, and his Head of Courſe in the upper Part, Mr. *Triewald* has contrived, that even there, when he has breath'd the hot Air as long as he well can, by means of a ſpiral Copper Tube *b*, *c*, placed cloſe to the Inſide of the Bell, he may then draw the cooler and freſher Air from the lowermoſt Parts; to which End, a flexible Leather Tube, about two Feet

Preſſure of the Air on the Parts without the Glaſs accelerates the Motion of the Blood and Fluids, towards the Part where it is diminiſhed or taken off by the Glaſs.

SINCE we know that *Heat* augments the repellent Power in the Particles of a Fluid, and by that means increaſes its Elaſticity, and thereby cauſes it to expand itſelf into a large Space; and that *Cold* has a quite contrary Effect; we learn the Uſe of the THERMOMETER in indicating the *various Degrees of Heat and Cold in the Air*, by the different Altitudes of the Spirit of Wine in that Inſtrument. (XCIII).

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long, is fixed to the upper End of the Tube at *b*, to the other End of which is a turned Ivory Mouth-piece, for the Diver to hold in his Mouth, to reſpire the Air from below by; and this he may do in any Poſture of ſtanding, ſitting, bowing his Body, *&c.*

16. I muſt not omit in this Place to mention the Invention of a Gentleman at *Newton-Buſhel* in *Devonſhire*, who contrived a large Caſe of ſtrong Leather, to hold about half a Hogſhead of Air, and made it perfectly Water-tight, adapting to it proper Parts for his Legs and Arms, with a Glaſs before; ſo that when he put himſelf into this Caſe, he could walk very eaſily about at the Bottom of the Sea, and go into the Cabin, and other Parts of a Ship in a Wreck, and deliver out the Goods; and this Method he practiſed forty Years, and thereby acquired a large Fortune, and equal Fame.

(XCIII.) 1. A THERMOMETER being deſign'd to indicate the *various Degrees of Heat and Cold* by the elaſtic or expanſive Power of Bodies of the Fluid ſort, ſo many Ways, Methods, and Forms of conſtructing ſuch an uſeful Inſtrument have been thought of, and invented

Plate XXIX

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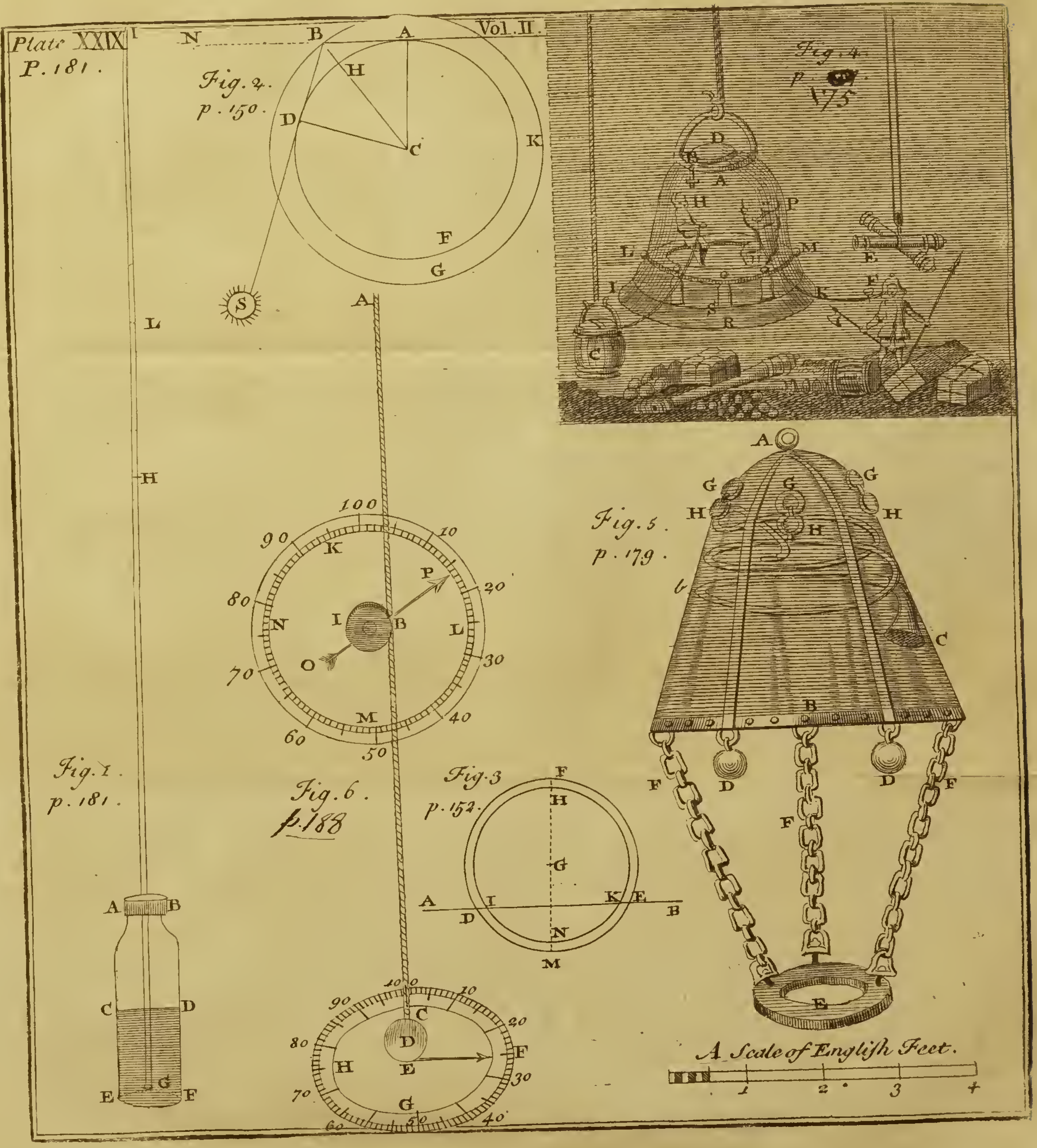

Fig. 1. p. 181.

Fig. 2. p. 150.

Fig. 3 p. 152.

Fig. 4. p. 175

Fig. 5. p. 179.

Fig. 6. p. 188

invented at ſeveral Times for this Purpoſe; at firſt *Air*, then *Oil*, then *Spirits of Wine*, and laſtly *Quickſilver* have been every Way attempted and tortur'd in this Experiment.

2. The Spring of Air being ſooner affected by *Heat* and *Cold* than that of any other Fluid, was firſt thought upon as the beſt Expedient to anſwer this End; and ſo it really would be, were it not that the Weight or Preſſure of the Atmoſphere affects it alſo at the ſame time; and by acting ſometimes with, ſometimes againſt it, renders the Effect by Heat or Cold very uncertain, and therefore the Inſtrument uſeleſs. For Example: The Air in the Bottle A F will, by its Expanſion, when the Air grows warmer, raiſe the Water higher in the Tube than the Point H, and if the Air be lighter at this time it will preſs leſs on the Surface of the Water at H, and ſo will ſuffer it to riſe ſtill higher. But if the Air be heavier it will act againſt the Spring, and not permit it to raiſe the Water ſo high. The ſame may be obſerved with reſpect to its Contraction by Cold; wherefore ſuch an Inſtrument, for common or conſtant Uſe, will not do at all, though perhaps none is better calculated for ſome extemporaneous Uſes, as meaſuring the Degree of Coldneſs in different Cellars, or of Warmth in divers Rooms upon the ſame Floor.

Plate XXIX. Fig. 1.

3. It was upon this Account found neceſſary to have Recourſe to ſome other Fluid, which, ſecured from the Preſſure of the Air in a Tube hermetically ſeal'd, might expand and contract ſolely by the Heat and Coldneſs of the Air about it. And becauſe moſt Fluids are ſubject to freeze or thicken in great Degrees of Cold, it was ſoon conſider'd that Spirits of Wine, a little tinged with Cochineal would beſt anſwer the Purpoſe, and accordingly Thermometers were generally made therewith, and became of common Uſe.

4. Though the Spirit of Wine Thermometers would do very well to ſhew the comparative Heat of the Air, yet this was far ſhort of the Virtuoſo's Views, who wanted to explore the various and vaſtly different Degrees of Heat in other Bodies, as *boiling Water*, *boiling Oils*, *melted Metals*, and even *Fire* itſelf, and Degrees of Cold too, beyond what the Spirit Thermometer can

ſhew. For Spirit in a moderate Degree of Heat will burſt the Tube ; and in an intenſe Degree of Cold will freeze, as the *French* Philoſophers found, who went to meaſure a Degree upon the Surface of the Earth under the North Polar Circle.

5. It having been found by Experiment, that Linſeed Oil required four times the Degree of Heat to make it boil as Water did, it was quickly ſubſtituted inſtead of Spirits for Philoſophic Uſes. This Sir *Iſaac Newton* always uſed, and by it diſcovered the comparative Degree of Heat which makes Water boil, which melts Wax, which makes Spirit of Wine boil, and melts Tin and Lead ; beyond which we do not find the Oil Thermometer has been applyed ; for which reaſon (as alſo for its ſullying the Tube) it has been leſs uſed of late, and given way to.

6. The MERCURIAL THERMOMETER which will ſuſtain any Degree of Heat or Cold, as far as any Inſtrument of this Kind can be expected to do. Mr. *Farenheit* of *Amſterdam*, was the Contriver of this Thermometer, and though ſeveral Artificers made them as well as he, yet they ſtill go by his Name. Dr. *Boerhaave* uſed only this Thermometer. As the Mercury very freely and uniformly expands itſelf from hard Froſt to the Heat of Summer, ſo one Sort of thoſe Thermometers are contrived with a Scale, to include thoſe Extremes only, and the Beginning of the Diviſions, or 0, is fixed to that Altitude of the Quickſilver, as is obſerved when Water juſt begins to freeze, or Snow to thaw ; for which reaſon that is call'd the *Freezing Point* in the Scale. This Thermometer is ſmall, ſhort, put in a neat Frame, and carried in the Pocket any where.

7. But the *Grand Thermometer* of FARENHEIT is graduated after a different Manner, as deſign'd to a more critical and extenſive Uſe. In this the Bulb, or large Part at the Bottom, is not *ſpherical* (as in common ones) but *cylindrical*, to the End, that the Heat may penetrate and reach the inmoſt Parts as ſoon as poſſible, ſo that the whole may expand uniformly together. Hence it is, that in the cylindric Bulb, the Fluid will expand and riſe immediately, whereas in the ſpherical Bulb, it is ſeen firſt to fall (by the ſudden

Expanſion of the Ball, before the Fluid is heated) and then to riſe, by the Expanſion of the Fluid when heated. I have here given a Figure, both of *Farenheit's* *Mercurial Thermometer*, and alſo of Sir *Iſaac Newton's* made with Linſeed Oil. Plate XXX. Fig. 3.

8. I take this of Sir *Iſaac's* to be the beſt fitted of any for a *Standard Weather Thermometer*; and even for any Degree of Heat which the various States of the human Body exhibit; and alſo for thoſe different Degrees which Vegetation requires in the Green Houſe, Hot-Bed, *&c.* In all which Caſes 'tis neceſſary there ſhould be one common, unerring, and univerſal Meaſure, or Standard, which at all times, and in every Place, will ſhew the ſame Degree of Heat, by the ſame Expanſion of the Fluid, according to which the Scale ſhould be made in every Standard Thermometer.

9. In order to this, the Tube propoſed ſhould be very nicely weigh'd when empty, and then the Bulb, and about a tenth Part of the Length of the Tube above it, is to be fill'd with Quickſilver; then it is to be weigh'd again, and the Exceſs of this, above the former Weight, will give the Weight of the Quickſilver pour'd in; this will give the Weight of 100dth Part. Let a Mark be made with a File upon the Tube at the Surface of the incloſed Quickſilver.

10. Then weigh out 9 or 10 Parcels of Quickſilver, each equal to 100dth Part of that firſt put in the Tube, and having pour'd the ſeveral Parcels in one after another upon the incloſed Quickſilver, and marked the Tube ſucceſſively at the Surface of each Parcel, you'll have the Tube divided into proper Intervals, which, if the Bore of the Tube be every where the ſame, will be equal to each other; if not, they will be unequal; and each of theſe Intervals is to be divided into 10 others, increaſing or decreaſing as the Intervals do.

11. When this is done, the Capacity of the Tube is divided into *Thouſandth Parts* of that of the Ball, and the contiguous Part of the Tube reaching up to the firſt Mark. The Tube is now to be put into a Frame, and by the Side of it is to be placed a Scale, divided into *Thouſandth Parts*, exactly correſponding to thoſe

on the Tube; and writing 1000 over-againſt the firſt Mark, you write 1010 over-againſt the ſecond, 1020 againſt the third, and ſo on, as you ſee in the Figure.

12. The Standard Thermometer-Tube, and its Scale, being thus conſtructed, is then to be fill'd with ſome proper Fluid, as *Linſeed Oil*, where great Degrees of Heat are not propoſed; and Mercury is to be uſed, when they are. When the Fluid is poured in, it is to be adjuſted in ſuch a Quantity, that it may ſtand juſt at the principal Point, marked 1000, in *Water juſt freezing*. And here great Precaution is to be uſed; for many Trials muſt determine this Point to which the Fluid muſt always riſe by ſlow Degrees, and with an uniform Motion.

13. When this Point is well ſecured, all the Trouble is over, the Ball being then immerſed in *boiling Water*, *Spirits*, *Oils*, *melted Metals*, &c. in *Snow*, *Freezing Mixtures*, &c. the Expanſions, by all the various Degrees of Heat and Cold, will be ſhewn by the Numbers againſt the Heights to which the Fluid riſes in the Tube in each Caſe, theſe are to be wrote on the Side of the Scale; and ſince the ſame Degree of Heat will cauſe the ſame Expanſion of the ſame Fluid at all Times, 'tis evident, if Thermometers were every where conſtructed in this Manner, the Obſervations made by them in any Part of the World, may be compared together, which cannot otherwiſe be done; whence this Part of Philoſophy would receive its final Perfection.

14. By one of thoſe Standard Thermometers well made, many more might ſoon be conſtructed with any expanding Fluid, without the Trouble of graduating their Tubes by equal Quantities of Quickſilver. For having fill'd the Balls, and a convenient Part of the Tube, with the propoſed Fluid, place them altogether in a Veſſel of cold Water; and while it is warming as gently as poſſible, when the Oil in the Standard Thermometer ſhall arrive ſucceſſively at the ſeveral Diviſions of its Scale, at the ſame Inſtant of Time mark the new Tubes at the ſeveral Heights of their Fluids, and form a Scale for every Tube, that ſhall correſpond to thoſe Marks. Then, while the Liquors ſubſide by cooling

gently, examine whether they nicely agree at the ſeveral Marks. To determine the *Freezing Point* in all, they are to ſtand together in the Water till it juſt begins to freeze: Or, having all the other Points duly, that may be deduced very exactly by the Rule of *Proportion*.

15. A Thermometer that ſhall vary very ſenſibly by every ſmall Variation of Heat and Cold, as thoſe of the Atmoſphere, muſt have a large Ball in Proportion to the Bore of the Tube; and that the Heat or Cold may ſooner penetrate the innermoſt Parts of the Liquor, the Ball ſhould not be ſpherical, but oblong and flatted like a *French Flaſk*; and the Lengths of the Tubes ſhould be proportioned to the Degrees of Heat they are intended to diſcover.

16. Sir *Iſaac Newton* graduated his Standard Thermometer on both Sides, as ſhewn in the Figure. Thoſe on the Right Hand meaſured the *Heat of the Oil*; as thoſe on the Left meaſured the Bulk thereof: But ſince the latter, as well as the former, begins from a Cypher at the Freezing Point, and is regularly continued upwards by the common Diviſions 10, 20, 30, 40, &c. it will equally ſerve both Purpoſes; ſince the Degree of Heat will always be proportioned to the Expanſion of the Bulk of the Fluid above or below the Freezing Point.

17. By this Diviſion therefore on the Left Hand, I ſhall expreſs ſome of the principal Articles of Sir *Iſaac Newton*'s Scale of the various Degrees of Heat, as in the Tablet below.

D. of Heat.	
0	Water juſt freezing, and Snow juſt thawing.
1 to 4	The Heats of the Air in Winter.
4 to 8	The Heats of the Air in Spring and Autumn.
8 to 12	The Heats of the Air in Summer.
13	The greateſt Summer-Heat.
26	The greateſt Heat of the external Parts of the Human Body.
31	Water juſt tolerable to the Hand at Reſt.

D. of Heat.	
$36\frac{1}{2}$	Water hardly tolerable to the Hand in Motion.
43	Melted Wax juſt growing ſtiff and opake.
$51\frac{1}{2}$	Melted Wax juſt before it bubbles or boils.
54	Spirit of Wine juſt begins to boil.
72	Water begins to boil.
75	Water boils vehemently.
86	A Mixture of $\frac{1}{10}$ of Lead, $\frac{2}{5}$ of Tin, and $\frac{1}{2}$ Biſmuth, melts.
103	A Mixture of equal Parts of Tin and Biſmuth melts.
122	A Mixture of $\frac{3}{5}$ of Tin and $\frac{2}{5}$ of Lead melts.
154	The Heat which melts Tin.
174	The Heat which melts Biſmuth.
206	The leaſt Heat which melts Lead.
290	The Heat with which burning Bodies ſhine in a dark Night.
410	The Heat of a ſmall Coal-Fire.
450	The Heat of a ſmall Wood-Fire.

18. Dr. *Hales* conſiders the Freezing Point as one Boundary to Vegetation, *viz.* on the Side of Cold; and the other Boundary he fixes to that Degree of Heat with which Wax will begin to melt, becauſe a greater Degree of Heat will, inſtead of collecting and aſſimilating the nutritive Particles, diſſipate them, even thoſe which are moſt viſcid and glutinous; and therefore the Plant will rather fade than vegetate in ſuch Degrees of Heat.

19. This Space the Doctor divided into 100 equal Parts in his Thermometers: But his Numbers expreſs'd in thoſe of the Standard Thermometer, are for ſeveral Particulars mentioned by the Doctor as follows. For *Myrtle*, $4\frac{1}{2}$; *Oranges*, $6\frac{1}{2}$; *Ficoides*, $7\frac{1}{2}$; Indian *Fig*, $8\frac{1}{4}$; *Aloe*, 10; *Cereus*, 11; *Euphorbium*, 12; *Piamento*, 13; *Ananas*, $14\frac{1}{2}$; *Melon-Thiſtle*, $15\frac{1}{2}$; *Air under the Glaſs of a Hot-Bed*, 17; *the Hot-Bed itſelf*, 28. If the Hot-Bed exceed the Heat of 40 or thereabouts, it will ſcorch the Plants and kill them. The Heat of Milk from the Cow is 28, that of Urine 29, and of Blood in a Fever nearly 40.

20. As *Farenheit*'s Thermometer is come into ſuch general Uſe, I have here placed it by the Standard Thermometer,

THE *Moiſture* and *Dryneſs* of the Air are ſhewn by the HYGROMETER, which is made ſeveral ways, but that with a *Cord* is moſt common and uſeful; for that by ſhrinking with Moiſture will turn an Index one way, and extending with Dryneſs will turn it the contrary way, over the graduated Limb of a Circle. (XCIV) (XCV).

meter, that the Diviſions on each may be reduced to the other's reſpectively by bare Inſpection, and the Uſe of both be ſtill preſerved. If the Reader would ſee all the different Sorts of Thermometers, or rather all the different Methods of graduating them, he may be fully ſatisfied by conſulting Dr. *George Martine*'s Treatiſe on this Subject.

21. To conclude this Head, it muſt be obſerved that the Right Hon. the Earl of *Cavendiſh* has very lately invented a curious Conſtruction of *Thermometers*, of two different Forms; one contrived to ſhew the greateſt Degree of Heat, and the other the greateſt Cold, that may happen in a Perſon's Abſence at any Time. But this Improvement cannot be underſtood ſo well by Deſcription as by viewing the Inſtruments themſelves.

(XCIV.) 1. An HYGROMETER, ſometimes call'd a NOTIOMETER, is an Inſtrument or Contrivance, by which we can eſtimate the Quantity of Moiſture or Vapours in the Air; or by which we can compare the various Degrees of its Humidity and Siccity at different Times. For this Purpoſe different Subjects have been at times eſſay'd, but none as yet have been found ſatisfactory or laſting.

2. Thus Cotton, Spunge, *&c.* hung at the End of a nice Balance, in an exact Equilibre, will by contracting Moiſture from the Air become heavier, which will therefore be ſhewn by its deſcending; and when the Air becomes drier, it ought to part with the Moiſture and

I SHALL finiſh this Lecture with giving you an Account of the Structure

become lighter; but this it will not readily do, and is therefore of little Uſe. Salts have been likewiſe uſed this way, but to no Purpoſe.

3. It would be endleſs to take notice of all the Methods that have been attempted by Philoſophers, and moſt of them without Succeſs. However, as ſome are better than others, and will endure for a conſiderable Time very well, I ſhall here give an Account of one which is the beſt of any I have hitherto thought of. It is made of a String either of Hemp or Cat's-Gut, (as all the beſt Sort are) and ſhews the increaſing *Moiſture* of the Air by its *Twiſting* and *Shortening*, and the *Dryneſs* by *Untwiſting* and *Lengthening*.

Pl. XXX. Fig. 6. 4. Thus let A B C be the lower Part of a twiſted Line or Cord, hanging from the Height of the Room againſt one Side thereof on the Wall or Wainſcot; let there be deſcribed a large Circle, graduated into 100 equal Parts, ſuch as K L M N; in the Centre of which is a Pin, with a ſmall Pulley I B, carrying an Index O P. If now a Cord be put round the Pulley, and a ſmall Weight or Ball D be ſuſpended at the lower End to keep it ſtrait, then as the Cord gathers Moiſture from the Air, it will twiſt and become ſhorter; the Conſequence of which will be, that in contracting it will turn the Pulley I B, and this by its Index will point to the Numbers on the graduated Circle, which will ſhew the Degree of Moiſture or Dryneſs by the Contraction or Relaxation of the Cord.

5. Again: If the Ball D hang over the Centre E of another graduated Circle C F G H placed horizontally, carrying an Index E F upon its Diviſions, it will ſhew the ſame Thing by the twiſting and untwiſting of the Cord B C, as in the Circle above; ſo that this may be looked upon as a *Double Hygrometer*, and ſo ſimple in its Structure, that any Perſon may make it; and that it will anſwer very well for a conſiderable Time, I am

RPJCB

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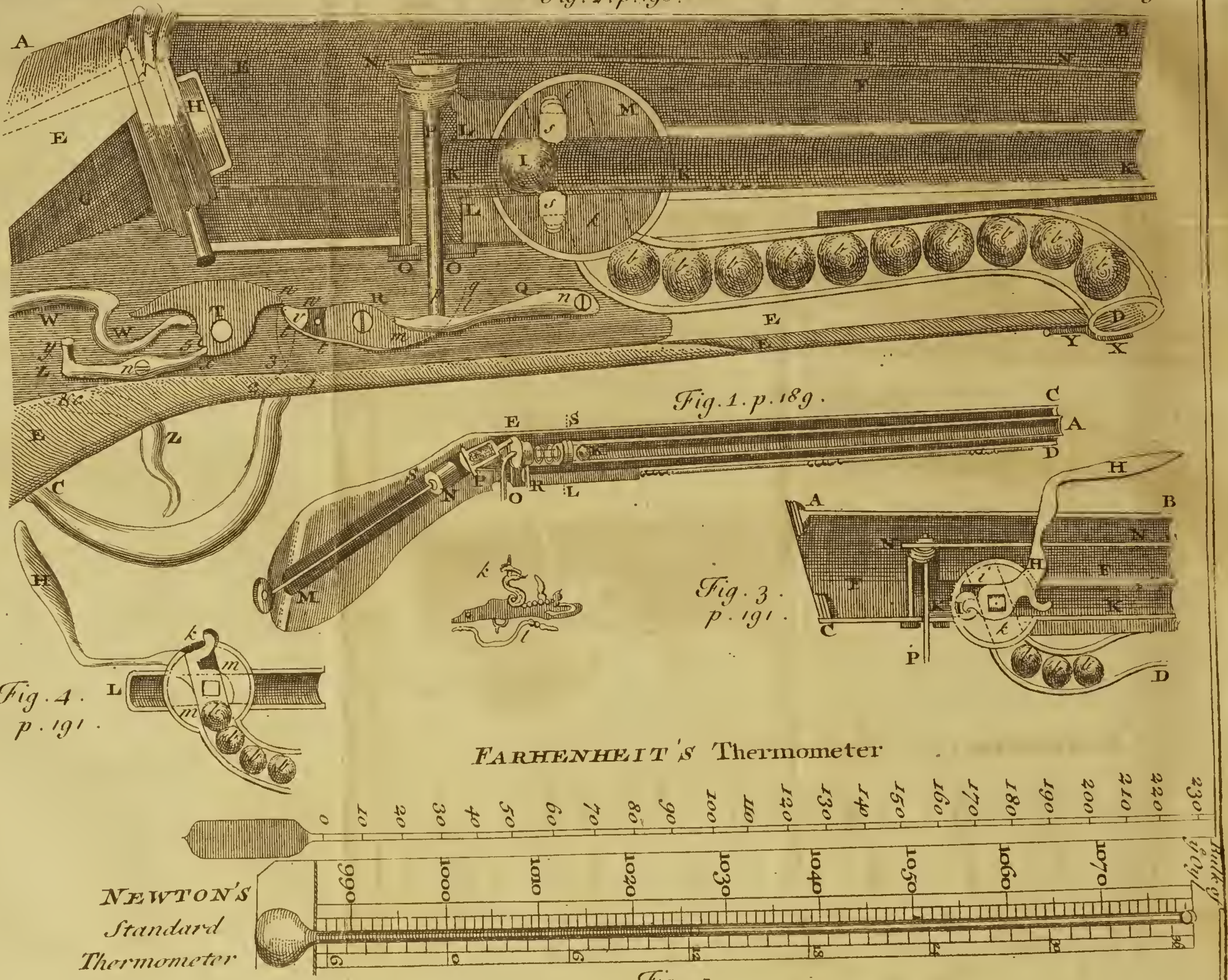

FARHENHEIT's Thermometer

NEWTON's
Standard
Thermometer

and Uſe of the common AIR-PUMP, and of one of a *new Invention* of my own.

fully ſatisfied by Experience: And I believe a better than this was never made.

Note, In the Conſtruction of my *Aerometrum Magnum* or Tripple Weather-Glaſs, I have a *Barometer*, *Thermometer*, and *Hygrometer* all in one Frame; all of the beſt Sort; which makes a Piece of Furniture equally uſeful and elegant.

(XCV.) 1. There remains yet one more Pneumatic Machine to be deſcribed, which has made conſiderable Noiſe in the Philoſophic World, but has never been of any Uſe in Civil Life; I mean, the famous Invention of the AIR-GUN, of which there are two Sorts; one the *Common Air-Gun*, the other the *Magazine Air-Gun*: Of both which I ſhall give the following ſhort Account.

Plate XXX. Fig. 1.

2. The *Common* AIR-GUN is made of Braſs, and has two Barrels; the Inſide Barrel K A of a ſmall Bore, from which the Bullets are ſhot; and a larger Barrel E C D R on the Outſide of it. There is a Syringe S M N P fixed in the Stock of the Gun, by which the Air is injected into the Cavity between the two Barrels through the Valve E P. The Ball K is put down into its Place in the ſmall Barrel with the Rammer, as in another Gun. At S L is another Valve, which being drawn open by the Trigger O, permits the Air to come behind the Bullet, ſo as to drive it out with great Force.

3. If this Valve be open'd and ſhut ſuddenly, one Charge of condenſed Air may make ſeveral Diſcharges of Bullets; but if the whole Air be diſcharged on one ſingle Bullet, it will drive it out more forcibly. This Diſcharge is effected by means of a Lock *k l* placed here, as uſual in other Guns; for the Trigger being pull'd, the Cock *k* will go down, and drive a Lever *o*, that will open the Valve, and let in the Air upon the Bullet K.

Pl.XXII. Fig. 4. The common or large Air-Pump is repre-ſented where *a a*, *a a*, are the two Braſs

Plate XXX. Fig. 2. 4. The *Magazine* AIR GUN is the Invention of an ingenious Artiſt, whoſe Name is *L. Colbe*. By his Contrivance ten Bullets are ſo lodged in a Cavity near the Place of Diſcharge, that they may be drawn into the ſhooting Barrel, and ſucceſſively ſhot ſo faſt as to be nearly of the ſame Uſe as ſo many ſeveral Guns. In the Figure you have a Section of the Gun as big in every Part as the Gun itſelf; and ſo much of the Length as is neceſſary to form a compleat Idea of the Whole.

5. AEE is Part of the Stock; G is the End of the injection Syringe, with its Valve H opening into the Cavity between the Barrels, as before. KK is the ſmall ſhooting Barrel, which receives the Bullets from the Magazine ED, which is of a ſerpentine Form, and cloſed on the End D when the Bullets *b*, *b*, *b*, *b*, are lodged in it. The circular Part *s* I *s k* M *i* is the Key of a Cock, having a cylindric Hole through it IK, which is equal to the Bore of the ſmall Barrel, and makes a Part of it in the preſent Situation.

6. When the Lock is taken off, the ſeveral Parts Q, R, T, S, W, *&c.* come into View, by which means the Diſcharge is made by puſhing up the Pin P*p*; which raiſes and opens a Valve V, to let in the Air againſt the Bullet I from the Cavity F, F, F; which Valve is immediately ſhut down again by means of a long Spring NN of Braſs. This Valve V being a conical Piece of Braſs, ground very true in the Part which receives it, will of itſelf be ſufficient to confine the Air.

7. To make a Diſcharge you pull the Trigger *z z*, which throws up the Seer *y x*, and diſengages it from the Notch *x*, upon which the ſtrong Spring WW moves the Tumbler T, to which the Cock is fixed. This by its End *u* bears down the End *v* of the tumbling Lever R, which by its other End *m* raiſes at the ſame time the flat End *l* of the horizontal Lever Q; and

Barrels, in which the Piſtons *cc*, *cc*, move by Chains faſten'd to each of them, and

by this means, of courſe, the Pin P *p* is puſh'd up, which ſtands upon it, and thus opens the Valve V, and diſcharges the Bullet. This is all evident from a bare View of the Cut.

8. To bring in another Bullet to ſucceed I inſtantaneouſly, there is a Part called the Hammer H, which by a ſquare Hole goes on upon the ſame ſquare End of the Key of the Cock, and turns it about ſo as to place the cylindric Bore of the Key I K in any Situation required. Thus when the Bullet is in the Gun, the Hammer ſtands as in the Figure, where the Bore of the Key coincides with that of the Barrel K K; but when the Ball is diſcharged, the Hammer H is inſtantly brought down to ſhut the Pan of the Gun, by which Motion the Bore of the Key is turned into the Situation *i k*, ſo as to coincide now with the Orifice of the Magazine; and upon lifting the Gun upright, the Ball next the Key tumbles into its Cavity, and falls behind two ſmall Ends *s*, *s*, of two tender Springs, which like Fingers Ends detain it. The Key in this Poſition is ſeen in the Figure. Then opening the Hammer again the Bullet is brought into its proper Place near the diſcharging Valve, and the Bore of the Key makes again a Part of that of the ſhooting Barrel.

Plate XXX. Fig. 3, 4.

9. It evidently appears how expeditious a Method this is of charging and diſcharging a Gun; and were the Force of condenſed Air as great as that of Gunpowder, ſuch an Air-Gun would actually anſwer the End of many Guns, and prove the beſt Defence againſt Highwaymen or Robbers that People are aware of; becauſe when they have Reaſon to ſuſpect them, they might then make five or ſix Diſcharges before the Thief can come within Piſtol-ſhot.

10. In the Air-Gun, and in all other Caſes where the Air is required to be condenſed to a very great Degree, it will be requiſite to have the Syringe of a ſmall Bore, *viz.* not exceeding $\frac{1}{2}$ an Inch in Diameter; be-

to a Wheel moving on the Axle *f*, when the Engine is put into Motion by the Winch *bb*. *gg*, *gg*, are two Pillars or Pieces of Wood ſupporting the Frame of the Pump-Wheel, which is ſcrew'd upon them by Nuts under the little Pieces of Wood *e*, *e e*. The Tube or Pipe marked *h h* is called the *Swan-Neck*, made of Braſs: By this the Air paſſes from under the Receiver *o o*, through a ſmall Hole *k* in the Middle of the Braſs Plate *i i i* on the Top of the Pump, to a Braſs Piece in the Box *d d*, which being perforated length-ways to the Middle Point under each Barrel, does there, through a ſmall Hole, by a Bladder-Valve, tranſmit the Air from the Receiver into each Barrel to be pump'd out by paſſing through the Hole in the deſcending

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cauſe, as has been ſhewn, the Preſſure againſt every *ſquare Inch* is about 15 *lb.* and againſt every *circular Inch* it is therefore about 12 *lb.* If therefore the Syringe be one Inch in Diameter, when one Atmoſphere is injected, there will be a Reſiſtance of 12 *lb.* againſt the Piſton; when 2, of 24 *lb.*; and when 10 are injected, there will be a Force of 120 *lb.* to overcome; whereas 10 Atmoſpheres act againſt the circular Half Inch Piſton (whoſe Area is but $\frac{1}{4}$ Part ſo big) with a Force but a $\frac{1}{4}$ Part ſo great, *viz.* 30 *lb.*; or 40 Atmoſpheres may be injected with ſuch a Syringe, as well as 10 with the other. In a Word, the Facility of working will be *(cæteris paribus)* inverſely *as the Squares of the Diameters of the Syringe.*

Piſton. Theſe Holes in the Piſtons and Bottoms of the Barrels are cover'd with Valves; to prevent the Return of the Air into the Receiver. *l l l* is the Mercurial Gage, or common Barometer, immerſed in a Baſon of Mercury *m m* fix'd in the Bottom of the Frame, and at Top communicates with the Receiver; which therefore ſhews how much the Receiver is exhauſted by the Riſing of the Mercury in the Tube, by a graduated Scale affixed thereto. The Stop-cock *n n*, alſo, communicates with the Receiver, and conſequently with the Swan-Neck and Mercurial Tube: Its Uſe is, by turning the Cock, to re-admit the Air, when there is Occaſion. The Receiver is ground true on the Bottom, and is fixed on the Pump at firſt by means of wetted Leathers to exclude the Air, inſtead of Cement formerly uſed for that purpoſe.

BUT with how much more Conveniency, and leſs Expence, Pneumatical Experiments of all Kinds may be perform'd, by a *New*, *Elegant*, and *Portable Air-Pump*, which I have lately contrived and made, will be eaſy to apprehend from a bare View of the Figure thereof: In which A B is the Head or Part containing the Wheel, which alternately raiſes and depreſſes the Piſtons

Pl.XXII. Fig. 5.

Piſtons C D in the Barrels E F, which are ſtrongly preſs'd down by the ſaid Part AB, ſupported on the two Pillars G H, fixed into the Bed or Bottom of the Machine I K L. On this Bottom ſtands the Receiver M N on a large ſmooth Braſs Plate, in the Middle whereof is a Hole, by which the Air paſſes out of the Receiver into a ſmall Tube on the under Part of the Frame, and goes to the Piece O, which communicates with the perforated Braſs Piece on which the Barrels ſtand, and from which they receive the Air to be exhauſted. On the middle Part of this Braſs Piece is a Perforation, over which is placed a ſmall Receiver P Q, and under it a Baſon of Mercury R, in which a ſmall Tube R S (hermetically ſealed at one End, and fill'd with Quickſilver) is inverted; and therefore as the ſmall Receiver P Q is exhauſted, (at the ſame time with the large one M N) the Approach of the *Vacuum* will be ſhewn by the *Deſcent of the Quickſilver* in the Tube R S. By the Stop-cock T the Air is again let into the Receiver. I take this to be the *laſt Improvement* this Machine is capable of, as to its Form; for it conſiſts of only ſuch Parts as are Eſſential. And thus conſtructed, it may, together with its Receivers, be contain'd in a Box of a ſmall Size, and comes to but a ſmall Price in compariſon of the other Forms. (XCVI.)

(XCVI.) 1. I ſhall conclude this Subject with a few Articles relating to the Rarefaction of the Air in the Recipient in working the Machine; for the Reader muſt not ſuppoſe that all the Air can be exhauſted, if the Pump be ever ſo good, or worked ever ſo long. The Reaſon is evident when we conſider, that the Air which is exhauſted is only puſhed out by the Spring of that which remains behind: if therefore every Particle were ſuppoſed to be exhauſted, the laſt would be expell'd without an Agent, or there would be an Effect without a Cauſe, which is abſurd.

2. Let the Capacity of any Receiver be to that of the Barrel as C to 1. Alſo let the Rarefaction of the Air which remains in the Receiver be to the common Air as R to 1, after any Number of Turns or Exſuctions N. Then, upon raiſing the Piſton, the Air will ruſh into the Barrel, and ſo will now be rarefied in the Ratio of C to $C+1$, or of 1 to $\frac{C+1}{C}$; and ſince this is the Ratio of the Rarefaction by every Exſuction, 'tis evident it will be the common Ratio of a Geometrical Series of Rarefactions produced by the ſeveral Turns of the Winch, *viz.* The Series $1 : \overline{\frac{C+1}{C}}\Big|^{1} : \overline{\frac{C+1}{C}}\Big|^{2} : \overline{\frac{C+1}{C}}\Big|^{3} : \overline{\frac{C+1}{C}}\Big|^{4}$, *&c.* to $\overline{\frac{C+1}{C}}\Big|^{N} =$ laſt Term of the Series, which therefore as it expreſſes the laſt Rarefaction, will be equal to R, that is, $R = \overline{\frac{C+1}{C}}\Big|^{N}$.

3. Hence from the known Property of Logarithms we have $L.\,R = N \times L.\,\frac{C+1}{C} = N \times \overline{L.\,\overline{C+1} - L.\,C}$; whence, $\frac{L.\,R}{L.\,\overline{C+1} - L.\,C} = N$. Wherefore if $C =$ 1,

1, that is, if the Capacity of the Receiver be equal to that of the Barrel, we ſhall have $N = \frac{L.\,R}{L.\,2}$. Conſequently, if R expreſs any Degree of Rarity propoſed, as 1, 2, 3, 4, 5, 6, *&c.* we have N the Number of Turns or Exſuctions to effect it.

4. And from hence the following Table is conſtructed; in which the firſt Column expreſſes the Rarity of the Air in the Receiver, and the ſecond the Number of Turns to produce it.

Rarity.	*Number of Turns.*	*Rarity.*	*Number of Turns.*	*Rarity.*	*Number of Turns.*
1	0.	60	5,907	900	9,814
2	1.	64	6.	1000	9,966
3	1,585	70	6,129	1024	10.
4	2.	80	6,322	2000	10,966
5	2,322	90	6,492	2048	11.
6	2,585	100	6,644	3000	11,551
7	2,807	128	7.	4000	11,966
8	3.	200	7,644	4096	12.
9	3,170	256	8.	5000	12,288
10	3,322	300	8,229	6000	12,551
16	4.	400	8,644	7000	12,773
20	4,322	500	8,966	8000	12,966
30	4,907	512	9.	8192	13.
32	5.	600	9,229	9000	13,136
40	5,322	700	9,451	10000	13,288
50	5,644	800	9,644	16384	14.

5. From this Table we may obſerve, that if any Numbers in the firſt Column be taken in Geometrical Progreſſion, the correſponding Numbers of the ſecond will be in Arithmetical Progreſſion. Thus againſt 2, 4, 8, 16 in the firſt, you ſee 1, 2, 3, 4 in the ſecond Column.

6. When the Capacity of the Receiver exceeds that of the Barrel, then the Number of Turns N to produce a given Rarefaction R will be greater than before. Therefore if the Number of Turns in this particular

Caſe, which let us call n, be multiplied by ſome Number m, it will produce a Number of Turns N that ſhall affect the Rarefaction R in any Receiver propoſed. Now ſince $n = \frac{L.\,R}{L.\,2}$, we ſhall have $n \times m = \frac{m \times L.\,R}{L.\,2} = N = \frac{L.\,R}{\frac{L.\,\overline{C+1} - L\,C}{L.\,2}}$; whence we have $m = \frac{L.\,2}{L.\,\overline{C+1} - L.\,C}$.

7. From Hence a Table of Multipliers expreſſing the Value of m, when the Receiver is in any given Proportion larger than the Barrel, is eaſy to be conſtructed. Of which the following is a Specimen.

Capacity of Receiver.	*Multiplier.*	*Capacity of Receiver.*	*Multiplier.*	*Capacity of Receiver.*	*Multiplier.*
1	1.	20	14,207	300	208,291
2	1,710	30	21,139	400	277,605
3	2,409	40	28,071	500	346,920
4	3,106	50	35,003	600	416,235
5	3,802	60	41,934	700	485,549
6	4,497	70	48,866	800	554,864
7	5,191	80	55,798	900	624,179
8	5,885	90	62,729	1000	693,494
9	6,579	100	69,661		
10	7,273	200	138,976		

8. By means of theſe two Tables, thoſe who know nothing of *Algebra* may find how many Turns are neceſſary to rarify the Air in the Receiver to any given Degree, when the Ratio of the Receiver's Capacity to that of the Barrel is known. For Example: Let the Receiver be 10 times as big as the Barrel, and let it be required to find how many Turns of the Winch will rarify the Air 100 times. Firſt ſeek the Number that will do it when the Receiver is equal to the Barrel; which I find by the firſt Table is 6 Turns, and 644 Parts of 1000 of another. Then, againſt 10 in the

ſecond Table, I find the Multiplier 7,273, by which if I multiply 6,644, I ſhall have a Product 48,322, which will expreſs the Number of Turns required.

9. The Aſcent of the Quickſilver in the Gage of the Common Pump is proportional to the Quantity of Air drawn out, either upon the Whole, or upon any ſingle Turn of the Winch: And the Deficiency from the Standard Altitude of 29 $\frac{1}{2}$ Inches is always proportional to the Quantity of Air remaining in the Receiver; as may be eaſily deduced from what has been ſaid of the *Denſity*, *Spring*, and *Preſſure* of the Air.

10. The Gage of a *Condenſer* will have the Spaces unpoſſeſs'd of Quickſilver at the End decreaſing in *Harmonical Proportion:* For ſince equal Quantities of Air are injected by the Syringe at each Stroke of the Piſton, the Quantity of Air in the Condenſer will increaſe in Arithmetical Progreſſion, and ſo will its Denſity, and of courſe the Denſity of that in the End of the Gage, becauſe the Quickſilver is preſſed on each Side equally; but the Spaces diminiſh as the Denſities increaſe, as we have elſewhere ſhewn. Therefore *the Spaces are inverſely as a Series of Terms in Arithmetical Progreſſion*, and conſequently are in *Muſical Proportion*; for that this is a Property of *Muſical Terms* will be ſhewn in *Annotat.* CIX.

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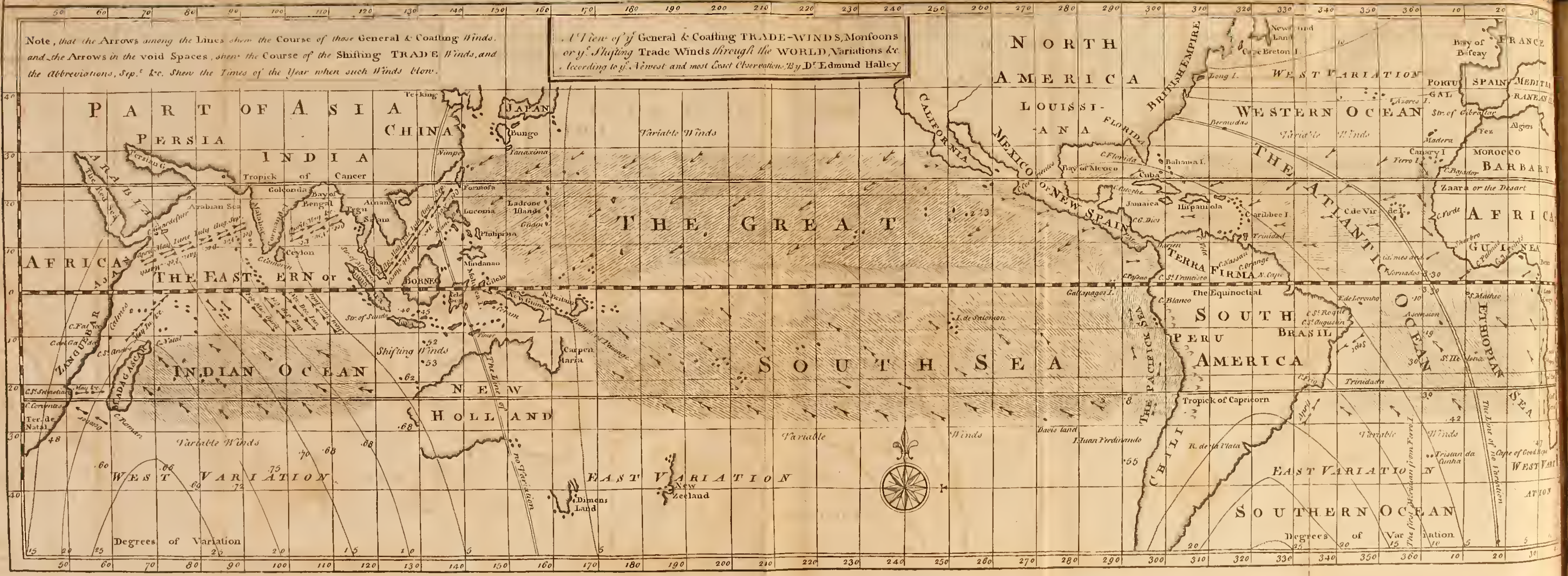

Note, that the Arrows among the Lines shew the Course of those General & Coasting Winds, and the Arrows in the void Spaces, shew the Course of the Shifting TRADE Winds, and the Abbreviations, Sep.t &c. Shew the Times of the Year when such Winds blow.
A View of ye General & Coasting TRADE-WINDS, Monsoons or ye Shifting Trade Winds through the WORLD, Variations &c. According to ye Newest and most Exact Observations By Dr. Edmund Halley
PART OF ASIA
PERSIA
INDIA
CHINA
JAPAN
Bungo
Tanaxima
Te-king
Nimpo
ARABIA
The Red Sea
Persian G.
Arabian Sea
Tropick of Cancer
Golconda
Bay of Bengal
Pegu
Siam
Ainan
Formosa
Lucconia
Philippina
Ladrone Islands
Guam
Ceylon
C. Comorin
Malabar
Cormandel
Str. of Malacca
Mindanao
Borneo
Gilolo
Str. of Sunda
Timor
AFRICA
AJAN
THE EASTERN or
INDIAN OCEAN
ZANGUEBAR
MADAGASCAR
C. Natal
C. St. Sebastian
C. Corientes
Ter. de Natal
C. Roman
C. Falso
C. del Gado
Shifting Winds
NEW HOLLAND
Carpentaria
New Guinea
N. Britain
Dampiers Passage
The Line of no Variation
Variable Winds
WEST VARIATION
Degrees of Variation
Dimens Land
New Zeeland
EAST VARIATION
THE GREAT SOUTH SEA
Variable Winds
I. de Salomon
Variable
Winds
Davis land
I. Juan Ferdinando
NORTH AMERICA
LOUISSIANA
CALIFORNIA
MEXICO
OR NEW SPAIN
FLORIDA
BRITISH EMPIRE
C. Florida
Bay of Mexico
Bahama I.
Cuba
Jamaica
Hispaniola
C. Gracias a Dios
Caribbee I.
Trinidad
Darien
TERRA FIRMA
C. St. Francisco
C. Nassau
C. Orange
N. Cape
Gallapagos I.
THE PACIFICK SEA
C. Blanco
The Equinoctial
SOUTH AMERICA
PERU
BRASIL
C. St. Roque
C. St. Augustin
C. Frio
Tropick of Capricorn
CHILI
R. de la Plata
Trinidada
Newfound Land
Cape Breton I.
Long I.
Bermudas
WESTERN OCEAN
THE ATLANTICK OCEAN
Variable Winds
Azores I.
Madera
Canary I.
Ferro I.
C. de Verd I.
C. Verde
Calmes and Tornados
E. de Lorenho
Ascension
St. Helena
Tristan da Cunha
Cape of Good Hope
Abrolhos
The first Meridian from Ferro
The Line of no Variation
SOUTHERN OCEAN
Degrees of Variation
FRANCE
Bay of Biscay
PORTUGAL
SPAIN
Str. of Gibraltar
MEDITERRANEAN
Algiers
Fez
MOROCCO
BARBARY
C. Bojador
Zaara or the Desart
AFRICA
GUINEA
Sherbro
C. Palmas
C. Lopez
St. Mathew
ETHIOPIAN SEA
WEST VARIATION

LECTURE VII.

The Doctrine of Winds *and* Sounds.

Of Wind *in general.* *The* Theory *of* Winds *by Dr.* Halley. *Of the conſtant, or General* Trade Winds; *of the* Monsoons; *the Cauſe of* Variable Winds. *Of* Aerial Tides. *Of the* Velocity *of* Wind. *Of the* Momentum *or* Force *of* Wind. *A* Calculation *thereof, and its* Application *to the* Sails *of a* Wind-Mill. *The beſt* Form *and* Position *of the* Sails. *A* Calculation *of the* Force *of* Bellows *in impelling* Wind. *A* new Invention *of* Water-Bellows. *The* Nature *of* Sound *in general.* *The* Sensation *of* Sound. *The* Organ of Hearing *deſcribed.* *Of the* Waves *or* Pulses *of* Air. *Their Various* Properties *explain'd.* *The* Newtonian Doctrine *of* Vibrations *and* Tremors *of* Sounding Bodies *explain'd.* *The* Waves *of* Water *accounted for.* *Of the* Velocity *of* Sounds. *The* Distance *to which they may be heard.* *Of* Echo's.

Of the Speaking Trumpet *of the beſt Form.* *Of* Otacoustic Instruments. *Of the* Note, Tone, *or* Tune *of* Sounds. *Of* Concords *and* Discords; *the* Rationale *of the* Diatonic Scale *of* Music. *The* Mathematical Theory *of* Musical Chords, *and of* Harmonic Proportions. *Of the* Sympathetic Vibrations *of* Musical Strings, *and other Bodies.*

IN this Lecture I ſhall conſider the Nature of Wind and Sound in general; and of the Vibrations of *Muſical Strings* and *Sonorous Bodies,* with Regard to the Science of MUSIC.

WIND is a *Stream* or *Current of Air* : As the Air is a Fluid, its natural State is that of *Reſt*, which it endeavours always to keep or retrieve by an univerſal *Equilibrium* of all its Parts. When, therefore, this natural *Equilibrium* of the Atmoſphere happens by any means to be deſtroy'd in any Part, there neceſſarily follows a Motion of all the circumjacent Air towards that Part, to reſtore it ; and this Motion of the Air is what we call *Wind.* (XCVII.)

(XCVII.) 1. I ſhall here give the principal Phænomena of the Wind, as they are deduced from Dr. *Halley*'s admirable Hiſtory thereof in the *Philoſophical*

HENCE, with reſpect to that Place where the *Equilibrium* of the Air is diſturb'd, we

Tranſactions, and illuſtrate the ſame by his Map of the World drawn up for that Purpoſe.

Plate XXXI.

2. The firſt is, That in the great *Pacific* or *Weſtern Ocean*, the *Atlantic* and *Ethiopic* Seas, there is a general Eaſterly Wind all the Year long, without any conſiderable Variation; excepting that it is ſubject to be deflected therefrom ſome few Points of the Compaſs towards the North or South, according to the Situation of the Place. The Reaſon is, becauſe the Parts under the Equator are more heated and rarified than any others, as above mentioned.

3. The Second is, That on each Side the Equator, to about 27 or 30 Degrees, the Wind does more and more decline from the Eaſt to the North-Eaſt on one Side, and South-Eaſt on the other; occaſion'd by the two contrary Motions of the Air, ariſing from Heat and Cold, as above explain'd. Theſe Winds are indicated by the Poſition of the Arrows in the *Atlantic* and *Pacific* Ocean in the Map.

4. Towards the *Caribbee* Iſlands, on the *American* Side of the *Atlantic* Ocean, the aforeſaid North-Eaſt Wind becomes ſtill more and more Eaſterly, ſo as ſometimes to be Eaſt, ſometimes Eaſt by South, but moſtly Northward of the Eaſt a Point or two, ſeldom more. It is likewiſe obſerved that the Strength of theſe Winds does gradually decreaſe as you ſail to the Weſtward.

5. All along upon the Coaſt of *Africa* on the Weſtern Side, the Wind ſets in upon the Land from various Points of the Compaſs, North-Weſt, Weſt, South by Weſt, South-Weſt, and almoſt South, eſpecially toward the *Cape of Good-Hope*, all which is eaſily ſeen in the Map.

6. In the *Atlantic* Ocean, towards the North of the Line, between 4 and 10 Degrees of Latitude, and 20 and 30 of Weſt Longitude, there is a Tract of Sea where the Winds are not properly ſaid to be *conſtant* or *variable*; for it ſeems to be condemned to perpetual

ſee the Wind may blow from every Point of the Compaſs at the ſame time; and thoſe who live *Northwards* of that Point have a *North Wind*; thoſe who live *Southwards*, a *South Wind*; and ſo of the reſt: But thoſe who live on the Spot, where all thoſe Winds meet and interfere, are oppreſſed with turbulent and boiſterous Weather, Whirlwinds and Hurricanes; with *Rain*, *Tempeſt*, *Lightning*, *Thunder*, &c. For *Sulphureous Exhalations* from the *South*, Torrents of *Nitre* from the *North*, and *aqueous Vapours* from every Part, are there confuſedly huddled and violently blended together; and rarely fail to produce the *Phænomena* abovementioned.

Calms, attended with terrible Thunder and Lightning, and Rains, ſo frequent that our Navigators from hence call this Part of the Sea the RAINS, as by others they are called the CALMS and TORNADOES, as in moſt of our common Maps. The Reaſon of this ſeems to be, that this being the Place where the Eaſterly and Weſterly Winds commence, the Air is divided and held as it were *in Equilibrio* between both; by which means it is render'd more rare than the reſt, and too light to ſuſtain the Vapours raiſed into it, ſo that it lets them deſcend in *continual Rains*. See the Parting of the Air in the Map.

7. In the *Indian* Ocean the Winds are partly *General*, as in the *Atlantic* and *Ethiopic* Oceans; and partly *Periodical*, that is, ſuch as blow one Half of the Year one Way, and the other Half of the Year near upon the oppoſite Points: And theſe Points and Times of ſhifting are different in different Parts of this Ocean.

MANY are the particular Cauſes which produce Wind by interrupting the Equipoiſe of the Atmoſphere; but the moſt general Cauſes are two, *viz.* HEAT, which, by *rarifying* the Air, makes it *lighter* in ſome Places than it is in others; and COLD, which, by *condenſing* it, makes it *heavier*. Hence it is, that in all Parts over the *Torrid Zone*, the Air being more rarified by a greater Quantity of the Solar Rays, is much lighter than in the other Parts of the Atmoſphere, and moſt of all over the Equatorial Parts of the Earth. And ſince the Parts at the Equator are moſt rarified which are near the Sun; and thoſe Parts are, by the Earth's diurnal Rotation *Eaſtward*, continually ſhifting to the *Weſt*; it fol-

Theſe Winds are called by Seamen *Monſons* or *Monſoons*.

8. Between 10 and 30 Degrees, from *Madagaſcar* to *New-Holland*, the general Trade-Winds about South-Eaſt by Eaſt are found to blow all the Year long in the ſame Manner, and for the ſame Reaſons as in the other Oceans above-mentioned.

During the Months of *May*, *June*, *July*, *Auguſt*, *September*, *October*, the aforeſaid South-Eaſt Winds extend to within two Degrees of the Equator; after this, for the other ſix Months, the contrary Winds ſet in, and blow from the North-Weſt from the Latitude of 3 to 10 Degrees South.

10. From about three Degrees South Latitude, over all the *Arabian* and *Indian* Seas and Gulph of *Bengal*, from *Sumatra* to the Coaſt of *Africa*, there is another Monſoon, blowing from *October* to *April* on the North-Eaſt Points; but in the other Half-Year, from *April*

lows, that the Parts of the Air which lie on the *Weſt* Side of the *Point of greateſt*

to *October*, from the oppoſite Points of South-Weſt and Weſt-South-Weſt, and that with rather more Force than the other, accompanied with dark rainy Weather, whereas the North Eaſt blows clear.

11. The Sea between *Madagaſcar* and *Africa*, and Southwards to the Equator, is ſubject to the ſame Change of Wind, or Monſoons, whoſe Courſe from *April* to *October* is South-South-Weſt, which, as you go more Northerly, becomes more and more Weſterly, till at laſt they fall in with the Weſt-South-Weſt Winds mentioned in the laſt Articles. What Winds blow the other Half-Year in thoſe Parts, the Doctor could not obtain any ſatisfactory Account of; only that they were Eaſterly, and as often to the North as to the Southward thereof.

12. To the Eaſtward of *Sumatra* and *Malacca*, on the North Side of the Equator along the Coaſt of *Camboia* and *China*, the Monſoons blow, and change at the ſame Times as before; only their Directions are much more Northerly and Southerly than the others, as is eaſy to obſerve in the Map. Theſe Winds reach to the *Philippine* Iſlands Eaſtward, and to *Japan* Northwards; and are not ſo conſtant to their Points as the others above mentioned.

13. Between the ſame Meridians, on the South Side the Equator, from *Sumatra* to *New Guinea* Eaſtward, the ſame Northerly and Southerly Monſoons are obſerved; only the Northerly are here North-weſterly, and the Southerly blow from the South-Eaſt. They are not more conſtant than the others; and beſides, they keep not the ſame Times, but change a Month or ſix Weeks later.

14. The Shifting of theſe contrary Winds, or Monſoons, is not all at once; and in ſome Places the Time of the Change is attended with *Calms*, in others with *variable Winds*, and particularly thoſe of *China*, at ceaſing to be Weſterly, are very ſubject to be tempeſtu-

Rareſaction, and, by flowing towards it, meet it, have leſs Motion than thoſe Parts

ous ; and ſuch is their Violence, that they ſeem to be of the Nature of the *Weſt-India* Hurricanes, and render the Navigation of thoſe Parts very unſafe at that Time of the Year. Theſe Tempeſts the Seamen call the *Breaking up of the Monſoons.*

15. The Cauſe of the *Monſoons*, or periodical Winds, is owing to the Courſe of the Sun Northward of the Equator one Half of the Year, and Southward the other. While he paſſes through the ſix Northern Signs of the Ecliptic, the various Countries of *Arabia*, *Perſia*, *India*, and *China* are heated, and reflect great Quantities of the Solar Rays into the Regions of the ambient Atmoſphere, by which Means it becomes greatly rarified, and has its Equilibrium of courſe deſtroyed ; to reſtore which, the Air, as well from the Equatorial Parts Southwards, where it is colder, as from the colder Northern Climes, muſt neceſſarily have Tendency or Motion towards thoſe Parts, and ſo produces the Monſoons for the firſt ſix Months, during which Time the Heat of thoſe Countries is greateſt.

16. Then for the other ſix Months, the Sun traverſing the Ocean and Countries towards the Southern Tropic, while in the ſix Southern Signs, cauſes the Air over thoſe Parts to be now moſt heated and rarified ; and conſequently the Equatorial Air to alter its Courſe, or the Winds to veer quite about, and blow upon the oppoſite Points of the Compaſs.

17. Theſe are the general Affections of conſtant and regular Winds ; none of which are found not ſubject to ſome Variation and Exception, on account of the different Circumſtances of Heat, Cold, Land, Water, Situation, *&c.* concerning all which I ſhall refer the Reader to the Doctor's own large hiſtorical Account of the Winds, publiſhed in the *Tranſactions*, or *Miſcellanea Curioſa*, Vol. I.

18. From what has been ſaid, 'tis eaſy to underſtand, that ſince ſo large a Portion of the Atmoſphere as

on the *Eaſt* of the ſaid Point, which follow it; and therefore the Motion of the *Eaſt-*

is over the Torrid Zone, and Parts about it, is in ſuch continual Agitation and alternate Motion, thoſe Agitations in an elaſtic Fluid muſt extend every way to a great Diſtance, and produce Effects of the ſame Kind in a various Manner; by which means the Air in all other Latitudes and Climes will ſuffer a Perturbation more or leſs, and have a perpetual Tendency to Motion in various Directions, depending on the Situation of Country, the Degrees of Heat and Cold in the Climate, the Poſition of Hills, Vales, &c. beſides what may be owing to the Accenſion and Exploſion of Meteors, the Eruption of Subterranean Air, and a hundred other Cauſes; I ſay, from all this it is eaſy to infer, *that our Climate, wherever we live, muſt neceſſarily be attended with variable Winds, almoſt perpetually.*

19. I ſhall only add farther, that ſince the Atmoſphere is a *gravitating fluid Subſtance,* it muſt be ſubject to the attracting Power of the Sun and Moon, as well as of the Earth: and therefore when the Influence of thoſe Luminaries, either ſingly or conjointly, is oppoſite to that of the Earth, the ſame Effects muſt follow in the Body of fluid Air, as we have ſhewn were produced in the ambient Fluid of Water, *viz.* that the Atmoſphere ſhall be of an oblong Figure, or of different Altitudes in different Parts; and that theſe *Tides of Air* have nearly all the ſame Affections with thoſe of the Ocean before explain'd, excepting only in this, that they muſt be as much greater as the Denſity of Water exceeds that of Air, *viz.* in the Ratio of 860 to 1.

20. Now becauſe of an Equality of Preſſure or Weight in the Atmoſphere in unequal Altitudes of Air, we can never be ſenſible of an *Aerial Tide,* either of *Ebb* or *Flood,* by the Barometer; and can only know it by the Poſition of the heavenly Bodies. However as this prodigious Protuberance of the Atmoſphere is conſtantly following the Moon, it muſt of courſe produce a Motion in all Parts, and ſo produce a Wind more or leſs to every Place, which, as it conſpires with, or is oppoſed

ern Air would prevail againſt that of the *Weſtern Air*, and ſo generate a *continual Eaſt Wind*, if this were all the Effects of that *Rarefaction*. But we are to conſider, that as all the Parts of the Atmoſphere are ſo greatly rarified over the Equator, and all about the Poles greatly condenſed by extreme Cold, this heavier Air from either Pole is conſtantly flowing towards the Equator, to reſtore the Balance deſtroy'd by the *Rarefaction* and *Levity* of the Air over thoſe Regions: Hence, in this reſpect alone, a conſtant *North* and *South Wind* would be generated (XCVIII).

to the Winds ariſing from other Cauſes, makes them greater or leſs. And I believe ſomething of this may be deduced from Obſervations made of the State of the Air at the Times of the *New and Full Moons*. And that this was the Caſe in reſpect to the two laſt great Storms, Dr. *Mead* has obſerved in his Tract *De Imperio Solis & Lunæ*.

(XCVIII.) 1. I find by Experience, that People have in general but an obſcure Idea or confuſed Notion of the Cauſe of this perpetual Current of Air from Eaſt to Weſt, or of a conſtant Eaſt Wind under the Equator; therefore in order to elucidate this Matter, I ſhall repreſent it in, and explain it by, a Figure. Let C B A D E be Part of a Section of the Atmoſphere over the Equator, C the Eaſt, E the Weſt, A the Point to which the Sun S is vertical, and R the Point of greateſt Rarefaction, or that where the Air is moſt of all heated, and conſequently lighteſt.

Plate XXXII. Fig. 1.

Now it is eaſy to underſtand, that by a Compoſition of theſe two Directions of the

2. That this Point R is on the Eaſtern Side of the Point A is not difficult to be conceived, when what is ſaid concerning the Tide in *Annot.* LXXXIV. is well conſider'd. And becauſe the Air at R is by Suppoſition lighter than where it is colder at C and D, it is plain that, in order to maintain an Equilibrium, (which is neceſſary in a fluid Body) the Air by its greater Weight will have a Tendency from C and D towards R, and riſe to a Height there greater than at C or D; in Proportion as its Denſity is leſs.

Plate XXXII. Fig. 1.

3. Now this being the Caſe, it is evident, the Sun being always between the Points R and D, will be heating the Air on that Part; and thoſe Regions between R and C, having been deſerted by the Sun, will grow cold: Conſequently, the Air between C and R, as it is colder, will likewiſe be heavier than that between R and D which is hotter, and ſo will have a greater *Momentum*, or Quantity of Motion, towards the Point R; and ſince this Point R is conſtantly moving after the Point A Weſtward, the Motion of the Weſtern Air towards it will be in Part diminiſhed by that Means; and being alſo inferior in Quantity to the Motion of the Eaſtern Air, the latter will prevail over it, and be conſtantly following the ſaid Point R from Eaſt to Weſt, and thus produce a *continual Eaſt Wind*.

4. It may perhaps be here ſaid, that though the Motion of the Air be leſs from D to R, yet it is ſomething; and ſo there ought to be a Weſtern Wind, at leaſt in ſome Degree, and to ſome Diſtance Weſtward of the Point R. To which I anſwer, That the Nature of a Fluid will not permit two contrary Motions to reſtore or ſuſtain an Equilibrium, (I mean in regard of the whole Body of it) for wherever one Part of the Fluid is determined to move, all the reſt muſt neceſſarily follow it; otherwiſe the Equilibre of the Air would be deſtroyed in one Part, to make it good in another; a Defect which Nature cannot be guilty of. Thus we ſee the

Air

Vol. II. Plate XXXII. P. 207.

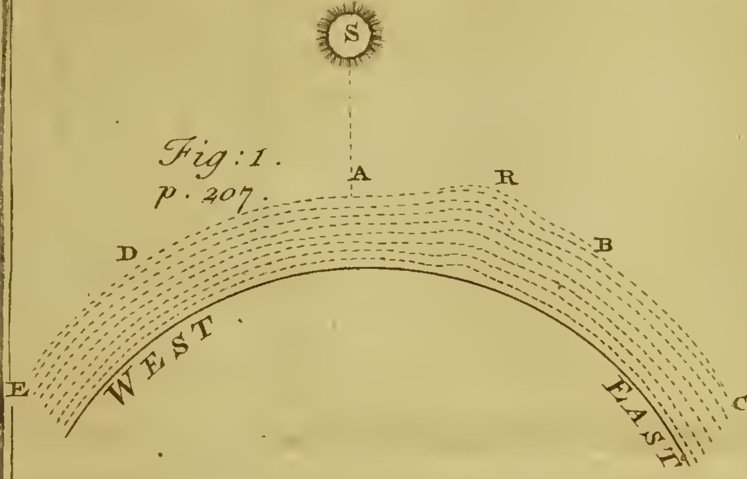


Fig: 1. p. 207.

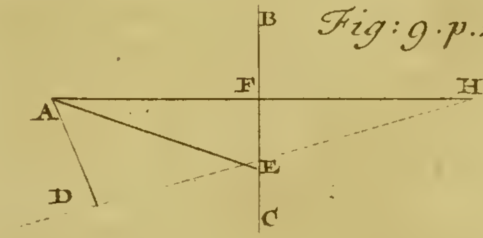


Fig: 9. p. 241.

Fig: 2. p. 211.

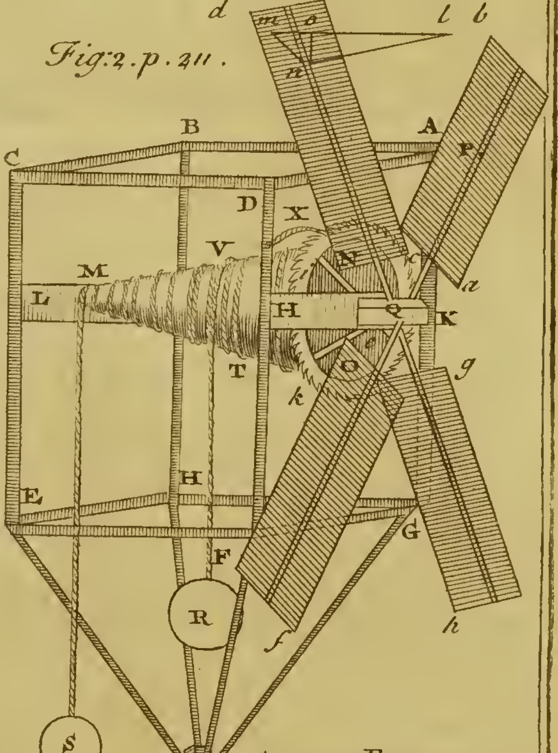
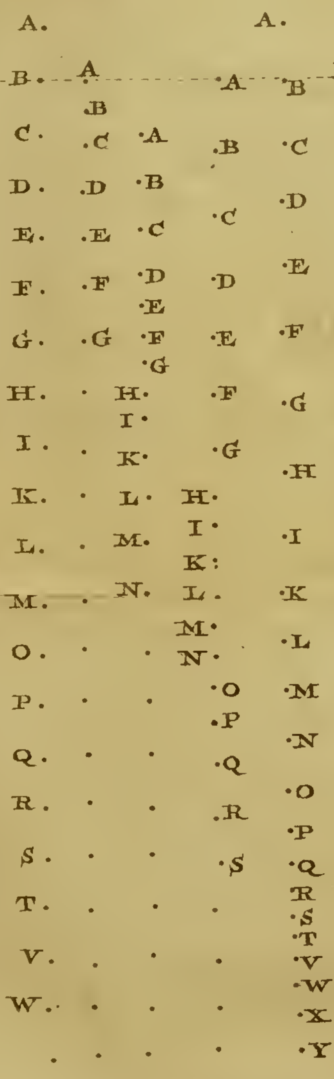


Fig: 5.



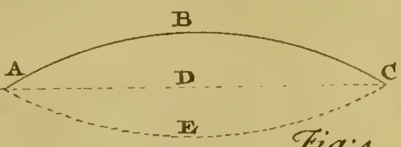


Fig: 4. p. 225.

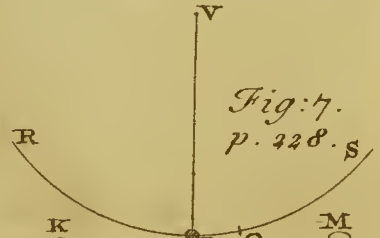


Fig: 7. p. 228.

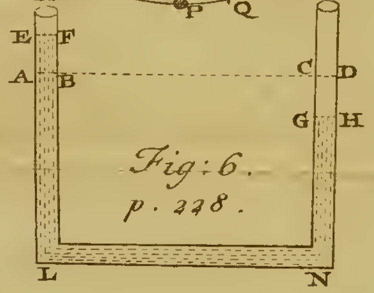


Fig: 6. p. 228.

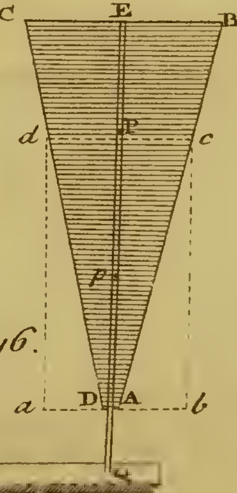
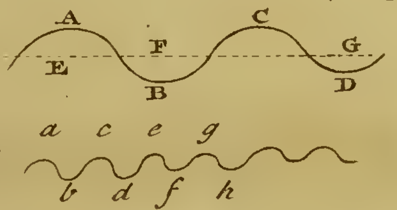


Fig: 3. p. 216.

Fig: 8. p. 230.



Air from the *Eaſt* and *North*, a *conſtant North-Eaſt Wind* will be generated in the *Northern* Hemiſphere, and a *conſtant South-Eaſt Wind* in the *Southern* Hemiſphere, to a certain Diſtance on each Side the Equator, all round the Earth. And this Caſe we find to be verified in the *General Trade Winds*, which conſtantly blow from the *North-Eaſt* and *South-Eaſt*, to about 30 Degrees on each Side the Equator, where thoſe Parts are over the open Ocean, and not affected with the Reflection of the Sun-Beams from the heated Surface of the Land; for in this Caſe the Wind will always ſet in upon the Land, as on the Coaſt of *Guinea*, and other Parts of the *Torrid Zone*, we know it does (XCIX.)

Tides of the Ocean always follow the Courſe of the Moon from Eaſt to Weſt, without any Motion of the Waters from the Weſt towards the Moon, in the open Oceans: And the Point R can only be conſidered as the *Aerial Tide*, or *Flood of High Air*; and has nearly the ſame Phænomena with aqueous Tides.

5. This being clearly underſtood, all the reſt is eaſy; for no one can find it difficult to conceive how the cold Air from each Pole muſt neceſſarily ſet in towards the Equator directly, where meeting, and interfering with the Eaſtern Current, it does with that compound a new Direction for the moving Air, which lies between both the former, *viz.* a *North Eaſt* Current on the North Side, and a *South Eaſt* one on the South Side: All which naturally reſults from the *Doctrine of the Compoſition of oblique Forces.* (See *Annot.* XXIV.)

(XCIX.) Mr. *Clare*, in his *Motion of Fluids*, has a very pertinent Experiment for illuſtrating this Matter.

As the Motion of the Air has a greater or leſſer Velocity, the Wind is *ſtronger* or *weaker* ; and it is found from Obſervation, that the Velocity of the Wind is various, from the Rate of 1 to 50 or 60 Miles *per* Hour (C).

It is thus : Let there be a very wide Diſh or Veſſel of Water, in the Middle of which is to be placed a Water-Plate fill'd with warm Water ; the firſt will repreſent the Ocean, the other an Iſland rarifying the Air above it. Then holding a Candle over the cold Water, blow it out, and the Smoke will be ſeen to move towards the warm Plate, and riſing over it will point out the Courſe of the Air from Sea to Land. And if the ambient Water be warm'd, and the Plate fill'd with cold Water, and the ſmoaking Wick of a Candle held over the Plate, the contrary will happen.

(C.) 1. The Experiment to prove this, is to chuſe a free open Place, where the Current of Air, or Wind, is not at all interrupted, but flows uniformly, or as much ſo as the undulatory State of the Atmoſphere will admit ; in ſuch a Place, a Feather, or ſome very light Body, is to be let go in the Wind, and then by a Half-Second Watch, or Pendulum, you obſerve nicely to what Diſtance it is carried in any Number of Half-Seconds ; or in how many Half-Seconds it has paſs'd over a given or meaſured Space ; this will give the Rate of Velocity in the Wind *per* Second, and of courſe *per* Hour.

2. The late Rev. Dr. *Derham*, who was moſt accurate in making Experiments of this Sort, approves of this Method before that of the *Mola alata* or *pneumatica* invented by Dr. *Hook* (of which ſee an Account in the *Philoſophical Tranſactions*, N°. 24.) And he tells us (in N. 313.) that he thus meaſured the Velocity of the Wind in that very great Storm of 1705, *Auguſt* 11. and by many Experiments he found, that it was at the Rate of 33 Feet *per* Half-Second, or of 45 Miles *per*

Thus much may ſuffice for a general Account of the *Nature and Origin of Winds*:

Hour; whence he concludes, that the moſt vehement Wind (as that of 1703 in *November*) does not fly at the Rate of above 50 or 60 Miles *per* Hour; and that at a Medium the Velocity of Wind is at the Rate of 12 or 15 Miles *per* Hour.

3. And ſometimes the Wind is ſo ſlow as not to exceed the Velocity of a Perſon riding or walking in it; and in that Caſe, if the Perſon goes with the Wind, he finds no Wind at all, becauſe there is no Difference of Velocity, or no relative Wind, which is that only that we are ſenſible of whilſt in Motion; the Reaſon of which we ſee in *Annot.* XX.

4. The beſt Method, that I know of, to bring the Force of the Wind to a Mathematical Calculation and Certainty, is by the following new contrived Anemoscope, of which I had the firſt Hint from the late ingenious Dr. *Burton.* A B C D E F G H I is an open Frame of Wood, firmly ſupported by the Shaft or Poſtern J. In the two Croſs Pieces H K, L M, is moved an horizontal Axis Q M, by means of the four Sails *a b*, *c d*, *e f*, *g h*, in a proper Manner expoſed to the Wind. Upon this Axis is fixed a Cone of Wood, M N O; upon which, as the Sails move round, a Weight S is raiſed by a String on its Superficies, proceeding from the ſmall to the largeſt End NO. Upon the great End or Baſe of the Cone is fixed a Ratchet-Wheel *i k*, in whoſe Teeth falls the Click X to prevent any retrograde Motion from the depending Weight.

Plate XXXII. Fig. 2.

5. From the Structure of this Machine, 'tis eaſy to underſtand, that it may be accommodated to eſtimate the variable Force of the Wind, becauſe the Force of the Weight will continually increaſe, as the String advances on the conical Surface, by acting at a greater Diſtance from the Axis. And therefore, if ſuch a Weight be put on, on the ſmalleſt Part at M, as will juſt keep the Machine in *Equilibrio* with the weakeſt Wind; then as the Wind becomes ſtronger, the Weight

We proceed now to the *Doctrine of Sounds.* We know by the Experiment of the Bell

will be raiſed in Proportion, and the Diameter of the Baſe of the Cone NO may be ſo large in Compariſon of that of the ſmaller End or Axis at M, that the ſtrongeſt Wind ſhall but juſt raiſe the Weight to the great End.

6. Thus for Example, let the Diameter of the Axis be to that of the Baſe of the Cone N O as 1 to 28, then if S be a Weight of *one Pound* at M on the Axis, it will be equivalent to 28 *lb.* or $\frac{1}{4}$ of an Hundred, when raiſed to the greateſt End. If therefore, when the Wind is weakeſt, it ſupports 1 *lb.* on the Axle, it muſt be 28 times as ſtrong to raiſe the Weight to the Baſe of the Cone. Thus may a Line or Scale of 28 equal Parts be drawn on the Side of the Cone, and the Strength of the Wind will be indicated by that Number therein from which the String ſhall at any time hang.

7. Futhermore, the String may be of ſuch a Size, and the Cone of ſuch a Length, that there ſhall be 16 Revolutions of the String between each Diviſion of the Scale on the Cone; ſo will the Strength of the Wind be expreſs'd in Pounds and Ounces. And if greater Exactneſs be required, let the Periphery of the Cone's Baſe be divided into 16 equal Parts, then whenever the *Equilibrium* happens, the String will leave the Conic Surface againſt one of thoſe Diviſions, and thus ſhew the Force of the Wind to a Dram *Averdupois* Weight.

8. Having premiſed thus much relating to the Structure and Nature of the Inſtrument, I ſhall now proceed to a more particular Examination of the Theory of Wind-Mills, by re-aſſuming what we have formerly ſaid on that Head (See *Annotat.* XLV.) Therefore let lm (parallel to the Axis QM) $= a$, repreſent the whole Force of the Wind on the Sail; this Force is reduced to ln, and this again to no, which acts normally to the Axis, and turns the Sail. Alſo we have ſhewn, that, putting $mn = x$, this Force which turns the Sail is expreſſed by $\frac{aax - x^3}{a}$; and that when it was a *Maximum*

in the exhauſted Receiver, that Sound has a neceſſary Dependance on the Air; and

$x = \sqrt{\frac{aa}{3}} = a\sqrt{\frac{1}{3}}$; and the Angle $lmn = 54^\circ\ 44'$.

9. Hence we obſerve, that when the Mill is in its greateſt Perfection, $ln = \sqrt{aa - xx} = \sqrt{a^2 - \frac{a^2}{3}} = a\sqrt{\frac{2}{3}}$; hence the whole Force in the Direction lm is to the ſame reduced, in the Direction ln, as lm^2 to ln^2, or as a^2 to $\frac{2}{3}a^2$, or as 1 to $\frac{2}{3}$, *viz.* as 3 to 2.

10. Again, the whole Force in the Direction lm is to the ſame a ſecond Time reduced in the Direction no, as a^2 to $\frac{a^2x - x^3}{a}$, that is, as a^2 to $aa\sqrt{\frac{4}{27}}$, or as 1 to $\sqrt{\frac{4}{27}} = \frac{200}{519} = \frac{5}{13}$ nearly; or, the Force thus reduced is to the whole Force as 5 to 13, when the Sails are poſited in the beſt Manner.

11. Now in order to determine the abſolute Force of the Wind, we muſt compare it with that of Water, as follows. Since Air and Water are both Fluids, if they move with equal Velocities, their Effects in a given Time will be as the Quantities of Matter, that is, (putting the *Roman* Letters for thoſe Particulars in Water, and *Italicks* for the ſame in Air) if $\mathrm{V} = V$, then $\mathrm{E} : E :: \mathrm{M} : M$. But in equal Quantities of Matter, *viz.* $\mathrm{M} = M$, their Effects will be as the Squares of the Velocities, *viz.* $\mathrm{E} : E :: \mathrm{V}^2 : V^2$ (See *Ann.t.* XLIX. 17.) Therefore, when neither the Velocity, nor Maſſes of Matter are known, the Effects will be in a given Time in a *Ratio* compounded of both; that is, $\mathrm{E} : E :: \mathrm{MV}^2 : MV^2$.

12. But we have ſhewn $\mathrm{M} : M :: \mathrm{DB} : DB$, (See *Annot.* LVI. 9.) therefore $\mathrm{E} : E :: \mathrm{DBV}^2 : DBV^2$ in

if we reflect on the Nature of the Particles of a ſonorous Body and thoſe of Air, we

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a given Time. Let us now ſuppoſe $\mathrm{B}=B$; then becauſe $\mathrm{D}:D::860:1$, (See *Annot.* LXXXIX. 6.) we have $\mathrm{E}:E::860\,\mathrm{V}^2=:V^2$; and laſtly, if we ſuppoſe the Effects to be equal, *viz.* $\mathrm{E}=E$, then we have $860\,\mathrm{V}^2=V^2$. Therefore if we put $\mathrm{V}=1$, we have $860=V^2$; and ſo $V=\sqrt{860}=29{,}326$; that is, *The Velocity of Air ought to be ſomewhat more than* 29 *Times greater than that of Water to ſtrike a given Surface with the ſame Force.*

13. Indeed Mr. *Belidor* makes $V=25\frac{1}{2}$, becauſe he has ſtrangely miſtaken the ſpecific Gravity of Air to be $\frac{1}{640}$, inſtead of $\frac{1}{860}$, on which Account all his Calculations on this Head are very faulty. If V denotes any equable Velocity of Water, the Height H of a Fall neceſſary to produce that Velocity is thus found, As $32:\sqrt{16}\,(=4)::\mathrm{V}:\sqrt{\mathrm{H}}$; or thus, As $1024:16::\mathrm{V}^2:\mathrm{H}=\frac{16\mathrm{V}^2}{1024}=\frac{\mathrm{V}^2}{64}$; or putting $\mathrm{V}=1$, we have $\mathrm{H}=\frac{1}{64}$. Now a cubic Foot of Water, whoſe Height is 1, ſtrikes with a Force $=62{,}5$ *lb*; therefore the Force of a Column, whoſe Height is H, ſtriking againſt a Surface of one Square Foot is $62{,}5\,\mathrm{H}=\frac{62{,}5}{64}$, or $=\frac{62{,}5\mathrm{V}^2}{64}$ when the Velocity is not given.

14. But (by *Art.* 12.) $\mathrm{V}^2=\frac{V^2}{860}$, therefore $\frac{V^2}{860}\times\frac{62{,}5}{64}=62{,}5\mathrm{H}=0{,}976$ *lb.* the Force of a Stroke of a Column of Water whoſe Velocity is $\mathrm{V}=1$, and of Air, whoſe Velocity is $V=29{,}3$, and Height $\mathrm{H}=\frac{1}{64}$ of a Foot; therefore $\frac{V^2}{860}\times 0{,}976=0{,}00113\,V^2$ will be a conſtant Multiplier to reduce the Force of Wind blow-

ſhall find that Sound is nothing but the Propagation of the Tremors and Vibra-

ing with any Velocity V, on any given Number of ſquare Feet or Area A, to Pounds *Averdupois* Weight. For Example, ſuppoſe the Velocity of the Wind at the Rate of 20 Feet *per* Second; here $V = 20$, and $V^2 = 400$, and $0{,}00113\, V^2 = 0{,}00113 \times 400 = 0{,}452$ of a Pound on a ſquare Foot; and therefore on 10 ſquare Feet it will be 4,52 *lb.*; on 100 ſquare Feet it will be 45,2 *lb.*; on 1000 452 *lb.*; and ſo on.

15. Hence, to compute the Force of Wind on the Sails of a Mill, we proceed as follows: Admit the Length of a Sail be 30 Feet, and Breadth 6 Feet, the Area or Surface will be 180 ſquare Feet, and $4 \times 180 = 720$ ſquare Feet, the Area of the 4 Sails; then admitting the Velocity of the Wind the ſame as before, *viz.* 20 Feet *per* Second, the Force on each ſquare Foot is 0,452, and therefore $0{,}452 \times 720 = 325{,}44$ *lb.* This is the abſolute or whole Force of the Wind blowing directly on the Sails: But ſince when the Sails are ſet right, this Force is diminiſhed in the Ratio of 13 to 5, therefore $\frac{5}{13} \times 325{,}44 = 125{,}17$ *lb.*

16. Suppoſe the Diſtance from the Axis Q to each Sail be 5 Feet, then will the Diſtance of the Centre of Gravity PQ be 20 Feet; therefore $20 \times 125{,}17 = 2503{,}4$ *lb.* the mechanical Force of the Wind on the Sails *per* Second to produce the Effects within the Mill, which may be computed as in the Example of the Water-Mill, *Annot.* XLIV.

17. To repreſent theſe Things more generally, let $A =$ Area of all the Sails, $V =$ Velocity of the Wind; then $0{,}00113\, V^2 A =$ abſolute Force, which multiplied by $\frac{5}{13}$ is $0{,}000435\, V^2 A =$ Force reduced by the oblique Poſition of the Sails. Now ſuppoſe a Weight W hanging from an uniform Axle, whoſe Semidiameter is d, keep the Sails *in Equilibrio* with the Force of the Wind; then D being the Diſtance of the Center of

tions of the former impreſs'd on the latter, to the *Tympanum* or Drum of the Ear, by

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Gravity of the Sails, we have D : d :: W : 0,000435 $V^2 A = F = \frac{W\,d}{D}$.

18. But becauſe, when the Machine is in its greateſt Perfection, the Weight it is charged with is but $\frac{4}{9}$ of W, (See *Annot.* XL.) therefore $\frac{4}{9} W \times \frac{d}{D} = \frac{4}{9} F =$ $0{,}000193\, V^2 A = P$; then putting $r = 0{,}000193$, we have $r V^2 A = P$, the reduced Force for the greateſt Effect; and $\frac{P}{r V^2} = A$, the Area or Surface of the Sails; and laſtly, $\sqrt{\frac{P}{r A}} = V$, the Velocity of the Wind, which therefore may be found by having A and P given.

19. Let $\frac{4}{9} W = w$, the Velocity of which Weight let be u; then $\frac{1}{3} V =$ Velocity of the Centre of Gravity of the Sails: then $\frac{V}{3} \times P = u \times w$, whence any one of the four Terms may be found, the reſt being given. Alſo $P = \frac{3\,uw}{v} = r V^2 A$, or $3\,u\,w = r V^3 A$; whence again any one of the four Quantities A, V, w, u, may be found, the others being known. See farther on this Subject my *New Principles* of GEOGRAPHY *and* NAVIGATION.

20. Since the Force of the Machine is as $A \times V^2 \times D$, it will be a *Maximum* when $A \times D$ is greateſt, the Velocity of the Wind V remaining the ſame; and if A be given, the *Maximum* will be when D is greateſt of all. Hence it appears, that if we are not confined to a given Diſtance from the Axis Q for adjuſting the Sails, we may diſpoſe the given Surface A into the Form of an *Iſoſceles Triangle* in each Sail, *viz.* ſuch as A B C D, inſtead of the equal Parallelogram Sail *a b c d* in common Uſe; for in the Triangular Sail the Center of Gravity is at P, and its Diſtance is QP; whereas in the Rec- Plate XXXII. Fig. 3.

the Action of whoſe Membrane they are communicated to the Air in the internal

tangular Sail *a b c d* the Centre of Gravity is in the middle Point *p*, and its Diſtance is Qp, much leſs than before.

21. For Example, let $ab = 6$ Feet, and $bc = 30$; then the Area $A=30$ Feet ſquare. Let QD equal 5, then is $Qp=20=D$, and $A\times D=600$. But in the Triangular Sail the Diſtance $QP=35=D$, and therefore $A\times D=1050$. The Force therefore of the ſame Wind upon the ſame Quantity of Sail, at the ſame Diſtance QD from the Axis, in the Triangular Sail A B C D, is to that on the common Sail *a b c d* as 1050 to 600, that is, $QP = 35$ to $Qp = 20$, or as 7 to 4, which therefore is nearly twice as great. The Truth of all is evident by Inſpection of the Figure, and *Annot.* XXXV. 8, 9.

22. I ſuppoſe it was ſome Conſideration of this Kind which led Mr. *Parent* to propoſe Sails in Form of *Elliptic Sectors*; for the Centers of Gravity in them alſo are removed to about two Thirds of their Length, and are moreover better adapted to fill a circular Space when placed oblique to the Wind, ſo that no Wind be loſt when you would take in all that falls on a given Space or Area. But for what Reaſon he ſhould declare the tranſverſe Poſition of the common Sail to be more advantageous than the longitudinal one, I am at a Loſs to gueſs. However, as I have not ſeen any thing he has wrote on the Subject, I ſhall ſay no more of the Matter.

23. As it would be endleſs to recount all the various Uſes which are or may be made of this moſt uſeful univerſal Element of Air, both for Natural and Mechanical Purpoſes; I ſhall content myſelf with ſetting before the Reader the Theory of that moſt uſeful domeſtic Inſtrument the BELLOWS, by whoſe means the Action of Fire, or Intenſity of its Heat, may be increaſed to a prodigious Degree. And for this Purpoſe I ſhall have recourſe to Dr. *Hales*'s *Statical Eſſays*. Vol. II. Page 329. where we find the following Computations.

Cavities of the Ear, where the *Auditory Nerve* receives the Impreſſion, and ex-

24. The Doctor meaſured the upper Surface of a Pair of Smith's Bellows, and alſo the Space they deſcended through in a Second of Time; by which he found the Quantity of Air expell'd in that Time was 495 Cubic Inches in its compreſs'd State. Now to find what Degree of Compreſſion it ſuffer'd, he fix'd a Mercurial Gage to the Noſe of the Bellows, and found the Force of the compreſs'd Air ſufficient to raiſe the Mercury *one Inch* high, at a Mean. Hence it appear'd, that the Force with which the Bellows impell'd Air into the Fire was $\frac{1}{30}$ of the Weight of the Atmoſphere.

25. Hence alſo it follows, that the Air driven thro' the Noſe of the Bellows in one Second was more than 495 Inches by $\frac{1}{30}$ Part of that Quantity, *viz.* by 16,5 Inches, which added to the former make $511\frac{1}{2}$ Inches of common Air. To find the Velocity with which this Air was impell'd, he meaſured the Area of the Orifice of the Noſe, and by that divided the 495 Inches, which gave for the Quotient 825 Inches, or 68,73 Feet, for the Length of the Cylinder of Air which ruſh'd *per* Second thro' the Noſe of the Bellows; which prodigious Velocity of Air acting conſtantly on the elaſtic re-acting Particles of Fire muſt immenſely increaſe their inteſtine Motion, and proportionably augment the Heat, which conſiſts therein, and from which all our Senſations of this Kind are derived.

26. The Doctor concludes with a Query, Whether if the Force with which the Air is impell'd by the Bellows into the Organ Pipes were taken in this Manner, we might not eſtimate the Velocities of the Undulations of Air required to form the various Notes or Sounds? The Velocity of undulating Air to that of Water being as their Denſities inverſely, nearly, *viz.* as 860 to 1, as will be ſhewn farther on.

27. Mr. *Martin Triewald* of *Sweden* has lately exhibited a new Invention for producing a continual Stream of Air, to blow the Fire of great Forges, Founderies, &c. and which may properly be call'd WATER-

cites the Senſation in the COMMON SENSORY in the BRAIN (CI).

BELLOWS; for the Contrivance is two hollow Bell-form Veſſels, ſuſpended from the Ends of a Lever, which is put into Motion by a Stream of Water running into two Troughs, both uniting or joining rather at the Stream, ſo that only one at a Time can receive the Water; which running to the larger and wider End, laid over the End of the Lever, does by its Weight carry the Lever down on that Part, till by deſcending the Water all runs out; and then the other Trough (which was filling in the mean time) preponderates, and forces down the other End of the Lever; and thus the Machine is conſtantly kept in Motion.

28. When one Arm of the Lever is raiſed, the Bell or Bellows hanging from it will be raiſed above the Surface of Water (in which the Machine is placed) that it may be fill'd with Air. Upon the Deſcent of the Lever, the Bell (by Weight affix'd to it) deſcends into the Water, by which means the included Air is greatly compreſs'd, and thereby forc'd to paſs through a long ſmall leathern Tube, going from the Top of the Bell to other metalline Tubes, which convey it to the Fire. Thus, by means of theſe two librating Bells, a conſtant Blaſt of Wind is ſupplied, whoſe Velocity may be increaſed or diminiſhed by proper Contrivances, which the Reader may ſee in the *Philoſophical Tranſactions*, together with a Print of the Engine.

(CI.) 1. The Structure of the EAR, with its admirable *Apparatus* to conſtitute an *Organ of Hearing*, is well worth the Attention of every Man. The external Part is adapted for taking in a large Portion of the tremulous Air, which is reflected ſtrongly by a fine, elaſtic, tremulous Cartilage, and by this means it is convey'd more denſe and elaſtic to the interior Cavity, or *Concha* of the outward Ear.

2. The free, hollow, elaſtic Aperture of this Cavity, conſtructed with proper Muſcles, is by that means capable of being expanded, contracted, and every way adapted to receive the various Tremors of the Air:

For the Parts of a *ſonorous* Body, being put into Motion by Percuſſion, do vibrate

And moreover it is ſo diſpoſed, that it is able more firmly to unite and condenſe, or more laxly to diſperſe or rarify, the ſame aerial Rays, ſo as to accommodate itſelf for attemperating a Sound too ſtrong, and augmenting it when too weak, as Occaſion requires.

3. The *Meatus Auditorius*, conſiſting partly of a cartilaginous and partly of a bony Pipe, conveys the Sound towards the interior Parts, and the Obliquity of the Canal increaſes the Superficies, and conſequently multiplies the Points of Reflection. Moreover, the triangular cartilaginous Tongue, by its elaſtic tremulous Texture, and erect Poſition in the Hollow of the *Concha*, juſt over the Orifice of the Auditory Paſſage, cauſes, by an egregious Mechaniſm, that all the Rays of Sound which arrive at the Ear ſhall enter the ſaid Paſſage ; and prevents their flying out again by any Reflections whatſoever. Its tubulous cylindro-elliptical Figure, by a ſerpentine Progreſs firſt aſcending, then deſcending, and then aſcending again till it terminates in the Membrane of the *Tympanum*, increaſes the Reflection and Sound, and cauſes that all the ſonorific Rays ſhall at laſt fall united upon the central Point of its End ; hindering at the ſame time all Senſation of a confuſed and clangorous Sound.

4. The *Membrana Tympani*, or fine Membrane at the End of the *Meatus Auditorius*, is ſo obliquely extended acroſs the Paſſage, as above to make an obtuſe Angle, and below an acute one with the ſaid *Meatus*. Hence the Surface is increaſed, and render'd more capable of tremulous Concuſſions, and of concentering the Rays upon its middle Point.

5. This Membrane being expanded upon and connected with the bony Margin of the *Meatus*, is on the fore Part (towards the *Meatus*) concave, and convex behind or on the internal Part, where it is applied to the Handle-Part of a little Bone call'd the *Malleus* or Hammer, whoſe Head is moveable in a bony *Sinus* on one Part,

F.ICE

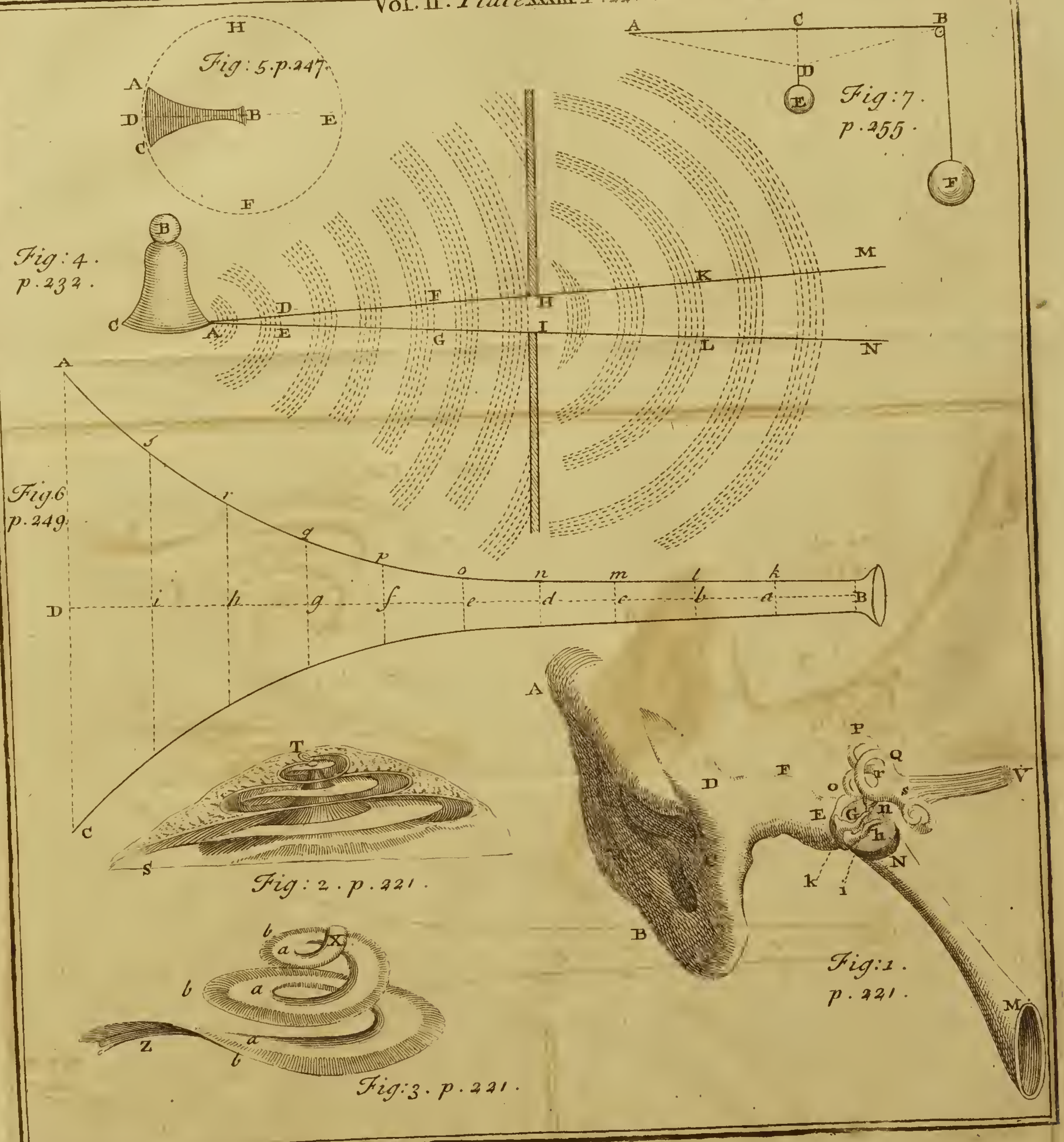

Fig: 5. p. 247.

Fig: 7. p. 255.

Fig: 4. p. 232.

Fig. 6 p. 249.

Fig: 2. p. 221.

Fig: 1. p. 221.

Fig: 3. p. 221.

forwards and backwards through very ſmall Spaces, by their elaſtic Quality. In

and on the other it is articulated with another little Bone call'd the *Incus* or Anvil, which freely moves in that Articulation; and on the other End it is again articulated with a little orbicular Bone, and with the *Stapes* or Stirrop, which on its Baſe Part is connected to a Membrane ſpread over the *Foramen Ovale* or elliptic Hole of another bony Cavity call'd the *Veſtibulum*.

6. But as it will be impoſſible to give an Idea of this wonderful Conſtruction without a Print, therefore let Plate XXXIII. Fig. 1. AB be the external Ear; C its *Concha*, or Cavity; DE the *Meatus Auditorius*, which in Length is 7½ Tenths of an Inch, in Breadth 3, and in Depth 4. G is the *Membrana Tympani*, h the Handle of the *Malleus*; k the *Incus*, and *i* the orbicular Bone; *n* the *Stapes*, and r the *Veſtibulum* hollowed out of the *Os Petroſum*, in the Cavity of the Labyrinth.

7. In the Veſtibule we obſerve the following particular Conſtruction of Parts. On the larger Part are three ſemicircular Canals or Conduits O, P, Q, which communicate by five Orifices with the Cavity of the Veſtibule; they are of a bony Subſtance, and of an elliptic Cavity. The leſſer Part of the Veſtibule communicates with the *Cochlea*, or ſpiral Fabric S.

8. This wonderful Part merits particular Notice, and is therefore repreſented by itſelf in two Figures; wherein is ſhewn the bony conical Canal ST, making 2½ Revolutions round a bony Cone from the Baſe to the Apex T. This ſpiral Cavity is, from the Baſe S to the Top T, divided by a tranſverſe *Septum*, or Partition, of a triangular Figure, repreſented by Z X. This on its Baſe Part adhering to the Cone is bony, (which is ſhewn by *a*, *a*, *a*,) Fig. 2, 3. and is of an elaſtic tremulous Texture, and exceeding ſmooth or polite. The exterior Part *b*, *b*, *b*, is of a membranaceous nervous Texture, whoſe Chords or Fibres lie as repreſented in the Cut: It is connected with the bony Baſe on one Part, and with the Canal on the other, ſo that the ſpiral Duct of the *Cochlea* is divided

this Action they affect the Particles of Air contiguous to them, and compel them up-

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into two equal Cavities without any Communication with each other; though the Orifice of the ſuperior Cavity opens into the *Veſtibulum*, and the other is ſhut cloſe by the Membrane of the *Foramen Ovale*.

9. The Auditory Nerve V enters the Veſtibule by ſeveral little Holes as at S, and forms a curious Lining or Tapis all over the inſide Surface both of the Veſtibule and its ſemicircular Canals O, P, Q. Theſe Nerves alſo paſs into the *Cochlea*, and entering between the two Membranes of the triangular Zone, or *Septum*, Z X, do there divide, and branch themſelves out into an exquiſite membranous Expanſion on each Side the ſame, which thus becomes the more immediate Organ of Hearing.

10. This Cavity of the Veſtibule is always fill'd with an elaſtic Air, though there appears no viſible Way by which it can enter. Alſo the Labyrinth or Cavity of the Drum is fill'd with common Air, by means of the *Euſtachian Duct* or *Tube*, as M N; the Orifice M opening into the Mouth, and N into the Cavity of the Labyrinth.

11. Having thus premiſed a Deſcription of the ſeveral Parts, we ſhall the better apprehend how Sounds are excited in the Mind. Thus the Pulſes of Air entering the *Meatus Auditorius* DE are condenſed by various Reflections through the Paſſage to their Incidence on the *Membrana Tympani* at G, which is render'd more or leſs concave, or lax and tenſe, by the Handle h of the *Malleus*, actuated by its proper Muſcle. By this means the Air contain'd in the Labyrinth is admitted, expell'd, compreſs'd or rarified, according as the *Euſtachian Tube* is opened or ſhut.

12. The *Membrana Tympani* G being thus adapted for receiving the Sounds of tremulous harmonic Bodies, and modulating the internal Air of the Labyrinth, can eaſily communicate the Impreſſions to the *Incus k*, which tranſmits them to the *Os Orbiculare i*, this to the

on the firſt Impulſe to move forwards alſo; and thoſe propel the next, and ſo on,

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Stapes n, and that to the Membrane of the *Foramen Ovale* of the *Veſtibulum* r.

13. This Membrane, by ſuch an *Apparatus* of Parts, may be intended or remitted in infinitely different Degrees, ſo as to become adapted for the Tremors of every Sort or Degree of Sound; and for communicating them to the internal Air, which affects the Nerves every where expanded over its internal Surface, but more eſpecially the nervous Expanſion of the *Cochlea*.

14. For here, as we have ſhewn, the Fibres of the *Septum Tranſverſale*, *b*, *b*, *b*, are contrived like ſo many Strings of an Harpſicord, of various decreaſing Lengths, and different Octaves, that ſo ſome or other of them may be of a proper Length to be in Concord with the ſounding Body, or to tremble with the ſame Vibrations, which by means of the Nerves are convey'd to the Common Senſory in the Brain, where the Mind perceives and diſtinguiſhes the infinite Differences of harmonious and diſcording Tones.

15. Thus, though we are admitted to view the amazing Mechaniſm of the Organ of Hearing, yet can we get but a general Notion of the Manner in which theſe Senſations are produced, or of the particular Functions performed by every Part, and the ſpecial Uſes to which they are ſubſervient, in the general Execution of this Senſe; with reſpect to which there remains many things yet to be enquired after, even by the learned *Boerhaave*, as we find in Page 250 of his *Inſtitutes*, which ſee.

16. From this Account of the Ear, we have a Solution of ſome Difficulties; as, Why the Ear is affected with great Pain in going down into the Sea in a Diving-Bell: Why People generally open their Mouths when they liſten with great Attention: Why Deafneſs enſues on a Rupture of the *Membrana Tympani*, or from an Obſtruction of the *Euſtachian* Tube: Why we hear but one Sound with two Ears. But how ſome People taking Smoke into the Mouth can emit it by their Ears,

to a very conſiderable Diſtance, according to the Intenſity of the percuſſive Force. By this means the Particles of Air are compreſs'd nearer together, than in their natural State.

BUT when the Particles of the ſonorous Body make the ſecond Part of the Vibration, by returning back again, the Particles of Air alſo, by their repulſive Power, repel each other towards their proper Places, and thus again expand themſelves.

Now ſince Motion once generated in elaſtic Bodies continues ſome time before it can be deſtroyed by the Reſiſtance and Counteraction of contiguous Bodies, it follows, that the Particles of the ſonorous Body, and conſequently thoſe of the adjacent Air, have for ſome time a reciprocal vibratory Motion, by going forwards and backwards through very ſmall Spaces in an indefinitely ſmall Particle of Time; which Motion gradually decreaſes, till it be totally deſtroy'd (CII).

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is not eaſy to anſwer, there being as yet no Perforation of the *Membrana Tympani* diſcovered; though this ſeems a plain Demonſtration that there is one or more, though not perceptible to the Eye.

(CII.) 1. The Doctrine of Sounds is the moſt intricate and perplex'd of any thing we find in Philoſophy; and perhaps this is the only Subject which the greateſt of Men has (in his *Principia*) treated in a

FROM the Nature of a Fluid, whatever Motion is generated in any one Particle, it

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Manner not quite ſo phyſical and mathematical as the Nature of the Thing required. I ſhall refer the Reader to the Commentaries of Meſſ. *Le Seur* and *Jacquier* on the *Principia*, where they will find Sir *Iſaac Newton*'s Hypotheſis relating to the Motion of the Particles of an elaſtic Medium to be fallacious; and other Methods propoſed, by which the *Newtonian* Doctrine of Sound is reſtored. I ſhall here add an Explication of ſuch *Phænomena* only, as are of principal Concernment, and at the ſame time pretty eaſy to be underſtood.

Plate XXXII. Fig. 4.

2. Let ABC be an elaſtic String or Chord, fix'd in the Points A and C, and drawn out of its natural right-lin'd Situation ABC. Such a Chord, in its State of Tenſion, will, when let go, return by its natural Reſort, not only to its natural Situation A D C, but with the Motion it there has will go on to E, ſo that D E is nearly equal to B D; and from thence it will return again nearly to B; which Motion from B towards E, and from E towards B, will be reciprocated a great Number of Times before the Chord will come to a State of Reſt: And each Motion through the Space BE is call'd a *Vibration* of the Chord.

3. When the Chord begins its Motion at firſt from B, it ſtrikes the Particle of Air contiguous to it in B, and that will by its Approach towards the next affect it, by means of the repulſive Power, which keeps them all at equal Diſtances from each other; and ſo on, through ſuch a Number of Particles as can receive the Motion while the String moves from B to D. Let A, B, C, D, E, F, G, *&c.* repreſent ſuch a Series of Particles of Air at an equal Diſtance, and the firſt Particle A contiguous to the middle Point B of ſuch a String, and agitated by it in its Motion.

4. The String beginning to move, all the Particles A, B, C, will begin to move forwards alſo; and ſince this Motion is propagated in Time, let E be the remoteſt Particle moved while the Chord is moving from

is by that Particle communicated equally to all around it, as from a Centre; conſe-

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B to D; during which Time the Chord, having an accelerated Motion, will cauſe the Particles to approach each other with an accelerated Motion likewiſe; and becauſe thoſe accelerated Approaches begin at A and reach to E, in the Time the Chord is going from B to D, therefore the Diſtance AB will be leſs than B C, and this leſs than C D, and that leſs than D E, and the Diſtance EF will begin to be leſſen'd when the String is arriv'd to the Site A D C, and the Particles A, B, C, D, E, F, *&c.* will have the Arrangement repreſented in the ſecond Line.

5. But now the Chord, having acquired the Situation ADC, will be no farther accelerated, but on the contrary retarded, as it will now go on from D to E; the Effect of which upon the Particles of Air before it will be as follows. They will all go on forwards till the Chord comes to E, and the Particle A to its Situation in the third Line: But ſince the Force upon A begins to abate as the String begins to move from D, the elaſtic Force now between A and B will, by acting both Ways, continue to accelerate the Motion of B, and retard that of A. Thus the Diſtance BC will ſtill diminiſh till B come to be nearly equidiſtant between A and C; and C will be accelerated till it be equidiſtant between B and D; and ſo on. So that as the Acceleration is continued forwards, the Diſtances will diminiſh towards F; and by the Time the Chord is arrived at E, the Particles E E will be at their neareſt Diſtance. And ſince the Motion of A is continually retarded, it will loſe what before it had gained in the ſame Time; and will therefore now be at the ſame Diſtance from B as at firſt nearly. So that the Particles from A to G will have the Situations as repreſented in the third Line.

6. The Chord now returning from E to D, gives Liberty to the repulſive Power between A and B to ſeparate them to a greater Diſtance than in their natural State, and which they at preſent have. By this means all the other Intervals B C, C D, D E, E F, will al-

quently the Tremors of the ſounding Body will be propagated all around from

ſo increaſe, and become ſucceſſively greater than the natural Diſtance; but that Exceſs will be leſſer in each, till you come to FG, which will be equal to the natural Diſtance, at preſent between A and B. The Motion at the ſame Time continuing in all the Particles from H to N, they will all move forwards, and the preſent contracted interval between H and I will ſucceed between all the reſt, till it arrives to the Particle N, when the Interval MN will be the ſame as at preſent is H I. And thoſe Particles beyond N to S, will, by the preceding ones, be put into the ſame reſpective Diſtances, but in an inverſe Order, as thoſe have between G and N. And the whole Series (now the String is at D) will have the Intervals of the Particles reſembling thoſe in the 4th Line.

7. The Chord not ſtopping at the Situation A D C, but going on towards A B C with a retarded Motion, the Velocity of the contiguous Particle A will alſo be retarded and become leſs than that of B; upon which the Diſtance between them will be leſſen'd, and the more ſo as the String approaches to B. Hence all the Intervals, now dilated beyond their natural State, will, by Degrees, contract; but gradually ſlower, till you come to F, where the preſent largeſt Interval between A and B will be found between F and G, and that between A and B will have acquired its natural Extent when the Chord is arrived at B. Then likewiſe the Particles from G to N will acquire the ſame Situation as thoſe now have between A and G; and from N to S, the ſame as now is ſeen between G and N; and from S forwards, the ſame as is now before the Particle N, the Point S being now the middle Point of Condenſation; all which is clearly ſeen in the 5th Line of the Figure.

8. Thus the Condenſation which began at A, by the firſt Part of the Vibration, was propagated to G by the ſecond, from thence to N by the third, and laſtly to S by the fourth Part of the whole Motion of the String in going and returning; and this Extent of Air, thus agitated by the Chord in going and returning, is call'd

the Point of Percuſſion, as a Centre, in *concentric hollow Superficies or Shells of Air*, which are not improperly called *Aerial Pulſes* or *Waves of Air :* Analagous to which are the circular Waves generated on the Surface of Water all around the Point where any Impreſſion is made, in any Manner or Direction whatſoever (CIII.)

by Sir *Iſaac Newton* a *Wave* or *Pulſe of Air*. In which Wave the Particles from A to N are in a dilated State, and from N to X in a contracted or condenſed State ; which two Parts of the Wave anſwer to the concave and convex, or low and high Part of a *watry Wave*.

9. As the Chord goes on to make another Vibration, it will not only continue to agitate the Air at preſent in Motion, but will ſpread the Pulſation of the Air as much farther, and by the ſame Degrees as before ; and the like will happen after every compleat Vibration of the String. Thus the Air being a fluid Body, and the Impreſſion made on any one Part affecting all the Particles alike around it, 'tis plain, thoſe Pulſes will be propagated in every Direction all around in concentric Aerial Shells or ſpherical Waves of Air.

10. That the Motion of the Pulſes in an elaſtic Medium is analagous to that of Waves generated in the Surface of ſtagnant Water, is evident, when we conſider that the Condenſation of the Parts of the elaſtic Medium is in Lieu of the Elevation of the Water; the elaſtic Force effects the ſame in the Medium as Gravity does in the Water, and the denſeſt Parts of the Pulſes correſpond to the higheſt Parts of the Waves. Wherefore as there is ſo great an Affinity between theſe two Phænomena, it will be requiſite, before we go farther, to explain the Nature and Properties of aqueous Waves, which will therefore be ſhewn in the next *Annotation*.

Plate XXXII. Fig. 6, 7. (CIII.) 1. Sir *Iſaac Newton* explains the Nature of Waves in Water after the following Manner. Let AB

THESE Pulſes or Waves of Air are affected with the following Properties, *viz.*

and C D be the Surface of Water quieſcent in the upright Legs K L, M N, of a recurv'd Tube. And if the Water be put into Motion, and aſcends in the Leg KL, to E F, it will deſcend in the Leg M N to G H; ſo that $EF=GH$. Again, let P V be a Pendulum vibrating in the Cycloid RPS; its Length VP, from the Point of Suſpenſion to the Centre of Oſcillation, is equal to half the Length of the Water in the Tube; let P be the loweſt Point, and P Q an Arch of the Cycloid equal to the Altitude A E.

2. The Force by which the Water is alternately accelerated and retarded in its Motion in the Tube, is the Exceſs of the Weight of Water in either Leg above the Weight in the other; and therefore when the Water in the Leg KL aſcends to EF, and in the other Leg deſcends to GH, that Force is equal to the Weight of the two equal Quantities of Water $AEFB+CGHD=2AEFB$; and therefore is to the Weight of the whole Water as E A to V P, or as P Q to P R, becauſe the Semi-cycloid P R is equal to the Length of the Pendulum which deſcribes it, from the Nature of the Curve.

3. Alſo the Power by which the Weight P is in any Point Q accelerated or retarded in the Cycloid, is to its whole Weight as the Diſtance P Q from the loweſt Point P to the Length of the Semi-cycloid PR. Wherefore the moving Forces of the Water and Pendulum, deſcribing equal Spaces AE, PQ, are as the Weights to be moved; and therefore, if the Water and Pendulum are at firſt quieſcent, thoſe Powers will move them equally in equal Times, and cauſe that they go forwards and backwards together, with a reciprocal Motion. All which is eaſily deduced from what has been ſaid of the Nature of the Cycloid, the Motion of heavy Bodies, and the Forces of Bodies in Motion.

4. Hence it follows, that whether the Diſtance A E be great or ſmall, the Reciprocations of the Water will be all perform'd in equal Times. Alſo it follows, that if the whole Length of the Water be 78,4 Inches, each

I. *They are propagated all around in a ſpherical undulatory Manner* (as I ſaid but now ;) and that not only from the tremulous Body, but from the Holes in any Obſtacles they meet with : Whence it comes to paſs, that *one and the ſame Sound*

Reciprocation, or Aſcent and Deſcent of the Water, will be perform'd in *one Second* of Time ; becauſe a Pendulum of half that Length vibrates in that Time. Laſtly, If the Length of the aqueous Canal be increaſed or diminiſhed, the Time of each Reciprocation will be increaſed or diminiſhed in the ſubduplicate Ratio of the Length.

Plate XXXII Fig. 8. 5. When the Nature of Waves in Water is conſidered, it will be found to agree very nearly with the Motion of the Water in the Tube above mentioned ; and conſequently their Motion will be ſimilar to that of a Pendulum. For let E F G repreſent the level Surface of Water when it is not agitated ſo as to produce Waves ; when it is thus agitated, let A, B, C, D, repreſent the wavy Surface ; A, C, the higheſt Parts of the Waves ; and B, D, the loweſt or concave Part. Then 'tis evident the Weight of the Water at A above E G will cauſe it to deſcend as far below the Level to B ; and with the Motion acquired by that Deſcent, it will again aſcend to the ſame Height C, and ſo produce a conſtant Succeſſion of Waves in the watry Surface, after the ſame Manner as was ſhewn in the Tube.

6. Hence it follows, that, becauſe the Length of the whole Water to be moved is from the higheſt Point A to the loweſt Point B, if the Length of a Pendulum be $\frac{1}{2}$ A B, it will oſcillate once while the Water deſcends from A to B ; and in another Oſcillation, it will aſcend from B to C, and ſo on. So that a Wave will paſs thro' its whole Length in the Time of two Oſcillations ; and therefore in the Time of one Oſcillation of a Pendulum four Times as long, or equal to ABC.

may be heard by ſeveral Perſons, in any different Situations with reſpect to the ſounding Body, if not at too great a Diſtance.

II. *The Denſity of theſe aerial Pulſes decreaſes, as the Squares of the Diſtances from the ſounding Body increaſe:* For ſince the Force or Motion in each Shell is the ſame, it muſt decreaſe as the Number of Particles increaſes in each Shell: But this Number of Particles is as the Superficies of the Shell, which is as the Squares of the Diameter or Semidiameter of the Sphere, that is, as the Diſtance from the ſounding Body. Hence the Diſtinction of Sounds into *loud* and *low*, *ſtrong* and *weak*, according as we are nearer to, or farther from, the ſounding Body. The utmoſt Limits of

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7. Whence becauſe A B C in very large and wide Waves is nearly equal to the Breadth AC; therefore when the Waves are 39,2 Inches broad, they will undulate in one Second of Time; and conſequently ſince the Times of all the Undulations are equal, there will be $39{,}2 \times 60 = 2352$ Inches, or 196 Feet run thro' by a Wave in one Minute, which is 11760 Feet *per* Hour. Hence alſo the Velocity of greater or leſſer Waves will be increaſed or diminiſhed in the ſubduplicate Proportion of their Breadth; that is, if V = Velocity of the greater Waves A B C D, and v = Velocity of the leſſer Waves *a*, *b*, *c*, *d*, *e*, *f*, &c. then it will be $V : v :: \sqrt{AC} : \sqrt{ac}$. Becauſe the Velocities and Times of Bodies moved in any Manner by Gravity, are proportional to the ſquare Roots of the perpendicular Altitudes; and thoſe Altitudes are as the Lengths of Pendulums, and therefore as the Breadth of Waves.

audible Sounds are about 180 or 200 Miles. (CIV.)

(CIV.) 1. Let A B C repreſent the ſonorous Body; by the tremulous Motion of its Parts, it will agitate the Air contiguous to every Point as A, where it will be condenſed to a certain ſmall Diſtance, and make a Pulſe or Wave of Air in the Manner as has been largely ſhewn (*Annotation* CII.) The firſt Wave or Pulſe will by its elaſtic Power in expanding itſelf produce a Second, that a Third, and ſo on; till the impreſs'd Motion be diffuſed thro' too large a Quantity of Air to be any longer ſenſible. Plate XXXIII. Fig. 4.

2. The Quantity of Motion produced by each Tremor of the ſonorous Body, being communicated ſucceſſively to larger Portions of Air, the Part thereof which each Particle will acquire will conſtantly decreaſe. This Decrement of the Motion will be as the Increment of the Number of Particles, which is as the Superficies of the ſpherical Shell; and ſince all Superficies are as the Squares of their Diameters or Semi-diameters, therefore the Force in the Particles of the Wave or Shell at D is to that in the Particles of the Shell at F as AF^2 to AD^2; that is, the Force of Sound decreaſes as the Squares of the Diſtances increaſe.

3. It is plain, the Diſtance to which Sounds may be heard will be proportional to the Magnitude or Intenſity of the Stroke made on the tremulous Body emitting the Sound; for the greater that Stroke is, the greater will be the Agitation of the Parts of the ſonorous Body, and of courſe the greater will be the Force with which they will ſtrike the Particles of Air. Laſtly, the greater Force is upon the Air, the more ſtrongly will it be condenſed and expanded; hence the greater will be the Stroke at any given Diſtance on the Drum of the Ear, and conſequently the greater will be the Diſtance at which the Agitation of the Air will be ſenſible.

4. The Experiments are numerous by which it has been found, that Sound is audible to the Diſtance of

III. *All the Pulſes, whether denſer or rarer, move with equal Velocities:* This Sir *Iſaac Newton* has demonſtrated *a priori,* and alſo that this Velocity is at the rate of 1142 Feet in one Second of Time; which moſt exactly agrees with the repeated and moſt accurate Experiments of the late Reverend Mr. *Derham.* The Velocity of Sound is therefore near thirteen times as great as that of the ſtrongeſt Wind: And ſince it muſt neceſſarily increaſe with the Air's Elaſticity, it will be greateſt in Summer when the Air is moſt heated, and *vice verſa* in the Winter: Alſo, as the Motion of the Wind conſpires with, or is contrary to that of Sound, the Velocity of Sound will be in ſome ſmall Degree augmented or diminiſhed thereby, though not diſcernible in Experiments.

IV. *The Interval or Diſtance of the Pulſes from each other is the ſame among all that are excited by the ſame Stroke:* For ſince each Pulſe is cauſed by a ſingle Vibration

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50, 60, or eighty Miles: But Dr. *Hearn,* Phyſician to the King of *Sweden,* tells us, that at the Bombardment at *Holmia, A. D.* 1658, the Sound was heard to the Diſtance of 30 *Swediſh* Miles, which make 180 of ours. And in the Fight between *England* and *Holland, A. D.* 1672, the Noiſe of the Guns was heard even in *Wales,* which cannot be leſs than 200 Miles.

bration of the ſounding Body, and ſince they all move with equal and uniform Velocities, 'tis plain they muſt ſucceed each other at Intervals proportion'd to the Times of the Vibrations: but the Times of the Vibrations of the ſame Body are all equal; conſequently, the Intervals of the Pulſes will be ſo too. (CV.)

(CV.) 1. Sir *Iſaac Newton* and other Mathematicians have ſhewn (in a Method too prolix and intricate to be here repeated) that if a Pendulum were conſtructed whoſe Length was equal to the Height of an homogeneal Atmoſphere, whoſe Denſity is every where the ſame with that of the Air upon the Surface of the Earth, in the ſame Time that ſuch a Pendulum makes one whole Oſcillation in going forwards and backwards, the Wave or Pulſe of Air will paſs through a Space equal to the Circumference of a Circle deſcribed with a Radius equal to the ſaid Pendulum.

2. Therefore while the Pendulum makes half an Oſcillation, or one ſingle Vibration, the Pulſe will move through a Space equal to half the Circumference: Whence the Space deſcribed by the Pulſe in the Time of a Vibration is to the Length of the Pendulum as the Semi-circumference to the Radius, or as the Circumference to the Diameter, that is, as 3,14159 to 1. Now the Length of ſuch a Pendulum is 30100 Feet, (as we have elſewhere ſhewn) but Sir *Iſaac* makes it 29725 Feet, whoſe Meaſure we ſhall here follow. The Circumference of a Circle whoſe Radius is 29725 is 186768, the half whereof is 93384 = Space a Pulſe paſſes through in one ſingle Vibration: But ſince a Pendulum 39,2 oſcillates in the Time of one Second, and the Times of Oſcillation in different Pendulums are in the Subduplicate Ratio of their Lengths; therefore, ſince in 29725 Feet we have 356700 Inches, we muſt ſay, As $\sqrt{39,2} : \sqrt{356700} :: 1 : 95\frac{3}{8}$ Seconds. But in

V. *The aerial Pulſes are propagated together in great Numbers from different Bodies*

that Time the Pulſe paſſes over 93384 Feet; conſequently, $95\frac{3}{5} : 93384 :: 1 : 979$ Feet = Space paſs'd through by a Pulſe in one Second of Time.

3. This then would be the Velocity of the Pulſes, were the Particles of Air ſo very ſmall as that their Magnitude ſhould bear no ſenſible Proportion to the Intervals between them; and alſo if the Medium had no Admixture of any other Particles but thoſe of pure Air: Neither of which is the Caſe; for the Particles of Air are ſo groſs that they will not paſs through the Pores of Glaſs any more than Water; and Sir *Iſaac Newton* ſuppoſes them to be of the ſame Magnitude with the Particles of Water or Salt. If this be ſo, let $D =$ Diameter of the Particles, $S =$ Space or Interval between them; then will $S + D =$ Diſtance of the Centres of the Particles. Let $N =$ Number of Particles in the Side of a Cube of Air, then will $NS + ND =$ Side of the Cube.

4. Again, let $M =$ Number of Particles of Water in the Side of an equal Cube, and $MD =$ Side of the Cube of Water; whence $NS + ND = MD$. Then if the Denſity of Air be to that of Water as 1 to A, we ſhall have $1 : A :: N^3 : M^3$; whence $1 : A^{\frac{1}{3}} :: N : M$; conſequently, $M = NA^{\frac{1}{3}}$. Wherefore ſince it is $NS + ND = MD = NDA^{\frac{1}{3}}$, it will be $S + D = DA^{\frac{1}{3}}$, and $S = D \times A^{\frac{1}{3}} - 1$; therefore $D : S :: 1 : A^{\frac{1}{3}} - 1$; whence $D : S \times D :: 1 : A^{\frac{1}{3}}$.

5. If therefore $A = 860$, (as we have ſhewn) then $A^{\frac{1}{3}} = 9$ nearly; if we put $A = 1000$, then $A^{\frac{1}{3}} = 10$. Whence $D : S + D :: 1 : 9$, or as $1 : 10$; whence the Diameter of a Particle of Air will in ſuch Caſe be to the Interval between the Particles as 1 to 8 or 9. And ſince the Motion is inſtantaneous through the ſolid Par-

without Diſturbance or Confuſion; as is evident from Concerts of Muſical Inſtruments, where divers Sounds, of different Intervals

ticles of Air, and they make up $\frac{1}{8}$ or $\frac{1}{9}$ Part of the whole Space 979 Feet paſs'd through in one Second by a Pulſe, therefore to allow for this we muſt add $\frac{979}{9}$, or 109 Feet to the former Sum; that is, $979 + 109 =$ 1088 Feet, for the Velocity of Sound *per* Second.

6. But ſince the Atmoſphere conſiſts not of pure Air, but has an Admixture of Vapours of a different Elaſticity and Tone; theſe Vapours will not participate of the Motion of pure Air, by which Sound is propagated; in like manner as an elaſtic String, if ſtruck, will not move another very near it, unleſs it be under the ſame Degree of Tenſion, and of the ſame Tone. Therefore the Quantity of Air producing Sound muſt be diminiſh'd in Proportion to the Quantity of Vapour, in a given Space; in which Sir *Iſaac* ſuppoſes the Air is to the Vapour as 10 to 1. Whence the Air and Vapour together in a given Space is to the pure Air as 11 to 10.

7. But the Velocity of the Pulſes will increaſe in the Subduplicate Ratio of the diminiſh'd Quantity of Matter, that is, in the Subduplicate Ratio of 11 to 10, or in the entire Ratio of 21 to 20, (as he has ſhewn, *Princip.* Prop. 48. Lib. II.) Therefore, if we ſay, As 20 : 21 :: 1088 : 1142; whence the real Velocity of Sound thus inveſtigated from the Nature of elaſtic Air by our great Author) is at length found to be at the Rate of 1142 Feet *per* Second.

8. The Truth and Accuracy of this noble Theory have been ſufficiently confirm'd by Experiments, particularly thoſe made by the late Rev. Dr. *Derham*, of which I ſhall give ſome Account by and by; but will firſt lay before the Reader a View of the different Eſtimates made of the Velocity of Sound by ſeveral eminent Philoſophers, as in the Table following.

and various Coincidences, ſtrike the Ear at once, yet with Diſtinctneſs and agreeable Conſonance.

VI. *The Particles of Air, and conſequently the Pulſes, ſtriking againſt an Obſtacle, will be reflected back under an Angle equal to that*

	Feet per *Second.*
The Honourable Mr. *Roberts*,	1300
The Honourable Mr. *Boyle*,	1200
Mr. *Walker*,	1338
Merſennus,	1474
The Academy at *Florence*,	1148
Royal Academy at *Paris*,	1172
Sir *Iſaac Newton*, *Flamſtead*, *Halley*, and *Derham*,	1142

9. As no Man ever had a better Opportunity, ſo none could improve it with greater Diligence, Aſſiduity, and Accuracy, in determining and ſettling the various Phænomena of Sounds, than the ſo often celebrated Philoſopher laſt mentioned. He proved by Experiments made with the Strokes of a Hammer, and the Exploſion of a Gun at the ſame time, at the Diſtance of a Mile, that the Velocity of Sounds produced from different Bodies was the ſame, or came to his Ear in the ſame Time.

10. That the Motion of Sound was equable and uniform, or that it paſs'd through Spaces proportional to the Times, he found by various Experiments made by the Exploſion of Guns at different Diſtances, as appears by the following Table which he has given us: Where the firſt Column ſhews the Places at which the Guns were fired; the ſecond the Number of Vibrations of an Half-Second Pendulum; the third the Diſtance of the Places in Miles and decimal Parts, as meaſured by Trigonometry; the fourth the Diſtances meaſured by the Velocity of Sound, admitting it to be at the Rate of one Mile every 9¼ Half-Seconds.

of Incidence; in the ſame manner as will be ſhewn in regard to the Rays of Light. Hence a *Repetition of the Sound*, heard by the

At *Hornchurch* Church,	9	— 0,9875 —	
North Okendon Church,	18½	— 2,004 —	2,000
Upminſter Mill,	22½ 23	— 2,4 —	2,4 2,48
Little Warley Church,	27½	— 3,0 —	2,97
Rainham Church,	33¼	— 3,58 —	3,59
Alvel Mill,	33	— 3,58 —	3,57
Dagenham Mill,	35	— 3,85 —	3,78
South Weal Church,	45	— 4,59 —	4,86
Eaſt Thorndon Church,	46½	— 5,09 —	5,03
Barking Church,	70½	— 7,7 —	7,62
Guns at *Blackheath*,	116	—12,5 —	12,55

11. The great Exactneſs of meaſuring Diſtances by Sounds appears from the above Table, as well as the Equability of the Motion; but to render this Matter ſtill more certain and indiſputable, the Doctor took a Journey to *Foulneſs* Sands on the Coaſt of *Eſſex*, which form a ſmooth large Plain for Miles. On this Plain he meaſured 6 Miles in a right Line, and cauſing a Gun to be fired at the End of each Mile, he found that his former Obſervations were very juſt and true, and that Sound paſs'd the firſt Mile in 9¼ Half Seconds, two Miles in 18½, three Miles in 27¾, and ſo on to the End of the ſix.

12. The *Academia del Cimento* made Experiments of this Sort, from whence they concluded, that the Velocity of Sounds was ſo far equable, as not to be accelerated or retarded by conſpiring or adverſe Winds; but in this they led themſelves and many others into a great Miſtake, which was owing to their firing Guns at too near a Diſtance; for in great Diſtances the Difference is ſenſible, as will appear by the following Table of many Experiments which the Doctor made on the Guns fired at *Blackheath*, at the Diſtance of twelve Miles from his Houſe at *Upminſter*.

direct Pulſes, will be made by thoſe which are reflected; which is what we call an ECHO.

	D.	H.	Vibrat.	Wind.
1704.	*Feb.* 13.	6 to 12 N.	120 122	N. E. by E. 1.
	21.	11½ M.	119	E. 2
1705.	*Mar.* 30.	10 M.	113	S. W. 7
	Apr. 2.	8½ pM.	114½	S. by W. 1
	3.	10 M.	116½	S. 4
	5.	1 pM.	111	S. W. by W. 7
	13.	8½ M.	120	N. by E. 2
	24.	5 pM.	116	S. W. by W. 0
	Sept. 11.	6½ pM.	115	W. 1
		7	115½	W. by N. 2
	29.	10½ M.	112	S. S. W. 6
	Oct. 6.	10 M.	117	E. S. E. 1, 2
	Nov. 30.	Noon	115	S. S. W. 4
	Feb. 15.	11 M.	116	S. by W. 1
1706.	*Nov.* 29.	11½ M.	116	S. W. 0
		Noon	118	S. W. by S. 1
	Feb. 7.	Noon	113	S. W. by W. 4

13. In the firſt Column of this Table M denotes the Morning, pM the Afternoon, and N Night: Alſo the Figures 1, 2, 3, 4, 5, 6, 7, affixed to the Points of the Wind in the third Column, denote the ſeveral Degrees of Strength with which the Wind blew at the Time when the Experiments were made. From this Table it is eaſy to obſerve, that in this large Diſtance (of near 13 Miles) the Velocity of Sound is ſenſibly affected with the Current of the Air or Wind; for ſince *Blackheath* lay near S. W by W. from *Upminſter*, we ſee that on *April* 5, 1705, when there was a ſtrong Wind conſpiring with Sound, it came in 111 Half-Seconds; whereas in *Feb.* 13, 1704, when the Wind was directly contrary, though but a gentle one, the Sound took up no leſs than 120 and 122 Half-Seconds in paſſing the ſame Diſtance. The ſame is confirmed alſo by the Experiment on *April* 13, 1705.

14. And it is farther obſervable, that the Acceleration of Sound depends on the Strength of the Wind; for on

The LOCUS *or audible Place of Sound*, will be there where the Particles of Air firſt be-

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April 24, 1705, a S. W. by W. Wind in the loweſt Degree permitted the Sound to arrive in 116 Half-Seconds; the ſame Wind on *Feb.* 4, 1706, blowing with 4 Degrees of Strength, brought the Sound in 113 Half-Seconds; and on *April* 5, 1705, the ſame Wind with 7 Degrees of Strength brought the Sound in 111 Half Seconds. The Winds which blow tranſverſely (as on *April* 3, *Feb.* 15, 1705.) ſeem not to affect the Velocity of Sound, it paſſing then in 116 Half-Seconds, which is mean Velocity, as appears by the former Table in *Article* 10.

15. The greateſt Difference we here obſerve in the Velocity of Sound, with or againſt the Wind, is 10 or 11 Half-Seconds, or $5\frac{1}{2}$ Seconds; whence $1142 \times 5{,}5 = 6281$ Feet, which is ſomewhat more than a Mile = 5280 Feet; and therefore for every 10 Miles we may allow Half a Mile, or 2640 Feet, when the Wind blows ſtrongly againſt the Sound, and deduct the ſame when it ſets with it; and ſo in Proportion for any other Diſtance. See a late Diſcourſe on this Subject in a Treatiſe entituled, *Obſervations Aſtronomiques et Phyſiques, par Don George Juan et Don Antoine De Ulloa.*

16. The Velocity of Sound being determined, the Intervals of the Pulſes are known by finding how many Vibrations the ſounding Body performs in one Second. Thus D. *Sauveur* found by Experiments, that an open Pipe, whoſe Length was about 5 *Paris* Feet, had the ſame Tone with a String that vibrates forwards and backwards 100 times in a Second; conſequently, of the Pulſes made by ſounding ſuch a Pipe, there are about 100 in the Space of 1142 Feet, or 1070 of *Paris*; and therefore a ſingle Pulſe occupies the Space of $11\frac{42}{100}$ Feet *Engliſh*, or $10\frac{7}{10}$ Feet of *Pars*; ſo that the Length of the Pulſe was about twice the Length of the Pipe. Whence it is probable, that the Lengths of the Pulſes excited by the ſounding of open Pipes are in all Caſes equal to twice the Length of the Pipes.

gin to diffuſe themſelves in Form of Waves.' Thus a Perſon ſpeaking in one End of a Tube, or Trumpet, will be heard as ſpeaking from the other. And as in the Caſe of Light, we ſee the Image of an Object always in the Direction of the reflected Ray; ſo in *Echoes*, we hear a Perſon ſpeak at the Place from whence the reflected Wave comes to the Ear. (CVI.)

17. This was farther confirmed by the ſame Gentleman by another Experiment he made afterwards, in which he found that an open Pipe of about two *Paris* Feet in Length, was in Uniſon with a String which vibrated forwards and backwards 243 times in a Second; wherefore $\frac{1070}{243} = 4\frac{2}{5}$ nearly: that is, the Length of a Pulſe was about $4\frac{2}{5}$ Feet of *Paris*, or nearly twice the Length of the Pipe.

(CVI.) 1. In order to account for the Nature of ECHOES, we muſt conſider, that Sound is perceived as coming from that Place, from which, as a Centre, the Pulſes are propagated. This is well known by Experience: But to illuſtrate this Matter, let A be the Centre from whence any Sound is directly propagated, and ſtrikes againſt any plain Obſtacle C B, ſufficiently large; draw A F perpendicular to B C, and produce it to H, ſo that it may be A F = F H; the Sound reflected will be perceived as coming from the Point H. Plate XXXII. Fig. 9.

2. For let A B be the incident Ray, impinging againſt the Obſtacle B C in the Point E; from E draw the Ray E D, in ſuch a manner that the Angle C E D may be equal to the Angle E F A, or that the Angle of Incidence may be equal to the Angle of Reflection; then will E D be the reflected Ray of Sound, and, if produced, will paſs through the Point H; for the Angle F E H = C E D = F E A. Therefore in the Triangles A E

BECAUSE the Sound is ſtronger in Proportion as the Air is denſer, it muſt fol-

A F E and E F H, ſince the Angles of one are reſpectively equal to the Angles of the other, and the Side F E common to both, the Sides of one Triangle will be reſpectively equal to the Sides of the other, and therefore H F = A F; wherefore the reflex Sound will be heard by a Perſon at D, as coming from the Point H.

3. As the Place of the Auditor or Point D approaches towards A, the Caſe will conſtantly be the ſame with Reſpect to the Centre of Sound H; the Triangles will ſtill be equal, and all their Angles and Sides reſpectively; therefore when D coincides with A, the reflex Sound, or Echo, will be heard from the Point H; which was to be demonſtrated.

4. The ſame Sound therefore is heard twice by an Auditor at D; firſt by the direct Ray A D, and ſecondly by the reflex Ray A E D; provided the Difference between A D and A E D be ſufficiently great, that the direct and reflex Sound do not in the ſame ſenſible Moment of Time affect the Ear: For if the reflex Sound arrives at the Ear before the Impreſſion of the direct Sound ceaſes, the Sound will not be double, only render'd more intenſe.

5. We know by Experience, if more than 9 or 10 Syllables are pronounced in a Second, the Sounds will not be diſtinct and articulate; therefore, that the reflex Sound may not be confounded with the direct Sound, there ought to be at leaſt the 9th Part of a Second between the Times of their Appulſe to the Ear. But in the 9th Part of a Second Sound runs through the Space of $\frac{1142}{9} = 127$ Feet; the Difference therefore between A D and A E D muſt not be leſs than 127 Feet, for the Echo to be diſtinctly heard in D.

6. Hence alſo it follows, that a Perſon ſpeaking or uttering a Sentence in A aloud, in order to obſerve the Echo by Reflection from the Obſtacle B C, ought to

low, that the Voice paſſing through a Tube or Trumpet muſt be greatly augmented by

ſtand at leaſt 73 or 74 Feet from it, that is, A F = 74. And ſince, at the common Rate of Speaking, we pronounce not above 3½ Syllables *per* Second (or read more than 20 Lines of *Engliſh* Poetry *per* Minute) therefore that the Echo may return juſt as ſoon as the three Syllables are expreſs'd, we muſt have twice A F equal to about 1000 Feet; or the Speaker muſt ſtand about 500 Feet from the Obſtacle B C; and ſo in Proportion for any other Number of Syllables.

7. In all the Experiments which Dr. *Derham* made with the Guns at *Blackheath*, there was always a reduplication of the Sound, particularly the firſt in the foregoing Table, on *February* 13, 1704; where the direct Sound came firſt in 120 Half-Seconds, and the reflex Sound or Echo in 122 Half-Seconds. The Difference in Time, being a whole Second, ſhews the Echo paſs'd over 1142 Feet more than the direct Sound; and that therefore the *Phonocamptic Object*, or Obſtacle which reflected the Sound, was very probably near the Guns; ſince after the Pulſes had paſs'd a great way, they would have been too weak, when reflected, to have made an Echo as ſtrong or ſtronger than the direct Sound, as the Doctor always found it was.

8. By ſome Experiments which he made on Guns fired on the River *Thames*, between *Deptford* and *Cuckold's-Point*, he obſerved the Sound was not only doubled, but tripled, quadrupled, and ſometimes repeated many more times, and each ſucceeding Echo was louder and louder; and often when he heard thoſe Fragors of great Guns, he obſerved a Murmur aloft in the Air, eſpecially if the Heavens were quiet and ſerene: And thoſe Pulſes of Air he has obſerved to ſtrike againſt a thin Cloud, and produce in it a Murmur for the Space of 15″. From hence he judged, that thoſe *Murmurs* in the Air proceed from the vaporous Particles ſuſpended in the Atmoſphere which reſiſt the Undulations of

the conſtant Reflection and Agitation of the Air through the Length of the Tube,

Sound, and reverberate them to the Ear of the Obſerver, in the Manner of indefinite Echoes.

9. Among the many pleaſant and ludicrous Phænomena of *Echoes*, thoſe which are *Polyphonous*, or repeat divers Syllables or Sounds diſtinctly, and are therefore call'd *Tautological* or *Prattling Echoes*, afford the moſt curious Amuſement. Of theſe there are ſeveral remarkable in different Parts of the World, and particularly here in *England*; concerning which I refer the Reader to *Harris*'s or *Chambers's* DICTIONARY under the Word *Echo*, or to my PHILOSOPHICAL GRAMMAR.

10. Nor is this merry Phænomenon of Sound without its Uſe; for by means of an Echo you may meaſure inacceſſible Diſtances, the Width of large Rivers, *&c.* Thus Dr. *Derham* ſtanding upon the Bank of the *Thames*, oppoſite to *Woolwich*, obſerved that the Echo of a ſingle Sound was reflected back from the Houſes in 6 Half-Seconds, or 3 Seconds; conſequently, 1142 × 3 = 3426 Feet: the Half of which, *viz.* 1713 Feet, is the Breadth of the River there; which is more than a Quarter of a Mile, or 1320 Feet.

11. After the ſame manner we find the Meaſure of any Depth, as that of a Well for Inſtance. To do this, let $a =$ Space an heavy Body falls freely in one Second of Time, $b =$ Space through which Sound moves in the ſame Time, and $c =$ Time given in Seconds from the firſt Deſcent of the Stone to the hearing of the Sound, and $x =$ Depth of the Well required.

12. Then to find how long the Stone is in deſcending to the Bottom of the Well, ſay, As $a : x :: 1''^2 : t''^2 = \frac{x}{a} =$ Square of the Time t in which the Deſcent is made, becauſe the Spaces deſcribed by falling Bodies are as the Squares of the Times, (*Annot.* XXVI.) wherefore $t = \sqrt{\frac{x}{a}}$.

by which it is condenſed, and its Action on the external Air greatly increaſed at its

13. Again, to find the Time t in which the Sound aſcends, ſay, As $b : x :: 1'' : t'' = \frac{x}{b} =$ the Time ſought in Seconds; but $t + \mathrm{t} = c = \sqrt{\frac{x}{a}} + \frac{x}{b}$; therefore $x + b\sqrt{\frac{x}{a}} = bc$. But $b\sqrt{\frac{x}{a}} = \frac{b}{\sqrt{a}}\sqrt{x}$; and ſince x is the Square of $\sqrt{x}$, the foregoing is a Quadratic Equation; and, by compleating the Square, we have $x + \frac{b}{\sqrt{a}}\sqrt{x} + \frac{bb}{4a} = \frac{bb}{4a} + bc = \frac{bb + 4abc}{4a} = \frac{ss}{4a}$ (by putting $ss = bb + 4abc$). Whence extracting the Roots on each Side we have $\sqrt{x} + \frac{b}{2\sqrt{a}} = \pm\frac{s}{2\sqrt{a}}$, that is, $\sqrt{x} = \frac{-b \pm s}{2\sqrt{a}}$. But x cannot be a negative Quantity; and therefore it cannot be $\sqrt{x} = \frac{-b - s}{2\sqrt{a}}$, but muſt be $\sqrt{x} = \frac{-b + s}{2\sqrt{a}}$. Therefore $x = \frac{\overline{s - b}^2}{4a} =$ the Depth of the Well required.

14. Now $a = 16{,}122$ Feet, and $b = 1142$; whence $4a = 64{,}488$, and $bb = 1304164$; alſo $4ab = 73646$; and if we ſuppoſe $c = 10''$, then $4abc = 736460$, and $bb + 4abc = ss = 2040624$. Whence $s = 1428{,}5$; and $s - b = 286{,}5$; and $\overline{s - b}^2 = 82082{,}25$. Conſequently $\frac{\overline{s - b}^2}{4a} = 1273 = x$; or the Depth of the Well is 1273 Feet.

15. Since $x = \frac{\overline{s - b}^2}{4a}$, we ſhall have $\sqrt{x} = \frac{s - b}{2\sqrt{a}}$;

Exit from the Tube; which from hence is call'd the *Stentorophonic Tube*, or *Speaking-Trumpet.*

FOR the ſame Reaſon, thoſe Funnel-like Inſtruments, which gather the larger and more languid Waves of Air, do greatly condenſe them, and heighten their Power and Action on the Drum of the Ear; by which means Voices and Sounds are render'd *ſtrong*, *loud*, and *audible*, which were not ſo before to a deafen'd Ear; and hence theſe Inſtruments come to be call'd *Otacouſtics.*

I SHALL only obſerve, in regard of thoſe Inſtruments which magnify Sounds, and aſſiſt the Hearing, that *the longer they are, the greater is their Effect*; and that of all the Forms or Shapes, none is ſo good as *that derived from the Revolution of the* Lo-

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and therefore $\frac{\sqrt{x}}{\sqrt{a}} = \frac{s - b}{2\sqrt{a}}$ divided by $\sqrt{a}$, that is, $\frac{\sqrt{x}}{\sqrt{a}} = \frac{s - b}{2a} = \frac{286{,}5}{32{,}24} = 8{,}89$ Seconds, the Time of the Stone's Deſcent to the Bottom of the Well. (See *Art.* 12.)

16. The Time of the Sound's Aſcent is $\frac{x}{b} = \frac{\overline{s - b}^2}{4ab} = \frac{82082{,}25}{73646} = 1{,}11$ of a Second. But $8{,}''89 + 1{,}''11 = 10''$, the whole Time, as it ought to be.

GARITHMIC CURVE *about its Axis.* (CVII.)

(CVII.) 1. The *Stentorophonic Tube*, or *Speaking-Trumpet*, is uſed for magnifying of Sound, particularly that of Speech, and thus cauſing it to be heard at a great Diſtance, how it does this will be eaſy to underſtand from the Structure thereof. Let A B C be the Tube, B D the Axis, and B the Mouth-Piece for conveying the Voice to the Tube. Plate XXXIII. Fig. 5.

2. Then 'tis evident when a Perſon ſpeaks at B in the Trumpet the whole Force of his Voice is ſpent upon the Air contained in the Tube, which will be agitated thro' the whole Length of the Tube; and by various Reflections from the Side of the Tube to the Axis, the Air along the middle Part of the Tube will be greatly condenſed, and its *Momentum* proportionably increaſed, ſo that when it comes to agitate the Air at the Orifice of the Tube A C, its Force will be as much greater than what it would have been without the Tube, as the Surface of a Sphere, whoſe Radius is equal to the Length of the Tube, is greater than the Surface of the Segment of ſuch a Sphere whoſe Baſe is the Orifice of the Tube.

3. For a Perſon ſpeaking at B, without the Tube, will have the Force of his Voice ſpent in exciting concentric Superficies of Air all around the Point B; and when thoſe Superficies or Pulſes of Air are diffuſed as far as D every way, 'tis plain the Force of the Voice will there be diffuſed through the whole Superficies of a Sphere whoſe Radius is B D; but in the Trumpet it will be ſo confined, that at its Exit it will be diffuſed through ſo much of that ſpherical Surface of Air as correſponds to the Orifice of the Tube. But ſince the Force is given, its Intenſity will be always inverſely as the Number of Particles it has to move; and therefore in the Tube it will be to that without, as the Superficies of ſuch a Sphere to the Area of the large End of the Tube nearly.

4. To make this Matter yet plainer by Calculation; Let B D = 5 Feet, then will the Diameter of the

FROM the fourth Property of the *aerial Pulſes* we have the Origin of the various

Sphere D E = 10 Feet, the Square of which is 100, which multiplied by 0,7854, gives 78,54 ſquare Feet for the Area of a great Circle A H E F C. And therefore 4 times that Area, *viz.* 4×78,54=314,16=ſquare Feet in the Superficies of the Aerial Sphere. If now the Diameter A C of the End of a Trumpet be one Foot, its Area will be 0,7854; but, 7854 : 314,16 :: 1 : 400, therefore the Air at the Diſtance of B D will be agitated by means of the Trumpet, with a Force 400 times greater than by the bare Voice alone.

5. Again, 'tis farther evident how Inſtruments of this Form aſſiſt the Hearing greatly; for the weak and languid Pulſes of Air being received by the large End of the Tube, and greatly multiplied and condenſed by the tremulous Motion of the Parts of the Tube and Air agitated by them, are conveyed to the Ear by the ſmall End, and ſtrike it with an *Impetus* as much greater than they would have done without it, as the Area of the ſmall End at B is leſs than the Area of the large End A C.

6. From what has been ſaid, 'tis evident the Effect of the Tube in magnifying Sound, either for ſpeaking or hearing, depends principally upon the Length of the Tube. But yet ſome Advantage may be derived from the particular Form or Shape thereof. Some very eminent Philoſophers have propoſed the Figure which is made by the Revolution of a *Parabola* about its Axis as the beſt of any; where the Mouth-Piece is placed in the Focus of the Parabola, and conſequently the ſonorous Rays will be reflected parallel to the Axis of the Tube. See the Figure of ſuch a Tube in *Muſchenbroek*'s *Eſſai de Phyſique.*

7. But this parallel Reflection ſeems no way eſſential to the magnifying of Sound; on the contrary, it appears rather to hinder ſuch an Effect, by preventing the infinite Number of Reflections and Reciprocations of Sound, in which, according to Sir *Iſaac Newton*, its

Degrees of what we call the NOTE, TONE, or TUNE of Sounds, in regard of which Augmentation does principally conſiſt. For all reciprocal Motion, in every Return, is augmented by its generating Cauſe, which is here the tremulous Motion of the Parts of the Tube. Therefore in every Repercuſſion from the Sides of the Tube, the Agitations and Pulſes of the confined Air muſt neceſſarily be increaſed; and conſequently this Augmentation of the *Impetus* of the Pulſes muſt be proportional to the Number of ſuch Repercuſſions, and therefore to the Length of the Tube, and to ſuch a Figure as is moſt productive of them. Whence it appears, that *the Parabolic Trumpet is of all others the moſt unfit for this Purpoſe*, inſtead of being the beſt.

8. But there is one Thing more which contributes to the Augmentation of thoſe Agitations of Air in the Tube, and that is the Proportion which the ſeveral Portions of Air bear to each other when divided by tranſverſe Sections, at very ſmall but equal Diſtances, from one End of the Tube to the other. Thus let thoſe ſeveral Diviſions be made at the Points *a*, *b*, *c*, *d*, *e*, &c. in which let the Right Lines *a k*, *b l*, *c m*, *d n*, &c. be taken in Geometrical Proportion. Then will the Portions of Air contained between B and *a*, *a* and *b*, *b* and *c*, *c* and *d*, &c. be very nearly in the ſame Proportion, as being in the ſame Ratio with their Baſes when the Points of Diviſion are indefinitely near together. Plate XXXIII. Fig. 6.

9. But it has been ſhewn already, that when any Quantity of Motion is communicated to a Series of elaſtic Bodies, it will receive the greateſt Augmentation when thoſe Bodies are in Geometrical Proportion. Therefore ſince the Force of the Voice is impreſs'd upon and gradually propagated through a Series of elaſtic Portions of Air in a Geometrical Ratio to each other, it ſhall receive the greateſt Augmentation poſſible.

they are diſtinguiſh'd into *low* and *high*, or *grave* and *acute*, by Muſicians call'd *Flats* and *Sharps*. Now the Tone of a Sound depends on the Time or Duration of the Stroke made on the Drum of the Ear, by a Wave or Pulſe of Air; for as that is longer or ſhorter, the Tone will be more *grave* or *acute*: And ſince all the Pulſes move equally ſwift, the Duration of a Stroke will be proportional to the Interval between two

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10. Now ſince by Conſtruction it is $Ba = ab = bc = cd$, &c. and alſo $ak : bl :: bl : cm :: cm : dn$, and ſo on; therefore the Points $k, l, m, n, o, p, q, r, s$, A, will in this Caſe form that *Curve Line* which is call'd the *Logarithmetic Curve:* Conſequently, a Trumpet formed by the Revolution of this Curve about its Axis will augment the Sound in a greater Degree than any other figured Tube whatſoever.

11. But to ſhew the Reaſon of the *Nature* and *Name* of this Curve, ſuppoſe the following Series of Quantities in Geometrical Progreſſion, viz. $a^0 : a^1 : a^2 : a^3 : a^4 : a^5$, &c. then it is plain the Ratio of a^1 to a^0 is 1, the Ratio of a^2 to a^0 is 2, the Ratio of a^3 to a^0 is 3, and ſo on; whence it appears, that the *Indices* of the ſeveral Terms expreſs the Ratios of thoſe Terms ſeverally to the firſt, and are therefore their *Logarithms*. Now if in the above-mentioned Figure we put the Ordinates $ak = a^0 = 1$, $bl = a^1 = a$, $cm = a^2 = aa$, &c. then will the intercepted Parts of the *Abſciſſæ* be $Ba = 1$, $Bb = 2$, $Bc = 3$, &c. and therefore the *Logarithms* or *Exponents* of the Ratios of theſe ſeveral Ordinates to the firſt or Unity. Hence the Curve which connects thoſe Ordinates is called the *Logarithmetical* or *Logiſtical Curve.*

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Fig. 2 .p. 251.

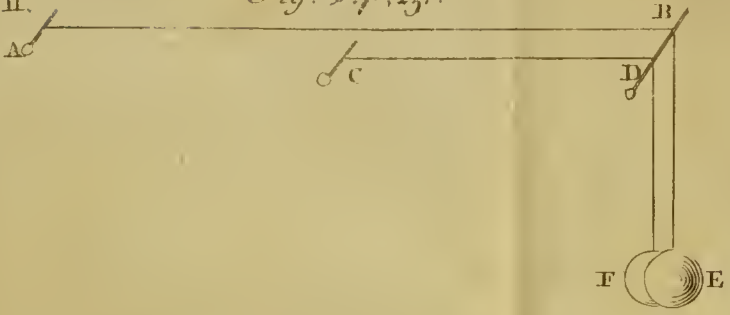


Fig. 2 .p. 251.

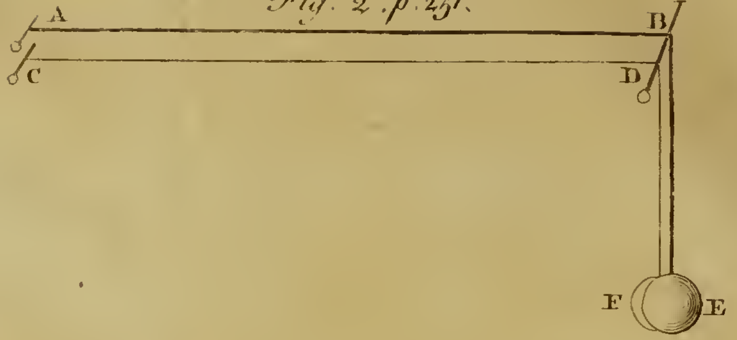


Fig. 3 .p. 252.

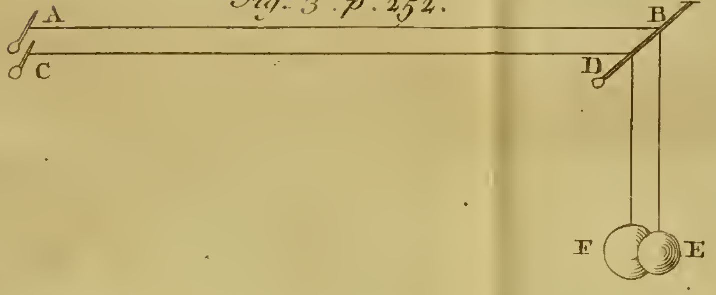


Fig. 5.

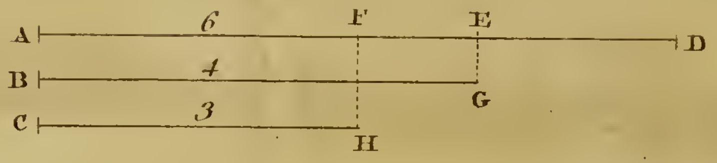


Fig. 4 .p. 292

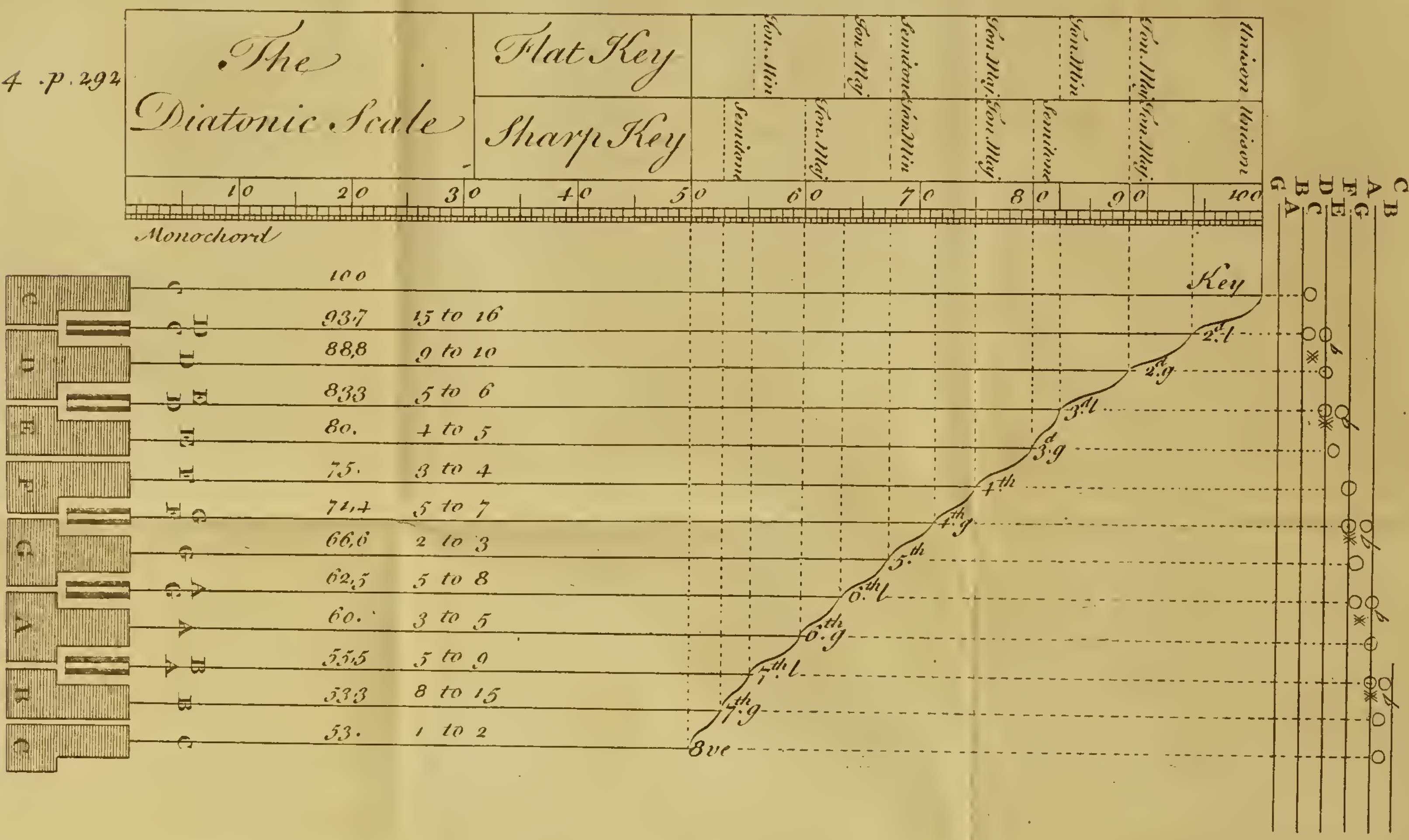
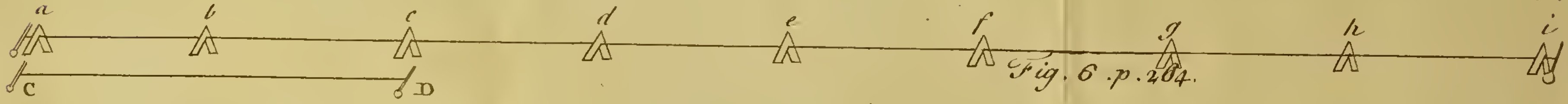


Fig. 6 .p. 284.



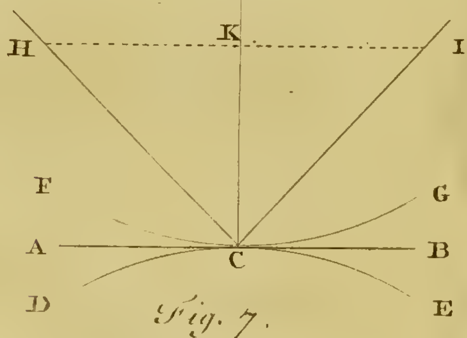


Fig. 7.

Fig. 8 .p. 297.

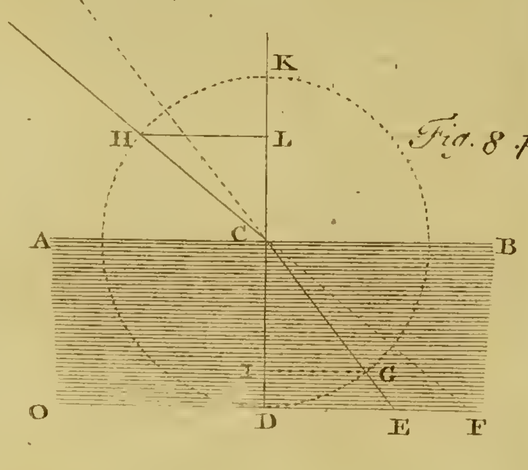
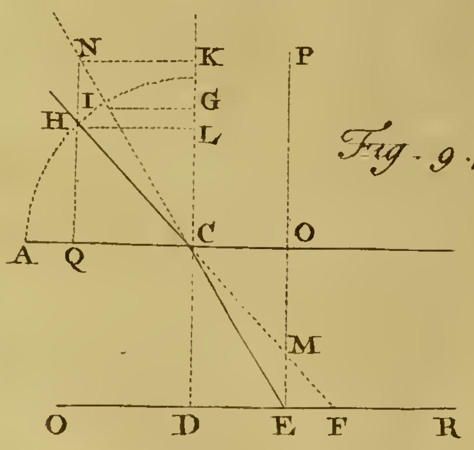


Fig. 9 .p. 310.



E. Bowen sc.

ſucceſſive Pulſes; and conſequently, *a Sound is more or leſs Grave or Acute in Proportion to the Length of that Interval.*

HENCE it follows, that all the Sounds from the *loudeſt to the loweſt*, which are excited by the Vibrations of the ſame Body, are of *one Tone*. It likewiſe follows, that all thoſe Bodies whoſe Parts perform their Vibrations in the ſame or equal Times, have the ſame Tone: Alſo, thoſe Bodies which vibrate *ſloweſt* have the *graveſt* or *deepeſt Tone*; as thoſe which vibrate *quickeſt* have the *ſharpeſt* or *ſhrilleſt Tone.*

THE Times of the Vibrations of *Muſical Strings*, and conſequently the Tones, vary in reſpect of the *Length*, the *Magnitude*, and the *Tenſion* of thoſe Strings. For if two Strings A B, C D, are of the ſame Magnitude, and ſtretch'd by equal Weights E, F, have their Lengths as 2 to 1, the Times of their Vibrations will be in the ſame Ratio. Hence the Number of Vibrations of the two Strings A B, C D, perform'd in the ſame Time, *will be inverſely as their Lengths*; or C D will make two Vibrations, while A B performs one. The Vibrations of two ſuch Strings will therefore co-incide at every ſecond of the leſſer. Plate XXXIV. Fig. 1.

AGAIN: If two Strings of the ſame kind A B, C D, have their Diameters as 2 to 1, Fig. 2.

and are of equal Length, and tended by equal Weights E, F; *the Times of the Vibrations will be as their Diameters*, *viz.* as 2 to 1; and ſo the Vibrations in a given Time, and the Co-incidences, as before.

LASTLY: If the Diameters and Lengths of the Strings be equal, *the Times of the Vibrations will be inverſely as the Square Roots of the Weights which ſtretch them.* If the Weights E and F be as 1 to 4 (the Square Roots of which are 1 and 2) then the Times of Vibration in A B and C D will be as 2 to 1. Hence in *conſtructing ſtringed Inſtruments*, as SPINETS, HARPSICHORDS, &c. a ſkilful Artiſt will compound theſe Proportions of the *Length*, *Diameter*, and *Tenſion* of the Strings to very great Advantage.

Plate XXXIV. Fig. 3.

IN *Wind-Inſtruments*, as the FLUTE, ORGAN, &c. where the Sound is made by the *Vibration of a Column of elaſtic Air* contain'd in the Tube, the Time of Vibration or Tone of the Inſtrument will alſo vary with the *Length and Diameter of the ſaid Column of Air*, *and Force of the Voice*, which compreſſes it; as will be eaſy to obſerve from Experiments.

IF one Body be made to ſound with another, their Vibrations will co-incide after a certain Interval; and the ſhorter the Interval of the Co-incidence, the more agreeable is the Effect or Conſonance to the Ear; conſequently, thoſe which are moſt frequent produce the moſt perfect Conſonance or *Concord*, as it is commonly called. When the Times of Vibration, therefore, are equal, the CONCORD is moſt perfect and more agreeable than any other, and this is called UNISON.

IF the Times of Vibration are as 1 to 2, the Co-incidence will be at every ſecond Vibration of the quickeſt, and ſo this is the next perfect Concord, and is what we commonly call a DIAPASON, or OCTAVE.

IF the Times of the Vibration be as 2 to 3, the Co-incidence will be at every third Vibration of the quickeſt; which therefore is in the next Degree of Perfection, and this is called a DIAPENTE, or FIFTH. If the Times of Vibration are as 3 to 4, the Co-incidence will be at every 4th of the leſſer; and this is called the DIATESSARON, or FOURTH. But this, and the next which follow in Order, are not ſo agreeable and pleaſant to the judicious Ear, and are therefore called *Imperfect Concords*. Nor are there above *ſeven Notes* in all the infinite Variety of Tones, which can merit a Place in Muſical Compoſitions, and they are exhibited in *Fig.* IV. Plate XXXIV. which repreſents the Strings in an Octave of a Harpſichord, with the *Semitones,* or *Half-Notes,* called *Flats* and *Sharps,* by which the natural Notes are made half a Note lower or higher, as the Air of the Song or Muſic requires. And this is called the DIATONIC SCALE of Muſic.

IN this Scale, the ſeven natural Notes are marked on the Keys by the ſeven Letters C, D, E, F, G, A, B. The firſt of which is call'd the *Fundamental* or KEY; the reſt in Order are the *Second Greater,* the *Third Greater,* the *Fourth Greater,* the *Fifth,* the *Sixth Greater,* the *Seventh Greater,* and then the *Eighth,* which begins the next Octave. Between theſe are interpoſed the five *Semitones, viz.* the *Second Leſſer,* the *Third Leſſer,* the *Fourth Leſſer,* the *Sixth Leſſer,* the *Seventh Leſſer.* Theſe ſeveral Tones and Semitones have the Lengths of the Strings adjuſted from the Diviſion of the MONOCHORD, or Line divided into 100 or 1000 equal Parts, as is very eaſy to apprehend from the Figure.

THE Number of thoſe Diviſions are alſo ſhewn for each String, by the firſt Series of Numbers on the Strings; the next Series ſhew the Proportion of the Length of each String to that of the *Key*, or *Monochord*; and conſequently the Number of Vibrations of the Fundamental and each String reſpectively, performed in the ſame Time.

OF theſe *twelve Intervals* or Ratios of Muſical Sounds, the *Octaves* and *Fifths* are *perfect Concords*; the third Greater, third Leſſer, the Greater and Leſſer Sixth are *imperfect Concords*; the Greater Fourth, the two *Seconds*, and two *Sevenths* are *Diſcords*; the *Fourth* is in its own Nature a perfect *Concord*, but lying between the Third and Fifth, it cannot be uſed as ſuch, but when joined with the Sixth, to which it ſtands in the Relation of a Third. All MELODY and HARMONY are compos'd of theſe twelve Notes; for the Octaves above or below are but the Replications of the ſame Sounds in a higher or lower Tone. MELODY is the agreeable Succeſſion of ſeveral Muſical Sounds in any ſingle Piece of Muſic; as HARMONY is the Effect of ſeveral of thoſe Pieces or Parts of Muſic play'd together. (CVIII.)

(CVIII.) 1. In order to account for the Motion and Tone of an elaſtic String, or *Muſical Chord* A B, it will be proper to conſider it as tended or ſtretched by a

Plate XXXIII. Fig. 7.

HARMONICAL PROPORTION is that which is between thoſe Numbers which aſſign the

Weight, as F, according to its Length, and drawn out of its right-lined Poſition A B, into an oblique Poſition A D B, by another Weight, as E. The former may be call'd the *Tending Force*, and the latter the *Inflecting Force.*

2. Now ſince the Tending Force F acts upon the String in the Direction D B, it may be repreſented by the Line C D, which Line or Force may be reſolved into two others, *viz.* C B and C D; of which the former draws the String horizontally from D to B, and the other acts in drawing the String directly upwards from D to C. Therefore the Part of the Force which acts in drawing the String perpendicularly upwards is to the whole Force as C D to D B; or, by ſuppoſing D C to be indefinitely ſmall, as C D to C B; becauſe in that Caſe D B = C B nearly. But the Force which acts in drawing the String upwards is equal to the Inflecting Force, becauſe they balance each other. Therefore the Inflecting Force E is to the Tending Force F as C D to C B, or $\frac{F \times CD}{BC} = D$.

3. Therefore, putting $CD = S$, and $2\,CB = L =$ the Length of the String, we ſhall have $\frac{F \times S}{L} : E$; hence it follows, that if F and L are given, that is, if the Tending Force and Length of the String remain the ſame, the Inflecting Force E will be always as the Line $CD = S$. This is confirmed by Experiment: For if A B be a Braſs Wire 3 Feet Long, ſtretched over the Pulley at B by a Weight $F = 3$ Pounds; if then E be firſt $\frac{1}{2}$ an Ounce, it will draw the Wire thro' $CD = \frac{2}{5}$ of an Inch; if E be an Ounce, it will draw it through $CD = \frac{4}{5}$ of an Inch; and ſo on.

4. The String being drawn into the Poſition A D B has an Endeavour to return, which is called the *Reſtitutive Force*, and which re-acts againſt the Inflecting

Lengths of Muſical Intervals, or the Lengths of Strings ſounding Muſical Notes; and of

Force; it muſt therefore be equal to it, and conſequently proportional to the Line C D. Wherefore the Point D is carried towards C with a Force every where proportional to the Diſtance or Space paſſed over. But we have ſhewn, that the Spaces paſſed by Bodies in Motion are as the Times and Velocities conjointly, that is, $S : TV$; (See *Annotation* XXII.) alſo that the Force of moving Bodies is as the Quantity of Matter and Velocity conjointly, *viz.* $M = QV$; therefore $\frac{S}{T} = V = \frac{M}{Q}$, or $TM = SQ$. But in the preſent Caſe Q is a given Quantity, therefore TM is as S; and becauſe it has alſo been ſhewn that M is as S in the preſent Caſe of the String, therefore T, or the Time in which the Vibrations are made, whether through greater or ſmaller Spaces, is ever the ſame, or a given Quantity.

5. The Reſtituent Force of the String, as it acts through very ſmall Spaces, may be looked upon as uniform; and then the Motion generated in the String will be as the ſaid Force and Time of its acting, that is, $M : ET$. Now in all Caſes it is $M : QV$; but here it is $Q : D^2 L$, (ſuppoſing $D =$ Diameter and $L =$ Length of the String) therefore $M : ET : D^2 LV$, and conſequently $T : \frac{D^2 LV}{E}$; but before, we had $E : \frac{FS}{L}$, which ſubſtituted in the above Ratio gives $T : \frac{D^2 L^2 V}{FS}$. But (ſince $S : TV$) we have $\frac{V}{S} : \frac{1}{T}$, therefore $T : \frac{D^2 L^2}{FT}$; that is, $FT^2 : D^2 L^2$, therefore $F^{\frac{1}{2}} T : DL$; conſequently, $T : \frac{DL}{F^{\frac{1}{2}}}$. That is, *the Time of a Vibration is as the Diameter and Length of the String*

three Numbers it is, *when the Firſt is to the Third, as the Difference between the Firſt*

directly; and as the Square Root of the Tending Force inverſely.

6. Hence if D and F be given, T is as L; that is, if the Diameter of the String and its Tending Force continue the ſame, the *Time of a Vibration will vary with the Length of the String, or be always proportional to it.* Thus $\frac{1}{2}$ of the Monochord vibrates in $\frac{1}{2}$ of the Time that it does, which is called an *Octave*; $\frac{2}{3}$ of the Monchord vibrates in $\frac{2}{3}$ of the Time, and is called a *Fifth*; $\frac{3}{4}$ vibrates in $\frac{3}{4}$ of the Time, and is called a *Fourth*, and ſo on.

7. If F and L be given, T is as D; that is, if the Tending Force and Length of the String remain the ſame, *the Time of a Vibration will vary with, and be proportional to, the Diameter of the String.*

8. If D and L be given, then T is inverſely as $F^{\frac{1}{2}}$; that is, if the Diameter and Length of the String be given, then *the Time of a Vibration will be as the Square Roots of the Tending Force.*

9. Now as the Tone of a String depends entirely upon the Time of a Vibration, it is eaſy to underſtand, that whatever the ſounding Body be, or how many ſoever there be together, if when they emit a Sound the Vibrations in each are of the ſame Duration, they will all be of the ſame Note, Tone, or Tune, which is call'd *Uniſon.*

10. In a Drinking-Glaſs, if a Perſon paſſes his wetted Finger briſkly round the Brim of the Glaſs, preſſing it at the ſame time, he will by degrees raiſe Tremors or Vibrations in the Parts of the Glaſs, which will produce a Tone or Sound, which will be conſtant ſo long as the Action of the Finger is continued, and more and more intended or heightened: So that if the Action be continued long enough, the Agitation will be ſo great as to diſengage the Particles, or break their Continuity, and thus reduce the Glaſs to pieces, if not too ſtrong.

and Second is to the Difference between the Second and Third, as the Numbers 3, 4, 6. Thus if the Lengths of Strings be as theſe Numbers, they will ſound an *Octave*, 3 to 6; a *Fifth*, 2 to 3; and a *Fourth*, 3 to 4.

AGAIN: *Harmonical Proportion* between four Numbers is, when *the Firſt is to the*

11. The Sound excited in the Glaſs ſeems one entire Effect, whereas it is in reality an Aggregate or Aſſemblage of an indefinite Number of Sounds, each effected by each ſingle Vibration of the Glaſs; but as the Times of the Vibrations are ſo quick and ſhort, their Intervals will be imperceptible, and conſequently the Diſtinction of the particular Sounds, which will therefore be loſt, and the Whole will appear but one entire Sound. After the ſame Manner a red-hot Coal whirl'd about makes the Appearance of a fiery Circle, becauſe the Coal ſucceeds to every particular Point of the Circle ſo quick, that a new Impreſſion is made upon the *Retina* before the Effect of the laſt is obliterated, and ſo the Coal appears in every Part of the Circle.

12. The Tremors of the Glaſs are made extremely ſenſible by putting a little Water into the Glaſs; for the Agitations of the Glaſs will by Degrees give Motion to the Water, which Motion will continually be increaſed till it be thrown up from the Surface in Form of a Miſt all over the Glaſs, and to a conſiderable Height above it every way. It is remarkable that the Motion of the Water is in Form of a *Vortex*, circulating round by the Sides of the Glaſs, and raging with impetuous Waves like the Sea after a prodigious Tempeſt.

13. Or otherwiſe theſe Vibrations of the Glaſs are made ſenſible by adjuſting a Screw very near the Rim of the Glaſs; then upon ſtriking the Glaſs, it will immediately be heard to ſtrike againſt the End of the Screw; which will ſhew not only the Vibration of the Glaſs, but alſo that in vibrating, the Form is alter'd from circular to elliptical.

Fourth as the Difference between the Firſt and Second is to the Difference between the Third and Fourth, as in the Numbers 5, 6, 8, 10: For Strings of ſuch Lengths will ſound an *Octave*, 5 to 10; a Sixth Greater, 6 to 10; a Third Greater, 8 to 10; a Third Leſſer, 5 to 6; a Sixth Leſſer, 5 to 8; a Fourth, 6 to 8.

IT may be here obſerved, *that a Series of Numbers in Harmonical Proportion are reciprocally as another Series in Arithmetical Progreſſion,*

$$\text{As}\begin{cases}\text{Harmonical } 10:12:15:20:30:60: \\ \text{Arithmetical } 6:\ 5:\ 4:\ 2:3:\ 1:\end{cases}$$

for here 10 : 12 :: 5 : 6; and 12 : 15 :: 4 : 5; and ſo of all the reſt. Whence thoſe Series have an obvious Relation to, and Dependence on, each other; which in ſome Problems of ſpeculative Philoſophy will be very uſeful to know (CIX.)

(CIX) 1. Let A, B, C, be three Numbers in Muſical Proportion; then becauſe we have A : C :: A — B : B — C, therefore A B — A C = A C — B C; whence if any two of the three be given, the other is immediately found by the following Canons, *viz.*

CANON I. If A and B be given, then $C = \frac{AB}{2A - B}$.

CANON II. If A and C be given, then $B = \frac{2AC}{A + C}$.

CANON III. If B and C be given, then $A = \frac{CB}{2C - B}$.

If the three Lines A D, B G, C H, be taken in Muſical Proprotion, or as the Plate XXXV. Fig. 5.

2. Thus, for Example, ſuppoſe you would find a Muſical mean Proportional between the *Monochord* 100 $= A$, and the *Octave* $50 = C$; then by Canon II. we have $B = \frac{2\,A\,C}{A+C} = \frac{1000}{150} = 66{,}6$, which is the Length of that Chord which is uſually called the *Fifth*.

3. Again, If there be four Numbers in Muſical Proportion, as A, B, C, D; then, ſince it is $A : D :: A - B : C - D$, we have $A\,C - A\,D = A\,D - D\,B$. From which Equation we have the following Canons.

Canon I. $A = \frac{D\,B}{2\,D - C}$.

Canon II. $B = \overline{2\,D - C} \times \frac{A}{D}$.

Canon III. $C = \frac{2\,A\,D - D\,B}{A}$.

Canon IV. $D = \frac{A\,C}{2\,A - B}$.

4. Hence, when any three of thoſe Numbers are given, the fourth may be found by the above Canons. Thus to the three Numbers 10, 8, 6, we find a fourth Harmonical Proportion, which is 5, the Octave; for thus the Theorem will ſtand, $\frac{A \times C}{2\,A - B} = \frac{10 \times 6}{20 - 8} = \frac{60}{12} = 5$.

5. But to carry this *Harmonical Theory* farther, and render it more general:

Let the Terms of an Harmonic Series be denoted by } A,B,C,D,E,F, &c.

And let the Difference between each two be denoted by } M,N,O,P,Q, &c.

6. Then will the Product of the two firſt Terms, *viz.* $A \times B$, be to the Product of any other two Terms immediately ſucceeding each other as $C \times D$, in the

Numbers 6, 4, 3; and in the Line A D we take A E equal to B G, A F equal to

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ſame Ratio with their reſpective Differences M and O. For by the Definition of Muſical Ratio we have $\left\{\begin{array}{l} A : C :: M : N \\ B : D :: N : O \end{array}\right.$

Therefore $A \times B : C \times D :: M \times N : N \times O :: M : O$.

$$\text{Alſo} \left\{\begin{array}{l} A : C :: M : N \\ B : D :: N : O \\ C : E :: O : P \end{array}\right.$$

Therefore $A \times B \times C : C \times D \times E$ ($:: A \times B : D \times E$) $:: M \times N \times O : N \times O \times P :: M : P$. That is, $AB : DE :: M : P$; and ſo on univerſally.

7. Again; the Difference between the two firſt Terms M is to the Difference between any other two, as O, in the Ratio of $B - 2M$ to D; or $M : P :: B - 3M : E$; or $M : Q :: B - 4M : F$; and ſo on continually. For, by the Nature of the Progreſſion, it is $A : C :: M : N$; and it is alſo $A = B - M$, (becauſe $B - A = M$) therefore it is $B - M : C :: M : N$; or, to put it in Form, we have $M : N :: B - 1M : C$. Again; $B - M : M :: C : N$, and by Diviſion $B - 2M : M :: C - N : N :: B : N$; but (by the Definition, *Art.* 1.) it is $B : N :: D : O$, therefore $M : O :: B - 2M : D$. Again; $B - 3M : M :: D - O : O :: C : O :: E : P$; therefore $M : P :: B - 3M : E$. And univerſally, let n = Number of Terms in the Series between the firſt and the laſt, and let the laſt Term be Z, and let the Difference between it and the next preceding Term be S; then will it be $M : S :: B - nM : Z$.

8. Becauſe (by *Art.* 6.) it is $M : S :: A \times B : Y \times Z$, ſuppoſing Y, Z, the two laſt Terms of the Series; therefore $A \times B : Y \times Z :: B - nM : Z$.

9. Becauſe the firſt Term of the Series is $A = \frac{A \times B}{B}$, and the ſecond Term $B = \frac{A \times B}{A}$, and $A = B - M$; therefore the ſecond Term is $B = \frac{A \times B}{B - M}$.

C H, *then will the Line* A D *be divided in Harmonical Proportion,* in the Points A, F, E, D; *viz.* $AD : AF :: DE : EF$. And in this manner is the Axis of a convex and concave Mirrour divided by the *Object*, the *Image*,

In the ſame Manner it is ſhewn, that the third Term is $C = \frac{A \times B}{B - 2M}$, the fourth Term $\frac{A \times B}{B - 3M}$; and univerſally, ſince $A \times B : Y \times Z :: B - nM : Z$, or, dividing the conſequents by Z, $A \times B : Y :: B - nM : 1$; therefore $Y = \frac{A \times B}{B - nM}$; and ſince $n =$ Number of Terms between A and Z, it will expreſs the Number or Place which the Term Y holds in the Series. Therefore *any Term* Y *is equal to the Product of the firſt and ſecond Term* B *of the Series divided by the Difference between that ſecond Term* B, *diminiſhed by ſo many times its Difference from the firſt, as is equal to the Number of the Terms from the firſt to the given Term* Y.

10. *All the Terms in a Muſical Progreſſion are among themſelves as Quantities whoſe Reciprocals conſtitute a Series in Arithmetical Progreſſion.* Thus the Terms of the firſt Series A, B, C, D, E, *&c.* are (by *Art.* 9.) as $\frac{A \times B}{B}$, $\frac{A \times B}{B - M}$, $\frac{A \times B}{B - 2M}$, $\frac{A \times B}{B - 3M}$, *&c.* to $\frac{A \times B}{B - nM}$; which Series divided by $A \times B$ gives the Series $\frac{1}{B}$, $\frac{1}{B - M}$, $\frac{1}{B - 2M}$, $\frac{1}{B - 3M}$, to $\frac{1}{B - nM}$. But the Reciprocals of this Muſical Series are B, $B - M$, $B - 2M$, $B - 3M$, to $B - nM$; which Terms are all in *Arithmetical Progreſſion.* If the *Harmonic Series* had been decreaſing, *viz.* $A - B = M$, $B - C = N$, *&c.* we ſhould have had $A = M - B$, *viz.* the Signs of M and B changed, but every thing elſe the ſame.

the *Vertex of the Mirrour*, and the *Centre*, as may be eaſily ſhewn by Experiment.

ALSO the Limits of the Colours of Light, as ſeparated by the Priſm, *fall upon the ſeven Muſical Diviſions of the Monochord*; as will be farther taken notice of, and exemplified in the next Lecture.

I SHALL conclude this with taking Notice of one ſingular Property of a Muſical Chord, *viz.* that it will be put into a vibratory Motion by the Pulſes of the Air proceeding from the Vibrations of another very near it, and in Concord with it: If the vibrating String be *Uniſon* with it, the other will tremble through its *whole Length*; if an *Octave*, it will vibrate by the Half-Lengths only; if the String which communicates the Motion be a *Double-Octave above*, or *one Fourth* of the Length of the other, the Motion will be ſtill correſpondent in that other String, for it will vibrate only by the *Fourths of its Length* from one End to the other. Thus if A B be a String four Feet long, and C D another of one Foot; if the latter be ſtruck with a Quill, the Vibrations will be communicated to the former in ſuch a Manner that it will vibrate only by a *Foot-Length* at the ſame time through the whole String; which

Plate XXXIV. Fig. 6.

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will be evident by the ſmall Pieces of Paper *b, d, f, h,* hung upon the Middle of every Foot-Length, ſuddenly leaping off; while the other Pieces *a, c, e, g, i,* remain unmoved upon the String at the End of every Foot, where the Vibrations ſeverally begin and end, and conſequently where the Line has no Motion at all (CX.)

(CX.) 1. What is here ſaid relating to Mirrours, and the Colours of Light, will be explained and demonſtrated in its proper Place. That one String A ſhould be put into Vibration by another B, by means of the Air, is not ſtrange, becauſe the Air will affect the String A with the ſame Impulſes it receives itſelf from the String B. If therefore the String A be under the ſame Circumſtances with the String B otherwiſe, (*i. e.* if it be of equal Magnitude, and equally tended) it muſt neceſſarily move in a ſimilar Manner, or vibrate in an equal Time.

2. If the String A be twice the Length of B, then (*cæteris paribus*) the Air by its Impulſe received from B cannot ſo affect A as to cauſe it to vibrate through its whole Length; but it will ſo affect each Half of A as to produce a ſimilar Effect, or equal Vibrations. Hence the String A will become divided in the middle Point, which will be at Reſt.

3. And if the String A were three times as long as B, it would be for the ſame Reaſon divided into three Parts, whoſe Vibrations are ſynchronous to thoſe of B, with two Points of Reſt between; and ſo on for any other Length. Alſo, if the Lengths of A and B are as 3 to 2, then if they are Concords, and one be ſtruck, the other will be put into Motion by Degrees, and in ſuch a Manner that will alter the Vibrations of the firſt String B, and each will vibrate by their aliquot Parts, and therefore in equal Times.

LECTURE VIII.

Of the Nature and Properties of LIGHT; *the* VELOCITY *thereof how diſcovered and computed. Of the Nature of* HEAT, FIRE, FLAME *and* BURNING. *Of the* IGNES FATUI, NOCTILUCÆ, *natural and artificial* PHOSPHORI. *The* THEORY *of* HEAT *and* COLD. *Of* ASBESTOS. *Of the* Nature *and* Effect *of* BURNING-GLASSES, *whether* Mirrours *or* Lenſes. *A Calculation of the* LIGHT *and* HEAT *of the* MOON. *Of the Cauſe of* TRANSPARENCY *and* OPACITY *in Bodies. Of the* REFLECTION *of* Light; *of its* INFLECTION; *of the* REFRACTION of Light. *The* Fundamental Laws *thereof demonſtrated. The different* REFRACTIVE POWER *of various Subſtances. The Ratio of the Sines of* INCIDENCE *and* REFRACTION *ſtated. Of the* TRUE *and* APPARENT PLACES *of Objects. Of the* ANALYSIS *of the* Solar Rays; *the* ſeveral KINDS *thereof, their different* REFRANGIBILITY *ſtated*; EXPERIMENTS *relating thereto by the* PRISM. *Of the various* COLOURS *of* Light *by the*

Priſm; *the* HARMONIC RATIO *of their Linear Extent in the* Sun's Image. *The* COLOURS *of* Natural Bodies *thence explain'd. Of the different* REFLEXIBILITY *of the* Solar Rays, *and Experiments relating thereto. The Manner and Cauſe thereof enquired into. Of* RINGS *of* COLOUR'D LIGHT *between* GLASS PLANES, *and* BUBBLES *of* WATER. *The different* ORDERS *and* DEGREES *of the ſeveral* Colours *explain'd. The Fits of* EASY REFLECTION *and* TRANSMISSION *explain'd. The* Artificial COMPOSITION *of* COLOURS. *Of the* RAINBOW; *its* Cauſe *explain'd; Calculations relating thereto. The Phænomena of* HALO'S *conſidered and accounted for.*

THAT *Light* is not a *mere Quality* of ſome Bodies, but is itſelf a real Body, or diſtinct Species of Matter, and endued with all the natural Properties thereof, will, I preſume, be ſufficiently manifeſt from the following Experiments relating thereto. We ſhall therefore, at preſent, take it for granted, *that Light conſiſts of inconceivably ſmall Particles of Matter of different Magnitudes, which are emitted or reflected from every Point in the Surface of a luminous*

luminous Body in Right Lines, and in all Directions, with an unparallel'd Velocity, and whoſe Power or Intenſity decreaſes as the Squares of the Diſtances increaſe.

THAT the Particles of Light are refracted through the Humours of the Eye to the *Retina*, or fine Expanſion of the Optic Nerve over all the interior hinder Part of the Eye; and there, by painting the Images of external Objects, become the immediate Means of Sight, will be fully ſhewn in the next Lecture.

WE ſhall now conſider Light under the various Characters and Qualities of a natural Body; and point out thoſe remarkable Affections and Properties ſo peculiar to itſelf, and the Cauſes of ſo many very curious and extraordinary Phænomena in Nature.

THAT the Particles of Light are *inconceivably ſmall*, is evident from hence, that the greateſt Quantity of Light, in the ſtate of greateſt Denſity, or *Flame*, is found to have ſcarce any ſenſible Gravity or Weight, which, we have ſhewn, is always proportional to the Quantity of Matter in all Bodies: Alſo, becauſe thoſe Particles pervade the Pores of all tranſparent Bodies, however hard or heavy, as Glaſs and Adamant.

mant. But we know it more eſpecially from hence, that the Stroke we receive by a Particle of Light has no ſenſible Force or *Momentum*, which, on account of its prodigious Velocity, would be very great, and inſufferable, were it of any aſſignable or conſiderable Magnitude.

YET, ſmall as they are, we find the Rays conſiſt of different Sorts of Particles in Light emitted from all Bodies; and that this Difference of the Rays of Light ariſes from the different Magnitude of the Particles, ſeems moſt evident from the different Directions the ſeveral Sorts of Rays move in, after they have paſs'd through a Body of Glaſs, Water, &c. of ſome ſpecial Figure, as that of a *Priſm* eſpecially.

THAT the Particles of Light are emitted from every Point in the Surface of a Body, is evident from hence, that any given Point in that Surface is viſible to the Eye in any Situation, from whence a Right Line can be drawn from the Eye to that Point; which could not be, if the Light were not propagated from that Point in all Directions.

THAT they proceed from the Body in Right Lines, is clearly ſeen by Experiments on the Sun-Beams, Candle-Light, &c. in a darken'd a darken'd Room; alſo from the Shadows which Bodies of every Figure caſt, being ſuch as would be determined by Right Lines drawn from the luminous Point touching the Extremities of thoſe Bodies (CXI.)

(CXI.) 1. Before Sir *Iſaac Newton*'s Time, ſcarce any thing of the Nature or Properties of Light was known. It had been eſteem'd a mere Quality or Modification of Matter, and was propagated by Preſſion, and I know not what of ſuch Kind of Stuff and ſenſeleſs Jargon; than which nothing can be more tireſome to read, or irkſome to repeat. Leaving therefore the idle Reveries of the *Carteſians*, we ſhall contemplate this glorious Phænomenon in the *Newtonian* Manner, which diffuſes Luſtre over the whole Face of Nature, and adds new Splendor even to Light itſelf.

2. That Light is a material Subſtance, and what we properly call *Body*, is not to be doubted; becauſe we find it is ſomething that has Motion, or is propagated in Time; ſomething that acts upon Bodies, and produces great Alteration and Changes in their Natures and Forms. It is ſomething that Bodies act upon, by reflecting, inflecting, and refracting it on their Surfaces, and in their Pores: And it would appear to have Weight, and all other ſenſible Qualities of common Matter, were it not that the Smallneſs of its Quantity renders them entirely imperceptible by us.

3. Nor are we to conſider Light only as a Body, but as the moſt active Principle or moſt general Agent in Nature. I greatly queſtion if it be not the true *Primum Mobile* in Nature, or the Spring of Motion and Action in all other Bodies. Were the Particles of Light to be annihilated, we ſhould ſee no Marks or Footſteps of Fire or Heat remaining, and therefore no Power of Motion in Bodies, but all Things would put on the Appear-

The Velocity of the Rays of Light ſurpaſſes that of all other Bodies we know of.

ance of lifeleſs inert Matter, rigid and inflexible, as it would be abſolutely cold and dark.

4. The Divine Wiſdom and Providence appears perhaps in nothing ſo remarkably as in the extreme Subtilty of the Particles of Light; without this Qualification it could not have pervaded the Pores of Bodies, and ſo we could have had none of thoſe which we call *diaphanous* or *tranſparent* Subſtances, and every thing but the Surface of a Body would have been concealed from the Sight of Mankind. Again; the Velocity of a Body is always as the Quantity of Matter inverſely; and therefore the ſmaller the Body, the greater Velocity it is ſuſceptible of from the ſame Force; whence it comes to paſs, that Light is thus qualified to be tranſmitted thro' immenſe Diſtance in a ſmall and inſenſible Part of Time; which Thing was quite neceſſary according to the preſent Frame and State of Nature.

5. But laſtly, it was abſolutely neceſſary that the Particles of Light ſhould be ſo exceeding ſmall, that when compounded with its Velocity it ſhould produce no ſenſible Force, as it muſt otherwiſe have done, and which therefore could not have been born by the tender and delicate Texture of the ſeveral Parts of Vegetable and Animal Bodies. To give an Example: The Velocity of a Particle of Light is found to be at the Rate of 897600000 Feet *per* Second; ſuppoſe its Matter to be but one Millionth Part of a Grain, then its Force to ſtrike an Object would be as $\frac{897600000}{1000000} = 897,6$ Feet *per* Second for one Grain; or it would ſtrike with the ſame Force that one grain Weight would do falling from half that Height, *viz.* through 448,8 Feet; which we ſhould find to be very great, were the Experiment to be made on the ſenſible Coats of the Eye.

6. Since the Weight of Bodies is proportional to the Quantity of Matter, it follows, that where the latter is diminiſhed indefinitely, the former will be ſo too; there-

By obſerving the Times of the Eclipſes of *Jupiter*'s *Satellites* when the Earth is near-

fore the Weight of Light muſt be inſenſible in ever ſo great a Quantity of it. Dr. *Boerhaave* cauſed a Globe of Iron 12 Inches in Diameter to be heated red-hot, and ſuſpended at the End of a very exact Balance, and counterpoiſed by Weights at the other End very nicely, and thus let it hang till all the Particles of Heat or Light were eſcaped, when he found the Equilibre of the Balance no ways alter'd; which plainly proves the above Theſis.

7. That the Particles of Light have not only Magnitude, but that in *different Degrees* alſo, is another and perhaps the moſt ſubtle Diſcovery of the *Newtonian* Philoſophy. The comparative Terms of *Greater* and *Leſſer* are now as applicable to the Particles of Light, as to any other Bodies. This is abſolutely proved by the different Refrangibility they are found to have in paſſing through a Priſmatic Figure of Glaſs or Water; for the Power of the Priſm detains the iſſuing Particle, and draws it a little towards the Surface; and ſince this Power is the ſame, it would have the ſame Effect on all the Particles of Light, if they were all of an equal Magnitude, becauſe they have all an equal Velocity. But ſince this Effect is different among the Particles, ſome being detained and drawn aſide to a greater Diſtance than others, it follows, they muſt be leſs in Magnitude, to become more ſubject to the Influence of the attracting Surface; in like manner as the electric Effluvia will act upon and agitate very ſmall and light Bodies, much ſooner and more eaſily than they can move thoſe which are larger. But of this more when we come to ſpeak of the Manner in which this Power acts refracting the Rays of Light.

8. If Light were not reflected from every Point in the Surface of a Body in all Directions every Way, there might be aſſign'd a Point of Space where a Ray of Light from ſuch a Point in the Surface does not come; and there the ſaid Point of the Surface could not be vi-

eſt,

RPJCB

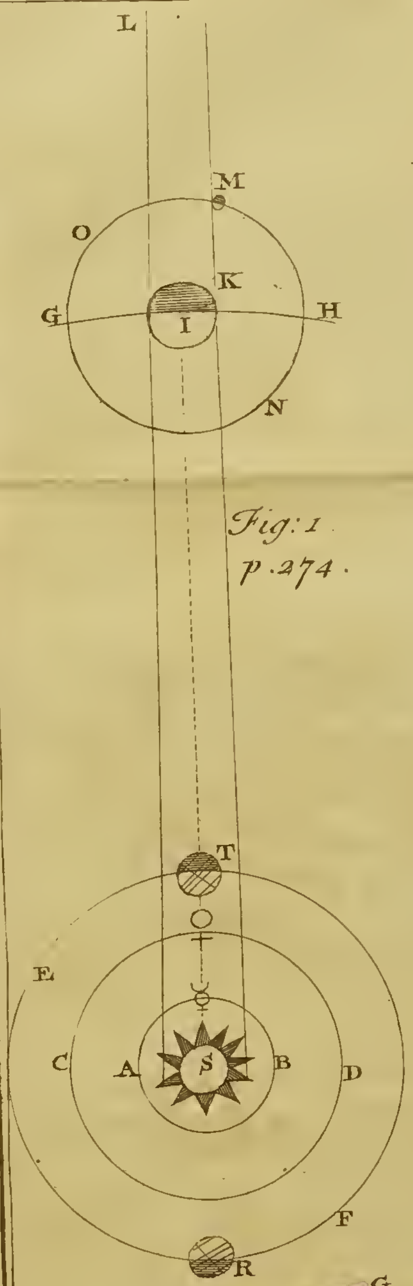

Fig:1 p.274.

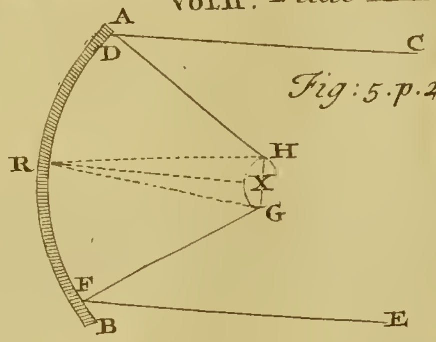

Fig:5.p.261.

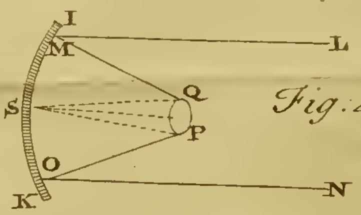

Fig:4.p.285.

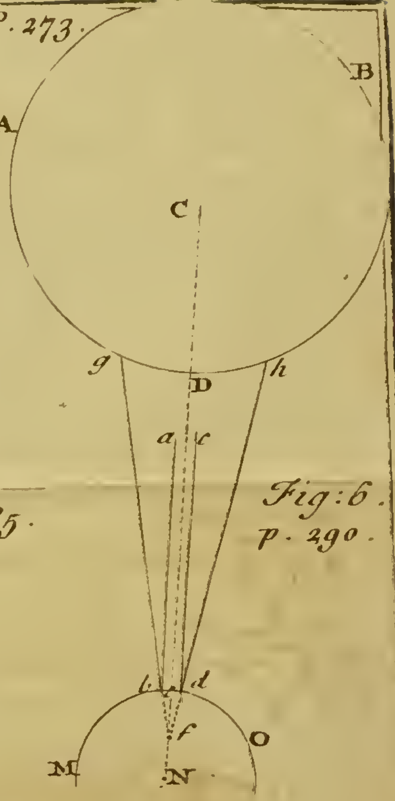

Fig:6. p.290.

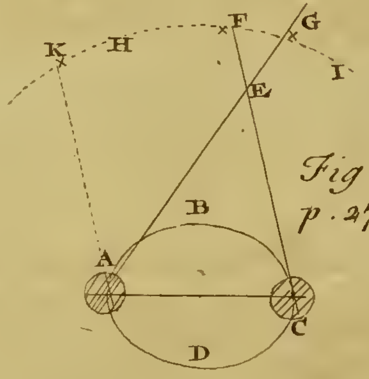

Fig:2. p.275.

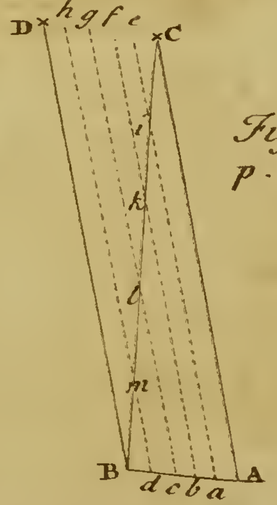

Fig:3. p.276.

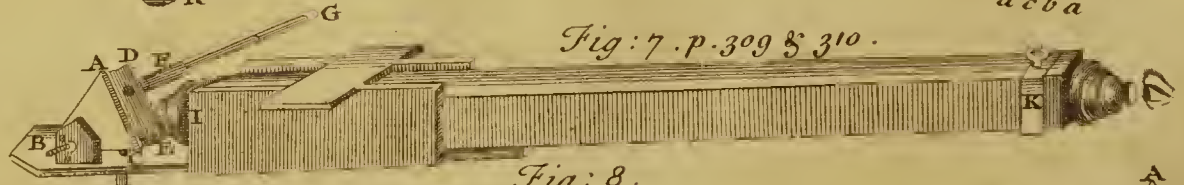

Fig:7.p.309 & 310.

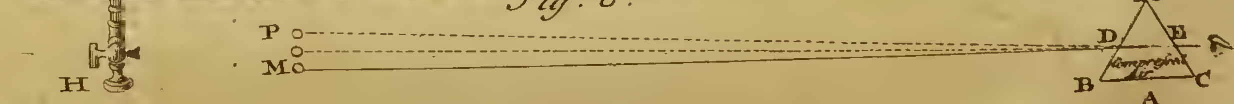

Fig:8.

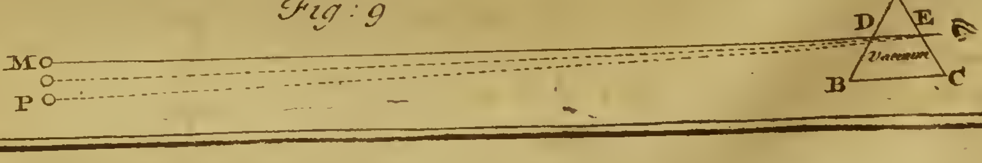

Fig:9

eſt, and again when it is fartheſt diſtant from that Planet, we ſhall find, that in the former Caſe thoſe Eclipſes happen *too ſoon*, and in the latter *too late*, by the Space of 8 Minutes and 13 Seconds; which ſhews, that in that Time the Light paſſes over the Semidiameter of the Earth's Orbit, which is about 82,000,000 Miles; which is at the rate of 170,000 Miles in a Second of Time, and which is therefore nearly 680,000 times greater than the Velocity of Sound. (CXII.)

ſible, but becauſe the Eye can find no Point of Space in all the viſible Hemiſphere reſpecting that Point, but where it is viſible, therefore a Ray of Light is reflected from that Point to every Part of Space, from whence a Right Line to that Point can be drawn.

9. That the Rays of Light proceed in Right-lined Directions, is evident from hence, that whatever the Figure of the Body be, if it be held perpendicular to the Rays of Light, it will always caſt a Shadow of the ſame Figure againſt a parallel Plane. Thus a Circle will produce a circular Shadow, a Triangle a triangular one, and ſo on. Which plainly ſhews, that the Rays of Light paſs by the Extremities of thoſe Bodies in Right-lined Directions, excepting thoſe only which paſs contiguous to the Edges of the Body, for they will be a little inflected, which will cauſe the Extremity of the Shadow to be not ſo diſtinct and well defined as it otherwiſe would be; of which we ſhall take farther Notice hereafter.

(CXII.) 1. As all the other Affections of Light, ſo that of Velocity, was utterly unknown to all the ancient, and moſt of the modern Philoſophers, who, before the Time of Mr. *Reaumur*, were of opinion that the Mo-

Again: Since Light is propagated in Right Lines, its Power or Intenſity will

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tion of Light was inſtantaneous, or that it was propagated through immenſe Spaces in an Inſtant. But Mr. *Reaumur* and other Philoſophers about this time, making frequent obſervations on the Eclipſes of *Jupiter's* Moons, found that the Time of thoſe Eclipſes did not correſpond to the Calculations founded upon the Aſtronomical Tables; where the Times are all calculated for the Diſtance of the Centre of the Sun, and conſequently, where the Eye of the Spectator muſt be ſuppoſed to be in viewing the ſaid Eclipſes, Occultations, *&c.* of *Jupiter's* Moons.

2. To illuſtrate this Matter; let S be the Centre of the *Sun*, A B the Orbit of *Mercury*, C D the Orbit of *Venus*, E F that of the *Earth*, and G H a Part of the Orbit of *Jupiter*. Let I be the Body of *Jupiter*, and K L its Shadow, O M N the Orbit of one of *Jupiter's* Moons M juſt entering the Shadow of *Jupiter*. Now a Spectator at S would obſerve the Moon M to enter the Shadow juſt at the Time which is calculated from the Tables; but a Spectator at the *Earth* at T always obſerves it to happen ſooner, and when the *Earth* is in the oppoſite Part of its Orbit R, he will always obſerve it to happen later, by the Space of about 7 Minutes in both Caſes. This Obſervation gave the firſt Proof that Light was progreſſive, and took up about 14 Minutes to paſs over the Diameter of the *Earth's* Orbit from T to R, or 7 Minutes to paſs from the *Sun* S to the *Earth* T.

Plate XXXV. Fig. 1.

3. But this, though a ſufficient Diſcovery or Proof of the progreſſive Motion of Light, was yet but an Experiment in the Groſs, and not accurate enough to determine or define the true Rate of Velocity which did really belong to Light. The Method by which it has been more nicely determined, was hit upon in the following Manner: Though Sir *Iſaac Newton* had demonſtrated the Motion of the Earth from the Laws of Gravity, yet as his Book was underſtood by few, thoſe

decreaſe as the Squares of the Diſtances increaſe; and therefore the Light and Heat

who could not comprehend his Method were willing to be ſatisfied of the Truth thereof otherwiſe, and rightly judged, that if the Earth did move about the Sun, it muſt neceſſarily cauſe an apparent Motion in any fixed Object at a Diſtance from it.

Plate XXXV. Fig. 2.

4. Thus if A B C D repreſent the Orbit of the Earth, and A and C the Place of the Earth at two oppoſite Times of the Year; then a fixed Object at E will be ſeen from the Earth at A in the Line A E, which will point out its apparent Place at G in the Concave Expanſe of the Sky HI. But at the oppoſite Time of the Year, it will be ſeen from the Earth at C in the Line C E, which will project its Place in the Heavens at F. So that while the Earth has paſs'd from A by D to C, the Object (though in reality fixed) has appeared to move through the Space G F; and the Angle which meaſures this apparent Motion of the Object, *viz.* the Angle A E C is called the *Parallactic Angle*, or *Parallax of the Annual Orbit*, becauſe it meaſures the viſible Appearance of the Diameter A C of the Earth's Orbit at the Object E.

5. This being the Caſe, it was applied to the fixed Stars, which they concluded would certainly have an apparent Motion, or Parallax, provided an Inſtrument could be made ſufficiently exact to obſerve it, and this would be a ſatisfactory Demonſtration of the Earth's Motion. Accordingly ſeveral Perſons addreſs'd themſelves to diſcover a Parallax of the fixed Stars; and in the Year 1725, the late Hon. *Samuel Molyneux*, Eſq; with an Inſtrument made by the accurate Mr. *Graham*, began to obſerve the bright Star in the Head of *Draco* as it paſs'd near the Zenith. Profeſſor *Bradley* alſo obſerved it along with him; and from many Obſervations made with great Care, it appeared that the Star was more Northerly 39 Seconds of a Degree in *September* than in *March*, juſt the contrary Way to what it ought to appear by the annual Parallax of the Stars. That is,

of the Sun at the Diſtances of the ſix Pla-
nets, *Mercury*, *Venus*, *Earth*, *Mars*, *Jupi-*

the Obſervers, who in *September* ſaw the Star at F, did in the *March* following obſerve it at K, in the Right Line A K parallel to C F, and not at G where it ought to have appeared by the parallactic Motion.

6. This unexpected Phænomenon perplexed the Obſervers very much, and Mr. *Molyneux* died before the true Cauſe of it was diſcovered. After this, Dr. *Bradley*, with another Inſtrument more exact and accurately adapted for this Purpoſe, obſerved the ſame Appearances, not only in that, but many other Stars; and being by many Trials fully aſſured that the Phænomenon was not owing to any Error in the Inſtrument or Obſervation, applied himſelf to conſider what might be the Cauſe thereof, and after ſeveral Reflections and Hypotheſes, which he ſtill found inſufficient to account for it, he at laſt found, that it was really owing to the progreſſive Motion of Light, and the ſenſible Proportion which the Velocity thereof bore to the Velocity of the annual Motion of the Earth.

Plate XXXV. Fig. 3.

7. This he was fully aſſured was the true Reaſon, not only becauſe nothing elſe could be thought of that would account for it, but becauſe ſuch an Appearance muſt neceſſarily reſult from the above-mentioned Hypotheſis, as may be thus ſhewn. Let A B repreſent a Part of the Earth's annual Orbit, and let C be a Star obſerved by a Spectator at the Earth at A; when the Earth arrives at B the Star will not be obſerved at C, as before, but at D in the Line B D parallel to A C; for let A B be divided into the equal Parts A *a*, *a b*, *b c*, *c d* and *d* B, then through thoſe Points draw the Lines *a e*, *b f*, *c g*, *d h*, parallel to A C and D B. Now let the Velocity of the Earth be to that of Light as A B to C B. When the Earth ſets out from the Point A, ſuppoſe the Ray of Light commences its Motion from the Star at C in the Direction C B perpendicular to A B; then 'tis plain when the Earth is arrived at *a*, the Particle of Light will be got to *i*, the Point where *a e* cuts B C,

ter and *Saturn*, will be nearly as 700, 200, 100, 43, 3, 1, ſuppoſing their Diſtances as the Numbers 4, 7, 10, 15, 52, 95.

and the Star will be ſeen in the Direction *a i*, and appear at *e*. In like manner, when the Earth is at *b*, the Particle of Light will be come to *k*, and will appear at *f*, and ſo on; when the Earth is at *c*, *d*, B, the Particle will be at *l*, *m*, and B, and the Star will appear at *g*, *h*, and D.

8. If therefore the Line C A repreſents the Axis of a Teleſcope, making the Angle B A C with the Direction of the Earth's Motion A B; when he comes to B he will ſee the Star at D, which he could not do if the Teleſcope was directed in the perpendicular Line B C; but the Difference of the Poſitions of the Lines D B and B C, or the Angle D B C, is ſo very ſmall, as to amount to no more than 20″ 15‴, which gives the Proportion of the Sides B C to C D or A B, as 10210 to 1; which ſhews that *the Velocity of Light is ten Thouſand two Hundred and ten Times greater than the Velocity of the Earth in her Orbit.*

9. But the Velocity of the Earth is known, which is about 500,000,000 Miles in 365 Days, or about 56,000 Miles *per* Hour, whence the Velocity of Light will be found to be ſuch as carries it through the Space of 170,000 Miles, or 897,600,000 Feet in one Second; and therefore it will paſs from the Sun to us in 8′ and 13″.

10. If a Cannon will throw a Ball 1 Mile perpendicular Height, or 5280 Feet, the Velocity with which it goes from the Mouth of the Cannon is the uniform Velocity of 10,560 Feet *per* $18\frac{1}{4}''$ (which is the Time of the perpendicular Aſcent or Deſcent,) and therefore the Velocity of the Cannon-Ball is 578 Feet *per* Second. Whence the Velocity of Light is to that of the Cannon-Ball, as 897,600,000 to 578, or as 1,550,000 to 1 nearly.

11. The *Doctor* found that the Parallax of the fixed Stars, inſtead of amounting to many Seconds, as many

FROM the ſtupendous Velocity of luminous Particles ariſe their prodigious Effects in regard of *Heat*, *Flame*, *Burning*, *Melting*, &c. Thus when they are conſiderably denſe, they act very forcibly on the Parts of an animal Body, and raiſe the *Senſation of Heat*, by the great inteſtine Motion which they produce in every Part. Hence all other Bodies are *hotter* or *colder*, as they contain a greater and leſſer Quantity of ignitious Particles, and ſo have a greater or leſſer Degree of inteſtine Motion of the Parts.

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have deduced from their Obſervations, does not make one Second; and from thence it follows that the abovementioned Star in *Draco*, is above 400,000 Times farther from us than the Sun; and conſequently, that the Light takes up above $493'' \times 400{,}000 = 197{,}200{,}000''$ Seconds, (which is more than ſix Years,) in coming from that Star to us. In the mean Time we may reflect how different are the Places of the Sun, Moon, and Planets in the Heavens from thoſe in which they appear. Thus, ſetting aſide the Refraction of the Atmoſphere, when the Centre of the Sun is really aſcending in the Horizon, it will be $8' 13''$ after, that we obſerve it there; in which Time the Sun will be far advanced in the Heavens.

12. The Motion of the Earth is by this Method abſolutely demonſtrated, and therefore put beyond all Doubt and Objection; they who deny it now muſt confeſs themſelves wholly ignorant of one of the fineſt and moſt important Diſcoveries that was ever made in *Aſtronomy*, and which was finiſhed in the year 1728; concerning which, ſee Dr. *Bradley*'s own Account in *Phil. Tranſ.* N° 406, which we ſhall farther explain in a future Part of this Work.

IF theſe lucific Particles are ſufficiently imbibed or generated in any opake Body, they cauſe it to ſhine, or glow, or become red-hot; and by their prodigious Activity will in Time diſunite, diſſolve, and deſtroy its natural Texture, and thus change its Form, and reduce it to another Species of Matter; even the *Aſbeſtos* not excepted. (CXIII.)

(CXIII.) 1. That *Heat*, *Fire*, *Flame*, &c. are only the different Effects and Modifications of the Particles of Light, is, I think, very evident; and the Particles of Light themſelves depend entirely on *Velocity* for their *lucific Quality*; ſince by many Experiments we know that the Particles of Bodies become lucid, or Particles of Light, by only producing in them a requiſite Degree of Velocity; thus the Particles in a Rod of Iron, being hammered very nimbly, ſhine and become red-hot; thus alſo the violent Stroke of the Flint againſt the Steel, in ſtriking Fire, puts the Particles of the Steel which it takes off into ſuch a Motion as cauſes them to melt, and become red-hot, which makes the Sparks of fire produced by each Stroke. The ſame Thing you may obſerve in many other Caſes.

2. As *Fire* conſiſts in the great Velocity of the Particles, ſo it may be communicated from one Body in which it is to another in which it is not, after the ſame Manner that one Body in Motion will communicate Motion to another Body that has none. Fire differs from Heat only in this, that Heat is a Motion in the Particles of a Body with a leſſer Degree of Velocity; and Fire a Motion with a greater Degree of Velocity, *viz.* ſuch as is ſufficient to make the Particles ſhine, though we often call ſuch a Degree of Heat as will burn, Fire, though it does not actually ſhine; and we ſeldom call thoſe lucid Bodies Fires which only ſhine,

If the ignific Particles of Light are ſufficiently condenſed, as the Rays of the Sun

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and do not burn. Theſe are a Sort of *Phoſphori*, which though they have no Heat, yet ſeem to owe their Lucidity to the Motion of the Parts.

3. This I think will appear for the following Reaſons; (1.) We obſerve ſeveral of thoſe *Phoſphori* are owing to Putrefaction, as rotten Wood, very ſtale Meat, eſpecially Veal, ſome Sort of Fiſh long kept, as *Oyſters*, *Lobſters*, *Flounders*, *Whitings*, &c. which Putrefaction is the Effect of a ſlow and gentle Fermentation, and that conſiſts in the Inteſtine Motion of the Parts as we have formerly ſhewn. (2.) Moſt of thoſe *Phoſphori* have their Light ſo very weak as to ſhine only in the Dark, which ſeems to indicate a leſſer Degree of Velocity in the Parts than what is neceſſary to produce Heat; for ſuch a Degree of Velocity will cauſe Bodies to ſhine in open Day-Light. (3.) Some of thoſe *Noctilucæ*, or Bodies which ſhine in the Dark, are the Parts of animated Bodies, as in the *Glow-Worm*, a ſmall Sort of *Centipede*, &c. but all the Parts of an Animal are undoubtedly in Motion. (4.) Other *Phoſphori* put on the Appearance of Flame, as the *Ignus Fatuus*, the Writing of common *Phoſphorus* made from Urine, Flaſhes of Lightning, *&c.* but all Flame is nothing but a kindled Vapour, whoſe Parts are all in Motion, but may be too weak to cauſe Burning. (5.) Several of thoſe innocent lambent Flames may have their Matter ſo agitated, or the Velocity of their Motion ſo increaſed, as to produce Heat and burn; thus the Writing of *Phoſphorus* on blue Paper, ſufficiently rubbed, will immediately kindle into an ardent Flame, and burn the Paper. (6.) Thoſe *Phoſphori* ſeem to have the eſſential Nature of Fire, becauſe they are ſo eaſily ſuſceptible of a burning Quality from Fire; thus common *Phoſphorus* is immediately kindled into a moſt ardent and inextinguiſhable Flame by common Fire. (7.) In ſtroking the Back of a black Horſe, or Cat, in the Dark, we produce innumerous *Scintillæ*, or lucid Sparks; in the ſame

by a *Lens* or *Burning-Glaſs*, they become *ardent*, and burn with an Intenſity propor-

Manner as rubbing a black Piece of Cloth, which has hung in the Sun to dry, will cauſe it to throw out the Particles of Light which it had imbibed from the Sun; whereas a white Piece of Cloth, which reflects moſt of the Sun's Rays, emits no ſuch lucid Sparks in the Dark. Many other Reaſons might be urged to ſhew that Light of every Kind is owing to one and the ſame Cauſe in a greater or leſſer Degree, viz. *to the Velocity of the Parts of the lucid Body.*

4. It has been juſtly obſerved by ſome of our modern Philoſophers, that *actual* or *abſolute Heat* is to *ſenſible* or *relative Heat* the ſame as *Motion* is to *Velocity*; for *abſolute Heat* is nothing but the whole Motion of all the Parts of the ignited Body, and *ſenſible or relative Heat* reſpects only the *comparative Velocity* of the Parts. Thus equal Bulks of Mercury and Water ſet in a Sand-Heat, where the Heat of the Fire may be uniformly communicated to both, will acquire in equal times equal Degrees of abſolute Heat, but the relative Heat of the Water, or that which is ſenſible to the Finger, will be near 14 times as great as that of the Mercury; becauſe the Water having 14 times a leſs Quantity of Matter, will admit of Velocity ſo much in Proportion greater.

5. Again, if Mercury and Water have the ſame relative or ſenſible Heat, that is, if both are heated in ſuch a Manner as to cauſe an equal Aſcent in the Thermometer; then a Quantity of Mercury will heat 14 Times as much Water as the ſame Quantity of Water will do; or it will make the ſame Quantity of cold Water 14 times hotter than the ſame Quantity of hot Water can. All which is eaſy to be ſhewn by Experiment, and abundantly proves the Truth of the foregoing Theory, viz. *That Heat and Fire are wholly owing to the Velocity of the Parts of the heated or ardent Body.*

6. The various Phænomena of *Heat* and *Cold*, *Fire*, *Burning*, &c. are rationally accounted for on this The-

tional to the Denſity of the Rays in the *Focus*, or Burning-Point of the Glaſs; which

ory. For firſt, we are to conſider that Cold and Heat are only comparative Terms, or that the ſame Thing may be either hot or cold according to the relative Idea, or Standard Degree; thus Ice or Snow is ſaid to be cold with reſpect to the Finger, but Ice or Snow is warm if compared with a freezing Mixture. So that if (as we commonly do) we make the Hand or any Part of the Body the Standard of Heat or Cold, or the Term of Compariſon; then 'tis evident, (1.) If the Parts of any Body applied to the Hand have the ſame Velocity as the Parts of the Hand, ſuch a Body we naturally pronounce is neither hot nor cold. (2.) If the Particles of the Body have a greater Velocity than thoſe of the Hand, we pronounce it *warm*, if the Exceſs be ſmall; but *hot*, if it be great. (3.) If the Velocity of the Parts of the Body applied be leſs than that in the Hand, the Senſation then is what we call *Cold*, which alſo may be in various Degrees. (4.) Hence it is plain there can be no ſuch Thing as *abſolute Cold*, but where the Particles of Matter are abſolutely quieſcent or at reſt. (5.) Hence alſo there can be no ſuch Thing as abſolute Heat, becauſe no Degree of Velocity can be aſſign'd, but a greater is aſſignable, till we come to Infinity, where we are quite loſt, as having no Idea of Infinite Velocity or Heat.

7. From this Theory of Heat and Cold, we may conclude that there is no Body in Nature whoſe Parts are not in Motion in ſome Degree, ſince we have yet been able to diſcover no ultimate Degree or Limit of Cold; and if any ſuch Thing were to be found in Nature, I believe it would be as impoſſible to bear or endure the Teſt as any extreme degree of Heat; both Heat and Cold naturally tending to deſtroy the animated Part, or Teſt, in the extreme Degrees; Cold, by deſtroying the vital Motion, and fixing the Part rigid and inflexible; but Heat, by putting the Parts in too great an Agitation, cauſing a greater Velocity in the Fluids and

Denſity of Rays in the Focus is always *as the Area of the Burning-Glaſs directly*, and

Diſſipation, and a Force of Tenſion in the Solids beyond what the natural State of the Body can bear; and therefore it will inevitably deſtroy it.

8. Whatever be the acting Principle in *Freezing* or *Congelation*, 'tis certain, the *Modus Agendi*, or *Manner of Operation*, muſt be to diminiſh the Velocity of the Parts of the congealable Subſtance to a proper Degree, by which means the Fluidity will be loſt, and the Parts become rigid and fixed. Thus if the Inteſtine Motion of the aqueous Particles be abated by the Admixture of any extraneous Body, the Parts will be no longer fluid, but remain to Appearance fixed in a Congelation, and become a Body of Ice. Whatever this Principle of freezing be, it is certainly of a *ſaline Nature*, becauſe it is well known Salt will greatly increaſe the Coldneſs of Water, Ice, or Snow; and *freezing Mixtures* are always made therewith, by equal Quantities of each.

9. On the other Hand, fixed Bodies are rendered fluid by Heat, only by increaſing the Velocity of the Parts; thus Ice becomes Water, thus Metals are put into Fuſion, and a greater Degree of Heat gives a ſtill greater Degree of Velocity to the Parts, and throws them off in the Form of a Steam or Vapour. This Steam or Vapour, if it conſiſts of ſuch Particles as will admit of a proper Increaſe of Velocity, will conceive it very readily, and kindle into a Flame, at the Approach of a Body whoſe Parts are thus in Motion; that is, of Fire or Flame.

10. There ſeems to be no other Difference between *Fire and Flame* than this, that *Fire* conſiſts in a glowing Degree of Velocity in the Parts of a Body while yet ſubſiſting together in the Maſs; but *Flame* is the ſame Degree of Velocity in the Particles diſſipated and flying off in Vapour; or to uſe Sir *Iſaac Newton*'s Expreſſion, *Flame is nothing elſe but a red-hot Vapour.*

the Square of the Focal Diſtance inverſely. Thus ſuppoſe the Surface or Area of one Glaſs contained 12 ſquare Inches, and its focal Diſtance were 8 Inches; and the Area of another Glaſs were 9 ſquare Inches, and its focal Diſtance 4 Inches; then the Effects or Intenſity of Burning would be as $\frac{12}{64}$ to $\frac{9}{16}$, or as 12 × 16 to 9 × 64, *viz.* as 197 to 576. (CXIV.)

11. The Effect of Fire in burning conſiſts in this, that the Velocity of the Particles of Fire ſo far increaſes the Velocity of the Parts of the Body to which it is applied, as to cauſe a Separation beyond the Sphere of corpuſcular Attraction, by which means the Body will be diſſolved, and the Particles which are volatile will fly off in the Form of Steam, Smoak, Fume, *&c.* while that which remains appears in the Form of *Coal*, *Calx*, *Aſhes*, *Caput Mortuum*, &c.

12. The Parts of ſome Bodies are extremely volatile, and will moſt of them be diſſipated by the Action of Fire; but others again are to be found whoſe Parts are of ſuch a Nature, or ſo fixed, as not to yield to the Force of Fire, or the Velocity communicated to them will not be able to diſſolve the corpuſcular Attraction; but when this glowing Velocity of the Parts is abated, or, in other Words, when the Fire in the Body is extinct, the Parts (and of courſe the whole Body) appear unalter'd. Of which Sort of Subſtance we have a noble Inſtance in that Foſſil call'd the *Aſbeſtos* or *Amianthus Stone.* This Stone is found in divers Parts of the World; particularly in *Wales* a great deal may be ſeen adhering to, and growing up with the Stone of many of their Quarries.

(CXIV.) 1. In order to account for the Nature of BURNING-GLASSES, whether *Mirrours* or *Lenſes*, we

WHEN Rays of Light fall on the Surface of an opake Body, part thereof are

muſt conſider the Area of their Surfaces, and the focal Diſtance; becauſe both theſe Quantities enter into the Expreſſion of their Power of Burning. Let A B, and I K, be two Mirrours expoſed directly to the Rays of the Sun C D, E F, and L M, N O; then will all the Rays, falling on the Surface of theſe Mirrours, be reflected to the Focus of the Glaſſes, where they will be concentered, not in a Point of Space, but into a ſmall round circular Area G H, and P Q.

Plate XXXV. Fig. 4, 5.

2. Now this circular Spot is the Image of the Sun inverted, in both Glaſſes; and the Angle under which the Image of an Object appears from the Centre of the Glaſs R and S, is equal to the Angle under which the Object appears; all which will be ſhewn hereafter. Therefore the Angle $GRH = PSQ$, and conſequently the Cones G R H and P S Q are ſimilar, and the Areas of their Baſe G H and P Q will be as the Squares of their Heights R H and S Q, that is, as the Squares of their focal Diſtances directly.

3. Let $A =$ Area or Surface of the large Glaſs, $a =$ that of the leſſer, F and f the focal Diſtances, and P and p the Power of Burning in each. Then, ſince while the focal Diſtance remains, the Power of Burning (P) will be as the Denſity of the Rays in the ſolar Spot H G, and this Denſity of the Rays will be as the Number of Rays reflected thither by the Glaſs, which Number of Rays will be as the Surface of the Mirrour (A); therefore P will be as A directly in a Mirrour of the ſame Concavity, that is, $P : p :: A : a$.

4. Again, if the Area of each Glaſs be the ſame, the ſame Quantity of Rays will be collected, and converged to the Focus's G H and P Q, and conſequently the Denſity of thoſe Rays will be greater, the leſs the Spot is in which they are contain'd; conſequently the Power of Burning (P) in this Caſe is inverſely as the

reflected to the Eye, which render it viſible; the other Part is tranſmitted, and variouſly

Area of the ſolar Spot, or the focal Diſtance, that is, P will be as $\frac{1}{F^2}$; or $P : p :: \frac{1}{F^2} : \frac{1}{f^2} :: f^2 : F^2$.

5. Conſequently when neither the Area of the Glaſs nor focal Diſtance is given, we have the Power of Burning compounded of the direct ratio of the Area and inverſe Ratio of the Square of the focal Diſtance of the Glaſs; or we have $P : p :: A\,f^2 : a\,F^2$; which is the Rule above laid down.

6. It has been ſhewn (*Annot.* XCIII.) that the Heat of Wood Fire is about 35 times greater than that of the Summer-Sun (becauſe it raiſes the Fluid in the Thermometer 35 times higher nearly); therefore that a Glaſs may be able to condenſe the Rays ſufficiently to burn, or to have the Heat of common Fire, the Sun's Image, or ſolar Spot in the Focus, ought to be at moſt but $\frac{1}{35}$ Part of the Area of the Glaſs; and as much as it is leſs than a $\frac{1}{35}$ Part of the Glaſs, ſo much the ſtronger will it burn. In this Caſe, if it be deſired to know in what Part of the Pencil of Rays the Denſity is 35 times greater than the common Denſity, and where the Power of Burning is equal to that of common Fire, it is found as in the following Example. Admit a Glaſs be 9 Inches in Diameter, and let the Diameter of the required Circle be (*a*); then ſince circular Areas are as the Squares of their Diameters, we have $35 : 1 :: 9^2 : a^2$; conſequently $\frac{81}{35} = a^2$, and ſo $a = \sqrt{\frac{81}{35}} = 1{,}5$ nearly; whence that Part of the Cone or Pencil of Rays, whoſe Diameter is 1½ Inches, has the Denſity and Power of Burning required; and that this is Fact, and that the Denſity of the Rays but a little leſs than that will not burn, I know from repeated Trials with ſuch a Glaſs, or concave Mirrour.

7. Of *Burning-Glaſſes* we have ſome extraordinary Inſtances and ſurprizing Accounts of their prodigious

reflected through the Pores of the Body, till it becomes totally ſuffocated and loſt therein;

Effects. Thoſe made of reflecting Mirrours are more powerful than thoſe made with Lenſes, (*cæteris paribus*) becauſe the Rays from a Mirrour are reflected all to one Point nearly, whereas by a Lens they are refracted to different Points, and are therefore not ſo denſe or ardent. Alſo the whiter the Metal or Subſtance is, of which the Mirrour is made, the ſtronger will be the Effect; and it is obſervable, that the great Mr. *Boyle* having made a very large Mirrour of black Marble, it would not ſo much as ſet Wood on Fire, though expoſed a long Time in the Focus, ſo ſmall a Quantity of Rays are reflected from black Surfaces, the Reaſon of which we ſhall hereafter explain.

8. Among a great Number of Mirrours made for *burning*, *melting*, *calcining*, and *vitrifying* Bodies, that of Mr. *Villette* is worth our Notice; it was 3 Feet 11 Inches in Diameter, and its focal Diſtance was 3 Feet 2 Inches. The following Experiments were made with it by Dr. *Harris* and Dr. *Deſaguliers*.

1. A red Piece of *Roman Patera* began to melt in 3″, and was ready to drop in 100″.
2. Another black Piece melted at 4″, and was ready to drop at 64″.
3. Chalk taken out of an *Echinus Spartagus*, fled away in 33″.
4. A Foſſile-Shell calcin'd in 7″.
5. A Piece of *Pompey*'s Pillar at *Alexandria* vitrified in the black Part in 50″, and in the white Part in 54″.
6. Copper-Ore vitrified in 8″.
7. Slag, or Cinder of ancient Iron-Work, ready to run in 29½″.
8. Iron-Ore fled at firſt, but melted in 24″.
9. Talc began to calcine at 40″, and held in the Focus 64″.
10. *Calculus humanus* was calcined in 2″, and only dropped off in 60″.
11. A great Fiſh's Tooth melted in 32½″.

therein; and ſince none of thoſe Rays come from the interior Parts to the Eye,

12. *The Aſbeſtos* ſeem'd a little condenſed in 28″, and Mr. *Villette* ſays, the Glaſs uſually calcines it.
13. Marcaſite of Gold broke to Pieces and began to melt in about 30″.
14. A Silver Six-pence melted in $7\frac{1}{2}''$.
15. A Copper Half-penny (of King *William*'s) melted in 20″, and ran with a Hole in 30″.
16. A King *George*'s, *ditto*, melted in 16″, and ran in 34″.
17. Tin melted in 3″.
18. Caſt Iron melted in 16″.
19. Slate melted in 3″, and had a Hole in 6″.
20. Thin Tile melted in 4″, had a Hole and was vitrified in 80″.
21. Bone calcined in 4″, and was vitrified in 33″.
22. A Diamond weighing 4 Grains loſt $\frac{7}{8}$ of its Weight.

9. The Power of Burning, in *Villette*'s Mirrour, may be computed, and compared with the Heat of Wood-Fire, as follows: Since the focal Diſtance R X is 38 Inches, and the Angle under which the Sun's Image in the Focus appears at R, is equal always to 32′ of a Degree; therefore if we ſay,

As Radius — — — 90° 00′ = 10,000000
Is to the Tangent of the Angle } HRX = 00° 16′ = 7,667849
So is the focal Diſtance RX = 38′ = 1,585461
To the Semidiameter of the ſolar Spot } HX = 0,179′ = 9,253310

Whence 2 H X = 0,358 of an Inch, the Diameter of the ſolar Focus; but the Diameter of the Mirrour was 47 Inches; now 47 × 47 = 2209, and 0,358 × ,358 = 0,128, &c. wherefore 2209 is to 0,128, as the Denſiy of the Rays in the Focus to their common Denſity; but 70,128)2209(= 17257; which ſhews that the Mirrour condenſed the Rays *Seventeen Thouſand Two Hundred and Fifty-ſeven times.*

we can ſee nothing of the internal Subſtance of ſuch a Body, which therefore is ſaid to be *opake*.

10. Since Rays but 35 times denſer than in their natural State with us, have a Power of Burning equal to Wood-Fire, if we divide 17257 by 35, the Quotient will be 493; therefore ſuch a Mirrour will burn with an Intenſity of Heat 493 times greater than common Fire. No wonder then that Bodies which remain unalter'd by the Force of our greateſt common Fires (as that of a Glaſs Houſe, where Gold has been found to lie ſeveral Days in Fuſion, without any ſenſible Loſs of Weight) ſhould immediately become fuſed, fume away in part, part be diſſipated and driven away in large Particles, and part remain in the Form of a *Caput Mortuum*; all which Phænomena have been obſerved of Gold in the Focus of a large Burning-Glaſs. And how rudely ſuch a Glaſs would treat the Principles of the *Chymiſts*, and what Confuſion it would induce in their Arithmetic of Elements, they will be better certified of, when they ſhall attempt to analyſe Nature, and reduce Subſtances to their original Principles, by more active and effectual Means than Laboratories at preſent afford.

11. Notwithſtanding the prodigious Denſity of the Rays in the Focus of thoſe large Burning-Glaſſes, yet it has been always obſerved, that the Rays reflected to us by the Moon when at Full, and concentered in the Focus of thoſe Glaſſes, produce no Heat that is ſenſible in the leaſt Degree, as is demonſtrated by holding a Thermometer in the Focus of lunar Rays, which always remains without the leaſt appearance of Motion. The Reaſon of this will appear by the following Calculation.

12. Let A B D be the Earth, C its Centre, M O the Moon, N the Centre, N *e* the Semidiameter of the Moon, which is equal to 1087,5 *Engliſh* Miles; the Semidiameter of the Earth D C = 4000 Miles; the

BUT when Rays of Light fall on *tranſparent Bodies*, part is reflected at the firſt

Diſtances of the Centres of the Earth and Moon N C = 240000 Miles. Then ſince the Rays of the Sun's Light at the Moon are of the ſame Denſity as with us (as being Parallel); and ſince the lunar Rays are only the ſolar Rays reflected to us by the convex Surface of the Moon; and laſtly, ſince parallel Rays are reflected by a ſpherical convex Surface, in ſuch a Manner as to go after Reflection diverging from a Point which is $\frac{1}{2}$ the Semidiameter of the Sphere diſtant from the Vertex (as will be ſhewn hereafter); therefore ſuppoſing the Surface of the Moon to be perfectly ſpherical and poliſhed, we may compute the Denſity of the ſolar Rays reflected from the Moon to the Earth as follows. Plate XXXV. Fig. 6.

13. Let $a\,b$, $c\,d$, be two parallel ſolar Rays falling on the Surface of the Full Moon, theſe Rays will be reflected to the Earth in the Directions $b\,g$ and $d\,h$ diverging from a Point f in the Radius N e, half way between N and e. Now the Denſity of the Rays falling on the Moon will be to thoſe reflected at the Earth's Surface, as the Square of $g\,h$ to the Square of $b\,d$, or as the Square of f D to the Square of $f\,e$; but $f\,e$ = 544 Miles, and fD (= N C − C D − N f = 240000 − 4544 =) 235456; and the Square of 235456 is to the Square of 544, as 187400 to 1 nearly; conſequently the Denſity of the lunar Rays is to that of the ſolar Rays at the Earth's Surface as 1 to 187400 nearly; therefore a Burning-Glaſs muſt condenſe the lunar Rays 187400 times to make them have the Heat of the common Sun-Beams. But this is 10 times more than *Villette*'s Mirrour can effect.

14. Now this is all upon Suppoſition that the Moon is a Sphere, and its Surface a perfect Poliſh, whereas neither of theſe Things have Place in Nature; for the Moon is not a Sphere but a Spheroid, and her Surface very unequal or uneven, on both which Accounts the Reflection of Light muſt be many times weaker than we have ſuppoſed it; and accordingly Mr. *Bouguer*, by Ex-

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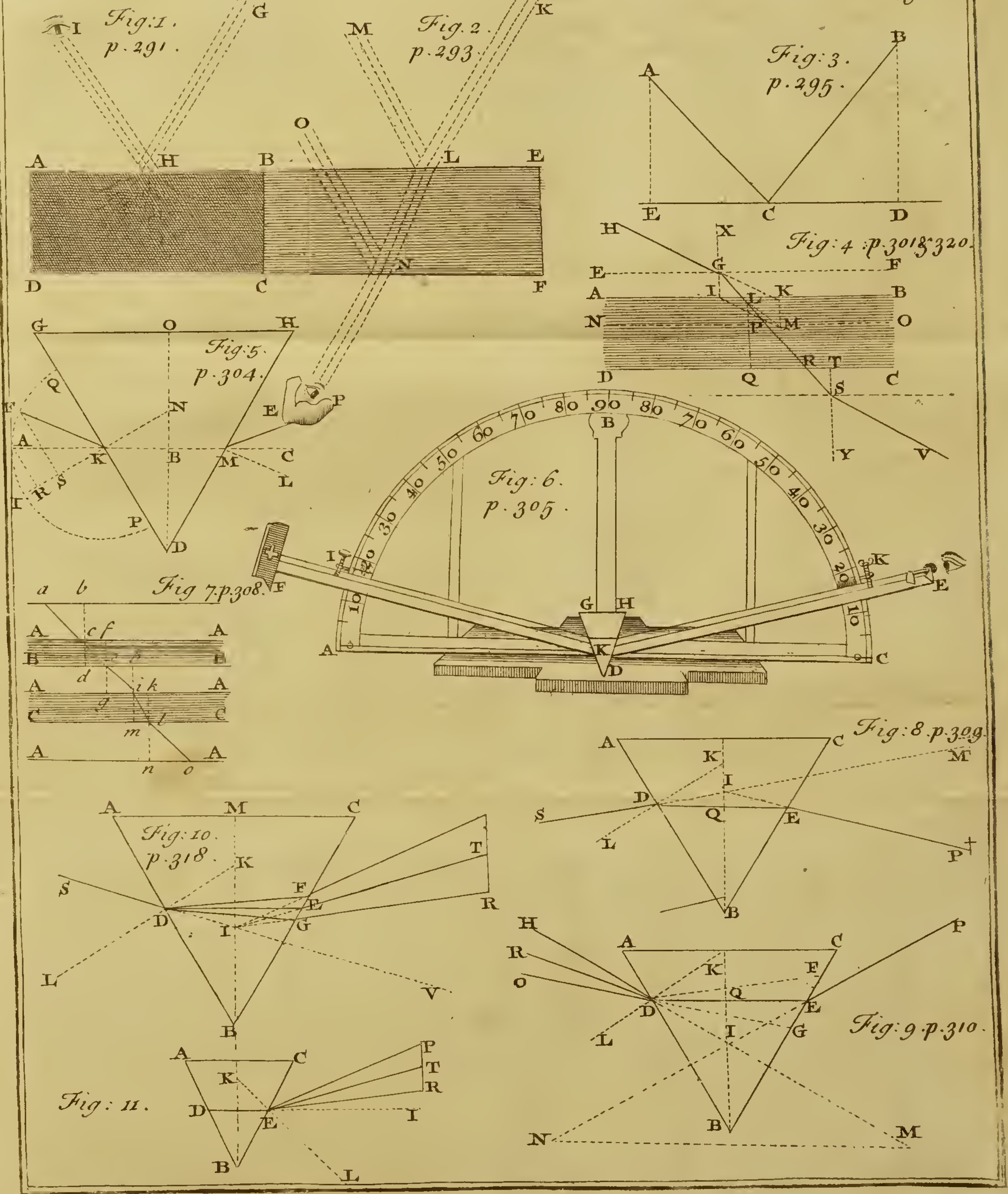

Fig:1. *p*.291.

Fig.2. *p*.293.

Fig:3. *p*.295.

Fig:4 :*p*.301 & 320.

Fig:5. *p*.304.

Fig:6. *p*.305.

Fig 7.*p*.308.

Fig:8.*p*.309.

Fig:9.*p*.310.

Fig:10. *p*.318.

Fig: 11.

Surface, and part is tranſmitted into the Body, which is refracted in right Lines to the ſecond or lower Surface, where it is again partly reflected and in part refracted into the Air, and coming to the Eye, renders the internal Parts of thoſe Bodies viſible, which for that Reaſon are ſaid to be *diaphanous* or *tranſparent* (CXV.)

periments, has found that it is about 17 times leſs, or that the Denſity of the lunar Rays is to that of the ſolar as 3000000 to 1; wherefore a Burning-Glaſs muſt condenſe the Rays of the Moon near 3000000, *i. e.* three Millions of times, to make them warm enough to raiſe the Liquor of the common Thermometer; which is an Effect almoſt 200 times greater than *Villette*'s Mirrour can produce.

(CXV.) 1. The Opacity and Tranſparency of Bodies in general is thus occaſioned: Let A B be the Surface of an opake Body A B C D, a Ray of Light G H falling thereon in the Point H will in part be reflected into the Ray H I, and by this reflected Ray the Point H becomes viſible to the Eye at I; and thus all the Points, and conſequently the whole Surface, is made viſible by that Part of the Light which it reflects. Plate XXXVI. Fig. 1.

2. But the other Part of the Ray entering into the Body, being irregularly refracted and reflected through its internal Subſtance of Particles and Pores, becomes divided, diſſipated, abſorbed and loſt therein; and therefore as none of the Rays can come from the internal Parts to the Eye, ſo none of thoſe Parts can be viſible, and the Body is in that caſe ſaid to be *opake.*

3. In order to this we muſt conſider, that though the whole Body be opake, yet the Particles of ſuch a Body are not ſingly opake, but freely tranſmit the Light without reflecting any Part between the Surfaces, and are therefore in themſelves tranſparent; and were thoſe

Plate XXXIV.
Figure 7

WHEN a Ray of Light HC falls on any plain, convex, or concave Surface, as AB,

Particles contiguous to each other, the Light would paſs from one to another (and ſo through the whole) without Reflection, as we find by Experiment it will paſs through ſeveral contiguous Pieces of poliſhed Glaſs, and thus produce Tranſparency.

4. But if the Particles do not touch in ſuch manner as to leave the Interſtices or Pores exceeding ſmall, there will be a Reflection of Light at every Pore from the Air which it there meets with, as being a Medium of different Denſity. For it is known by Experiment, that though a Ray of Light will paſs from one Piece of Glaſs to another, that is contiguous without Reflection, yet will it not paſs from the Glaſs through the contiguous Air without being in part reflected; conſequently where the Pores are large and very numerous, there the Reflection of the Light will be ſo great upon the whole, as to cauſe a total Diſſipation and Loſs of the Light that enter'd the Body, and ſo render it opake.

5. This is confirmed by taking ten Pieces of clear Glaſs, and laying them one upon another over a Leaf of Print, quite dry, and having only Air between them; then taking ten other Pieces of the ſame Glaſs, and putting them into Water, ſo that it may fill all their Interſtices, and then laying them on the ſame printed Paper by the other, a Perſon looking through each will ſee the Print or reading much more diſtinct, clear, and bright, through the latter Pieces than through the former; the Rays being more regularly tranſmitted through them where the Denſity of the Parts is not ſo unequal, and alſo with much leſs Reflection, than through the other, where the Light undergoes a conſiderable Reflection at every Interſtice or Plate of Air between the Glaſſes.

6. 'Tis hence alſo that tranſparent Bodies are rendered opake by ſeparating their Parts and rendering them more porous; thus Beer before it is raiſed into Froth

DE, FG, in the Point C, the Angle HCK, made by the incident Ray HC and the

is tranſparent, but the Froth, by reaſon of its Pores, becomes opake; thus dry Paper is more opake than that which is wetted with Water or Oil, becauſe more porous. Thus the *Oculus Mundi* Stone is more opake when dry than when ſteeped in Water; and Glaſs reduced to Powder is no longer tranſparent.

7. Hence it follows, that the Parts of Bodies and their Pores muſt not be leſs than a certain definite Bigneſs to render them opake. For the opakeſt Bodies, if their Parts be ſubtilly divided, become perfectly tranſparent. Thus Copper diſſolved in Aqua-fortis has all its Particles pellucid, and the whole Solution is tranſparent. Thus a Bubble blown of Soap-Water may become ſo thin on the Top as to reflect no Light, but will tranſmit the whole. Thus Water, Salts, Glaſs, Stones, *&c.* though they are as porous as other Bodies, yet their Parts and Interſtices are too ſmall to cauſe Reflections in their common Surfaces.

Plate XXXVI. Fig. 2.

8. Therefore in all tranſparent Bodies, as BEFC, a Ray of Light, as KL, falling on its Surface in the Point L, will there be in Part reflected (as before) into the Ray LM; the other Part will go regularly on in a rectilineal Direction from the upper to the lower Surface at N, where meeting with the Air (a Medium of a different Denſity) it will be in Part reflected again into the Ray NO; the other Part goes out to the Eye at P, by which Means all the internal Parts from whence that Ray comes will be rendered viſible to the Eye; and ſince this may be conceived of every Point in the Body, it is eaſy to underſtand how the whole becomes tranſparent.

9. I have often found Gentlemen reflect with great Surprize on the exceeding great Poroſity of Bodies neceſſarily required for the Tranſmiſſion of Light, and yet at the ſame Time on the Hardneſs and Firmneſs of the Parts of ſuch Bodies, as Glaſs, for Inſtance, and others. But Sir *Iſaac Newton* has put us into a Method

Perpendicular K C, is always equal to the Angle K C I, made by the ſaid Perpendi-

by which we may conceive this with as much Eaſe as it produces Surprize; and it is this: Suppoſe a Body be compoſed of ſuch Particles, and of ſuch a Figure, that when laid together, the Pores or Interſtices may be equal to the Particles themſelves; how this may be done, and the Body hard and firm, is not difficult to conceive; ſuch a Body then will be half ſolid and half porous.

10. Now if each of theſe conſtituent Particles, inſtead of being ſolid, ſhould be ſuppoſed to conſiſt of other Particles, equal in Bulk to their Pores between them, then would the ſolid Part of the whole Body be but half of what it was before ſuppoſed to be, that is, it will be but $\frac{1}{4}$ Part of the whole Bulk. In like manner if theſe Parts are ſuppoſed not ſolid, but to conſiſt of other Parts with equal Pores between them, 'tis then manifeſt the ſolid Matter will be but $\frac{1}{8}$ of the whole Bulk of the Body. And thus by continuing this Subdiviſion of the Parts, you diminiſh the Quantity of the ſolid Parts, and increaſe that of the Pores, till it ſhall be in any Proportion greater than that of the ſolid Matter, and yet the Parts, and conſequently the whole Body, ſhall be every where compact and hard.

11. Hence it follows that the leaſt aſſignable Particle of Matter may be conceived to be ſo minutely divided, that it ſhall be diffuſed through any aſſignable Space, how great ſoever, in ſuch a manner, as to be in Contact, and to conſtitute a hard and compact Body, whoſe Pores ſhall be leſs in Diameter than any aſſignable Length; or, in other Words inverſely, the ſolid Matter in the Globe of our Earth, yea of all Bodies in the Univerſe, may be no more than what may be reduced within the compaſs of a cubic Inch, or be contained in a *Lady's Thimble*. They who would ſee a Mathematical Demonſtration of this, may conſult Dr. *Keill*'s Introduction to *Natural Philoſophy*.

cular and the reflected Ray CI: Or the Angle of Incidence is equal to the Angle of Reflection in every Inclination of the Ray of Light. This is evidently ſhewn by Experiment; and it is very well worth our Obſervation, that in this Caſe only, the ſaid Ray takes the *ſhorteſt Way poſſible* from any Point H, to any other Point I, if it muſt, in its Paſſage, touch any of thoſe Surfaces (CXVI.)

12. Hence we ſee the Poſſibility of Bodies being ſo exceeding porous, as to be rare enough to tranſmit Light with all that freedom pellucid Bodies are found to do. Though what their real Structure or inward Frame may be, is yet unknown to us.

Plate XXXVI. Fig. 3.

(CXVI.) 1. The Demonſtration of this is as follows: Let A C be the incident Ray, and C B the reflected one; from A and B let fall the Perpendiculars A E, B D, and let A E $= a$, B D $= b$, E D $= c$, and E C $= x$; then C D $= c - x$, and A C $= \sqrt{aa + xx}$, and alſo C B $= \sqrt{bb + cc - 2cx + xx}$. Then ſince A C + C B is to be a *Minimum*, we muſt make the Fluxion of its Expreſſion $\sqrt{aa + xx} + \sqrt{bb + cc - 2cx + xx}$ equal to nothing, *viz.* $\frac{x\dot{x}}{\sqrt{aa + xx}} + \frac{x\dot{x} - c\dot{x}}{\sqrt{bb + cc - 2cx + xx}} = 0$; whence dividing by $\dot{x}$, and multiplying croſs-wiſe, we have $x \times \sqrt{bb + cc - 2cx + xx} + \overline{x - c} \times \sqrt{aa + xx}$, conſequently $x \times \sqrt{bb + cc - 2cx + xx} = \overline{c - x} \times \sqrt{aa + xx}$, that is, E C × C B = C D × A C; and ſo we have E C : A C :: C D : C B. Conſequently (by *Euclid*, 6 and 7.) the Triangles A E C and B D C are equiangular, and therefore the Angle of Incidence A C E = B C D the Angle of Reflection.

The Rays of Light reflected from the firſt Surface of a Glaſs are in a much leſs Quantity than thoſe reflected from the ſecond Surface, as is evident from hence, that the Image formed in the firſt Caſe is leſs bright and ſplendid than that of the latter; and if the ſecond Surface be contiguous to any tranſparent *Medium*, as Air, Water, *&c.* the Rays will be reflected from thence in greater Plenty, as the *Medium* is more rare; whence the Image by Reflection from the ſecond Surface is brighter when that Surface is contiguous to Air, than when it touches Water; and moſt bright when it is contiguous to a *Vacuum*.

If the ſecond Surface of Glaſs be cover'd with an opake Body impervious to the Rays of Light, they will then be reflected in much greater abundance from the ſecond than from the firſt Surface, and the Image will be proportionally more bright than that formed by Reflection from the firſt Surface;

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2. Since the concave Plane F C G, and convex Plane D C E do both touch the Plane A B in the ſame ſingle Point C on which the Ray of Light is ſuppoſed to fall; the ſame Law of Reflection muſt hold with reſpect to all the Planes equally; becauſe the Situation of any other Particles have nothing to do in the Cauſe of Reflection of Light, but that on which the Ray immediately impinges.

face; which is the Caſe of all Glaſſes foliated or quickſilver'd. Whence it appears, that the Light reflected from the firſt Surface bears a very ſmall Proportion to that which is tranſmitted into the Subſtance of the Glaſs.

WHEN a Ray of Light, as H C, paſſes out of Air into a denſer Medium, as A B F O, it will be ſtrongly attracted by the Particles of the Surface of the Medium A B, a little way on each Side; the Conſequence whereof is, that its Motion will be accelerated at the Entrance of the Medium, and its Direction ſomewhat altered; for ſince the Attraction of the Medium is perpendicular to its Surface, it will deflect or bend the Ray out of its firſt Direction H F, into a new one C E, (through the *Medium)* which lies nearer to the Perpendicular K D, drawn through the Point of Incidence C: And this is called the REFRACTION *of a Ray of Light*; H C K is the Angle of Incidence, and D C E the Angle of Refraction. Plate XXXIV, Fig. 8.

IF on the Point C be deſcribed a Circle D H K G, and from the Points H and G (where the Circle cuts the incident refracted Ray) be drawn the Lines H L, G I, at Right Angles to the Perpendicular K D, they will be the *Sines of the Angles of Incidence and Refraction*,

Refraction. And it is ſeveral ways demonſtrable, that in every Inclination of the Ray of Light HC to the Surface of the Medium AB, thoſe two Sines HL and GI will always have one certain or conſtant Ratio or Proportion to each other: And that HL : GI :: 4 : 3, if the Refraction be out of *Air* into *Water*; but HL : GI :: 17 : 11, or 3 : 2 nearly, if out of *Air* into *Glaſs*; and in general, the denſer the Medium, the greater its refractive Power, or Diſproportion of the Sines; all which Particulars are render'd very evident by Experiments.

IF a Ray of Light, as EC, paſs out of a denſer Medium into a rarer, as a *Water* or *Glaſs* into *Air*, it will, upon entering the rarer Medium at C, be refracted from its firſt Direction EN into a new one CH, which will be farther off from the Perpendicular KCD; and in this Caſe, IG will be the Sine of the Angle of Incidence, and HL that of the Angle of Refraction; and all other Particulars juſt the reverſe of what they were before under the ſame Names.

HENCE it follows, that if any Object be placed at E, and covered with Water to the Height CD, it will be ſeen by an Eye placed any where above the Surface A B, in a Situation lower than would be otherwiſe poſſible; and thus Objects which are inviſible may be render'd viſible by the Interpoſition of a denſer Medium, as is well known by a common Experiment. On this Account it is that we ſee the Sun, and other Luminaries, while they are yet below the Horizon, in a Morning before they riſe, and in the Evening after they are ſet, by the Refraction of the Atmoſphere. Hence alſo the Difference in the Diameters of the *horizontal Sun and Moon*, and their *elliptic Figure*, by the greater Refraction of the Rays coming from the lower Limb.

Again; it follows, that if an Object be viewed which is part in one Medium and part in another, as a Staff repreſented by N E, it will not appear *ſtrait*, but *crooked*; for if the Eye be in the rarer Medium, the Part of the Staff in the denſer, C E, will be refracted into the Line C F, and the whole Staff will appear in the crooked Form N C F.

Hence alſo all Objects in a denſer Medium appear raiſed or elevated above their real Situations: Thus the Part of the Staff C E is raiſed into the Situation C F; and the Bottom of all Veſſels, if covered with Water, appear raiſed, or higher by a fourth Part of the Depth of the Water, than what they really are (CXVII.)

(CXVII.) 1. If Bodies, on which Light falls, were ſuppoſed to affect it no other Ways than by giving Admiſſion to the Rays, or permitting them to paſs thro' their Subſtance, they would then perſevere in the ſame Right Line after their Immerſion, as before; and of Courſe there could be no ſuch thing as the Refraction above defined. But Bodies are not paſſive to the Rays of Light, but act upon them with a real and determinate Force, as is evidently proved by Experiments. Thus if a very ſmall round Hole be made in a thin Piece of Metal, and the Light of the Sun tranſmitted through it into a dark Room; if the Metal acted not on the Ray paſſing through the Hole, the Spot of Light would always be of the ſame Size with the Hole at all Diſtances from it; but becauſe we always obſerve the luminous Spot is larger than the Hole, and the more ſo as it is farther diſtant, is a plain Proof that the Particles of the Metal in the Periphery of the Hole act with an attracting Force on the Rays of Light, and inflect them in ſuch a manner as to cauſe them to proceed diverging from each other.

2. In like manner, if the Rays of Light are made to paſs between the parallel Edges of two Knives placed at the Diſtance of $\frac{1}{10}$ of an Inch, we ſhall obſerve on each Side the tranſmitted Beam a Glare of Light like that of the Tail of a Comet, if the Beam be received on a Sheet of Paper, at the Diſtance of about 4 or 5 Foot from the Knives. And if the Knives are placed with their Edges about $\frac{1}{100}$ of an Inch apart, inſtead of the Light above-mentioned, you'll obſerve on each Side the Beam of Light, three Fringes of colour'd Light parallel to the Edges of the Knives, which are more diſtinct as the Hole of the Window or Beam of Light is leſs.

THE Sun's Rays, as I have ſaid, are not homogeneous, but of different Kinds; and

3. If the Edges of the Knives be brought within $\frac{1}{400}$ of an Inch, no Light will appear on the Paper between the ſaid Fringes, ſo that all the Light which paſſes between the Edges is inflected on either Side, which plainly ſhews that Steel acts at the Diſtance of $\frac{1}{800}$ Part of an Inch upon the Rays of Light, by an *attractive Force* which is increaſed as the Diſtance of the Knives is diminiſhed.

4. On the other Hand, the Shadows of all Bodies placed in the Beam of Light in the dark Room are larger than they ought to be, were the Rays of Light to paſs by them unaffected by any Power from them; for then the Shadow would be at all Diſtances of one and the ſame Bigneſs, *viz.* equal to that of the Body; but ſince we obſerve the Shadow always larger than the Body, it follows, that the Rays muſt proceed diverging from the Surface of the Body, which they could not do but by virtue of a *repellent Power*, which cauſes them to ſeparate to a greater Diſtance after they have paſs'd by the Surface of the Body; thus the Shadow of a Hair has been obſerved 35 times bigger than the Hair itſelf.

5. This attracting and repelling Power in the Particles of Bodies, by which they inflect the Rays of Light, is the Cauſe of all Reflection and Refraction of Light, of which we ſhall now treat more particularly. Let there be two Mediums (ſuppoſe of *Air* and *Water*) and a Ray of Light H G in the rarer Medium (*Air*) tend towards a Point K in the Surface of the denſer Medium (*Water*) A B; the attracting Power of the Particles in the Surface of the denſer Medium extends to a certain ſmall Diſtance, as to the Line E F; as ſoon then as the Ray is arrived at the Line E F, it gets into the Attraction of the Medium, which acts perpendicular to the Surface.

Plate XXXVI. Fig. 4.

6. The Particle of Light in the Point G, begins to be acted upon by two Forces; one derived from its na-

each Sort has a different Degree of Refrangibility, that is, in paſſing through a denſe Medium,

tural Velocity in the Direction G K, the other derived from the attracting Medium in the Direction G I; let then the Parallelogram G K M I be compleated, and 'tis manifeſt (from what we have ſhewn already) that the Ray will move in the Diagonal of this Parallelógram, *viz.* in the Direction G M, and impinge on the Surface at L.

7. Now ſince the Ray of Light, after it comes to G, is influenced by the attracting Virtue of a Number of Particles continually increaſing till it comes to L, the Force therefore by which it is urged in the Direction G I, is a Force uniformly increaſing, like that of Gravity; its Motion therefore will be conſtantly accelerated, and its Direction G L not a Right Line, but a Curve. But ſince the Diſtance G I is indefinitely ſmall, the Curvature of its Path for ſo ſhort a Space is not ſenſible, and may therefore be repreſented by a Right Line.

8. Let N O be drawn parallel to the Surface A B, at the ſame depth below, as E F is above it; and then it is evident that ſince the Particle of Light is attracted every way equally within the Diſtance of I G all round, the Attraction will be greater towards the Line N O as it approaches nearer to it; conſequently its Motion will ſtill be accelerated from L to the ſaid Line, and will alſo be a Curve; therefore the Particle will not go on to M in the Diagonal G M, but will go to a Point P in the Curve L P, nearer to the Perpendicular Line L Q.

9. After it is arrived to the Line N O in the Point P, the Attraction will be on all Sides equal, its Motion or Velocity uniform, and its Direction a Right Line, till it comes within the ſame Diſtance G I of the under Surface of the Medium C D, where its Path will again begin to be incurvated into R S, and every thing will be the Reverſe of what we have now obſerved at its Im

Medium, they are differently diſpoſed to be refracted, being bent or turned out of

merſion, that is, R S will be ſimilar to G L, and S V parallel to H G, or the Angle H G X = V S Y.

10. The denſer any Medium is, the greater will be the Number of attracting Particles in a given Space, and ſo the greater will be the Force G I, or the refractive Power of the Medium; thus Water is leſs denſe, and therefore has a leſs refractive Power than Glaſs, and Glaſs leſs than Diamond. But *Oils*, though leſs denſe than Water, have yet a greater refractive Power, as containing a greater Proportion of *Sulphur* than other Bodies; for ſince Action and Re-action are mutual and equal between all Bodies, and ſince we ſee that Rays of Light congregated by a Burning-Glaſs act moſt upon ſulphureous Bodies in turning them into Fire and Flame, ſo on the contrary, Sulphurs, Oils, Spirits, &c. ought to act moſt upon Light, as we conſtantly find they do; and Sir *Iſaac Newton* thought it reaſonable to attribute the refractive Power of Bodies chiefly, if not wholly, to the ſulphureous Parts with which they abound.

Plate XXXIV. Fig. 8.

11. Since the Velocity of Light in different Mediums is different, let its Velocity in the rarer Medium from H to C be to that in the denſer Medium from C to E, as m to n; and ſince the Spaces deſcribed are as the Rectangles under the Times and Velocities, the Times will be as the Spaces directly, and the Velocities inverſely; whence the Time of deſcribing the Line H C will be to the Time of deſcribing the Line C G, as $n \times HC$ to $m \times CG$. Let $CI = a$, $CL = b$, $HL + IG = e$, and $IG = x$; then will $HL = c - x$, and conſequently $CG = \sqrt{aa + xx}$, and $HC = \sqrt{bb + cc - 2cx + xx}$; whence the Time in which $HC + CG$ is moved through is $m\sqrt{aa + xx} + n\sqrt{bb + cc - 2cx + xx}$.

12. Now admitting that Nature does every thing *in the ſhorteſt Way*, we have the foregoing Expreſſion of the Time a *Minimum*, and ſo its Fluxion equal to No-

their firſt Courſe to different Diſtances from the Perpendicular: And theſe ſeveral Sorts

thing, *viz.* $\frac{m x \dot{x}}{\sqrt{aa+xx}}+\frac{n x \dot{x}-n c \dot{x}}{\sqrt{bb+cc-2xc+xx}}=0$; whence we have $\frac{mx}{\sqrt{aa+xx}}=\frac{n\times\overline{c-x}}{\sqrt{bb+cc-2cx+xx}}$; that is, $\frac{m\times \mathrm{IG}}{\mathrm{CG}}=\frac{n\times \mathrm{HL}}{\mathrm{HC}}$. Hence, making H C = C G, we have $m\times$ I G $= n\times$ H L; and conſequently, $m:n$ :: H L : I G.

13. But the Ratio of *m* to *n*, that is, of the Velocity before and during the Refraction, is conſtant, or always the ſame in the ſame *Media*; therefore the Lines H L and I G are in a given or conſtant Ratio. Hence we have this fundamental Law of Refraction, *That the Sine of the Angle of Incidence is always in a conſtant Ratio to the Sine of the Angle of Refraction*, in all Inclinations of the incident Ray whatſoever.

Plate XXXVI. Fig. 5.

14. Since the Proportion of theſe Sines is conſtant, it remains that we determine what that Ratio is in different *Media*; and for that Purpoſe there are various Methods, one of the beſt of which I ſhall here deſcribe, but muſt firſt premiſe the following *Lemma*. Let G H D be an equilateral Triangle, and let the Angle D be biſected by the Right Line D O; let A K M C be drawn parallel to the Side G H, and through the Point K draw I K N cutting O D in N; then is the Angle A K I = N K B, as being vertical to each other. Alſo the Triangle N K D is divided into two ſimilar and equiangular Triangles N K B and B K D, by the Perpendicular K B; and therefore the Angle N K B is equal to the Angle K D B. All which is evident from *Euclid*'s Elements.

15. Suppoſe now that G H D be the Section of a Priſm of Water or Glaſs, or any pellucid Medium, and K M a Ray of Light paſſing through it parallel to the Side G H; aud let it go out of the Priſm and be refracted into the Air on each Side into the Directions K F

of Rays have each a peculiar Colour, *viz.* thoſe which are leaſt refrangible are red;

and M E; upon the Point K deſcribe the Semicircle P I Q; then is N K B (= K D B) = A K I, the Angle of Incidence out of the Priſm into Air, and F K I is the Angle of Refraction; conſequently, A R and F S are the Sines of the Angles of Incidence and Refraction out of the Priſm into Air.

16. On the contrary, we may conſider F K as the incident Ray falling upon the Priſm in the Point K, and refracted in the Direction K M parallel to the Side G H, which at the Point M emerges again into the Air in the Direction M E, making the Angle E M L with the Perpendicular M L equal to the Angle F K I. In this Caſe the Angle F K I is the Angle of Incidence, and N K B is the Angle of Refraction in the Priſm; which Angle of Refraction is therefore given, or conſtant, as it is always equal to the Angle K D B, or half the Angle of the Priſm.

17. The Angle of Incidence F K I conſiſts of two Parts, *viz.* of the given Angle A K I (= K D B) and the additional Angle A K F. Now the Angle A K I is known, as being equal to half the Angle of the Priſm; and the Angle F K A is known by placing the Priſm by the Centre of a graduated Semicircle, as A B C, carrying an Index, whoſe two Arms F K and K E are equally elevated above the horizontal Line A C, and correſpond to the incident and emergent Ray F K and M E in the other Figure. For here 'tis evident, if an Object be placed on the End of the Arm F, it will be ſeen by an Eye looking through the Sights at the other End of the Index E; and when the Object is thus ſeen, the Angle A K F is known by the Number of Degrees which each Arm cuts upon the Limb of the Semicircle.

Plate XXXVI. Fig. 6.

18. This Number of Degrees, added to the conſtant Number 30°, which is equal to half the Angle of the Priſm, gives the whole Angle of Incidence F K I; and thus the Angles of Incidence and Refraction being

the ſecond Sort, *Orange*; the third Sort, *Yellow*; the fourth Sort, *Green*; the fifth

found, the Proportion of the Sines FS and AR will be diſcovered, which Ratio is always the ſame while the Matter of the Priſm remains the ſame, as was before ſhewn from the Theory, and may by this Inſtrument be proved by Experiment. For Example, let the Priſm be of *Water*, it will be neceſſary to elevate each Arm 12 Degrees upon the Limb, before the Image of the Object at F can be ſeen by the Eye at E; then $12 + 30 = 40^\circ = FKA + AKI = FKI$, the Angle of Incidence. But the Sine F S of 42° is to the Sine A R of 30° as 4 to 3 very nearly.

19. Now it is plain, if the Ratio of the Sines A R and F S were not fixed, ſince F S might be in any Ratio greater or leſs than A R, the incident Ray F K may make an Angle F K I greater or leſs than 42°, and yet the Object at F be ſeen by the Eye at E; but this we find by Experiment to be impoſſible, becauſe there is no other Elevation of the Arms of the Index that will exhibit the Appearance of the Object, but the one abovementioned.

20. If G H D were a Priſm of Glaſs, as that is a denſer Body than Water, ſo its refractive Power will be greater, and conſequently it will act more ſtrongly upon the Ray K M at its Exit into the Air, and cauſe it to be refracted farther from the Perpendicular I K or M L. Therefore the Angle of Incidence out of Air into Glaſs, *viz.* the Angle F K I, ought to be greater, and ſo to require a greater Elevation of the Legs of the Index than before in the Priſm of Water: And this we find by Experiment is the Caſe; for then the Elevation, inſtead of 12°, muſt be about 22° or 23°.

21. Hence 'tis plain, the Sine of Incidence F S muſt be in a conſtant Ratio to the Sine of Refraction A R; becauſe, ſince the Angle A K L is invariable, (being always equal to G D O) and in the ſame Medium G D H, the Angle F K I muſt always be the ſame, becauſe the refractive Power is every where ſo; therefore, the

Sort, *Blue*; the ſixth Sort, *Indigo*; and the ſeventh Sort, *Violet*, which are moſt refrangible, or refracted to the greateſt Diſtance from the perpendicular.

To illuſtrate this Matter, let G F repreſent a Parcel of the ſolar Rays entering through the Hole H of a Window-Shutter, into a darken'd Room; and there let them fall on the Priſm ABC, in the Point F: In paſſing through the Priſm they will be ſeverally refracted in a different Degree, and thus ſeparated from each other, ſo that at their Exit on the other Side at E,

Pl. 48
Fig. 1

Angles being conſtant, the Sines will be ſo too, or their Ratio to each other always the ſame.

22. As by this Inſtrument the Angles of Incidence and Refraction are diſcovered, the Ratio of their Sines will be known of courſe, for each reſpective Medium. Thus in *Water* the Sine of 42° is to the Sine of 30° as 4 to 3 very nearly; and in *Glaſs* the Sine of 46° is to the Sine of 30° as 3 to 2, or more nearly as 17 to 11. By ſome Experiments it has been found, that the Sine of Incidence is to the Sine of Refraction in *Diamond* as 5 to 2.

23. But ſince in Phyſical Matters we have no Authority comparable to Sir *Iſaac Newton*, I ſhall here give a Table (from his *Optics*) of the Proportion of the Sines of Incidence and Refraction of Yellow Light (that being nearly a Mean between the greateſt and leaſt refrangible Rays, as we ſhall ſee farther on). This will be contain'd in the firſt Column; the ſecond expreſſes the Denſities of the Bodies eſtimated by their Specific Gravities; and the third the refractive Power of each Body in reſpect of its Denſity.

they will proceed at different Diſtances from the Perpendicular E P to the other

24.

The Refracting Body.	*Proportion of the Sines.*	*The Denſity.*	*Ref. Power*
Air	3201 to 3200	0,0012	5208
Glaſs of Antimony	17 to 9	5,2800	4864
A Pſeudo-Topaz	23 to 14	4,2700	3979
A Selenites	61 to 41	,2520	5386
Common Glaſs	31 to 20	2,5800	5436
Cryſtal of the Rock	25 to 16	2,6500	5450
Iſland Cryſtal	5 to 3	2,7200	6536
Sal Gemmæ	17 to 11	2,1430	6477
Alum	35 to 24	1,7140	6570
Borax	22 to 15	1,7140	6716
Nitre	30 to 21	1,9000	7079
Dantzick Vitriol	303 to 200	1,7150	7551
Oil of Vitriol	12 to 7	1,7000	6124
Rain-Water	529 to 396	1,0000	7854
Gum *Arabic*	31 to 21	1,3750	8574
Spirit of Wine rectified	100 to 73	0,8660	10121
Camphire	3 to 2	0,9960	12551
Oil Olive	22 to 15	0,9130	12607
Linſeed Oil	40 to 27	0,9320	12819
Spirit of Turpentine	25 to 17	0,8740	13222
Amber	14 to 9	1,0400	13654
A Diamond	100 to 41	3,4000	14556

25. The Refraction of the Air in this Table is determined by that of the Atmoſphere obſerved by Aſtronomers; for if Light paſs through many refracting Subſtances, or Mediums, gradually denſer and denſer, and terminated with parallel Surfaces, the Sum of all the Refractions will be equal to the ſingle Refraction it would have ſuffer'd in paſſing immediately out of the firſt Medium into the laſt; becauſe the immergent Ray will be parallel to the incident one in every Medium ſingly (by *Art.* 9.) if they were ſeparated; and their being contiguous can make no Alteration. Hence, if A A be the Medium of Air interceding two different Media, as B B of Water, and C C of Glaſs; then the emergent Ray *e i* out of the Water is parallel to the in-

Plate XXXVI. Fig. 7.

Side of the Room, where they will make a long and various-colour'd Image of the

cident Ray ac, and the emergent Ray lo out of Glaſs is parallel to the incident Ray ei; whence 'tis plain, the Refraction of the Ray il is the ſame as if the two Media B B and C C were contiguous, the Ray ei in that Caſe being loſt, which makes no Difference.

26. Hence, if the Sine of Incidence out of Air into Water be as $(ab : de)$ I : R, and that of Incidence to the Sine of Refraction out of Air into Glaſs as $(no : ik)$ $I : R$; then $no = \frac{ik \times I}{R} = ab = \frac{de \times \mathrm{I}}{\mathrm{R}}$; whence $\mathrm{R} \times I \times ik = R \times \mathrm{I} \times de$; but when the two Media B B and C C are contiguous, $de = cf$ will be the Sine of Incidence out of Water into Glaſs, and $ik = ml$ the Sine of Refraction; therefore $cf : ml :: \mathrm{R} \times I : R \times \mathrm{I}$ :: the Sine of Incidence out of Water : to the Sine of Refraction in Glaſs.

Plate XXXVI. Fig. 8.

27. I cannot here omit to mention the accurate Method which was made uſe of by Mr. *Hawkſbee*, at the Appointment of the Royal Society, to determine the refractive Power of the Air, which was thus: He made choice of a diſtinct erect Object P, at the diſtance of 2588 Feet; a Priſm ABC was exhauſted of its Air, and applied to the End of a 10 Foot Teleſcope with a Hair in its Focus. The Object was then viewed through the Vacuum by the Ray P E S; then admitting the Air into the Priſm, the Object was ſeen to riſe above the Hair gradually, as the Air entered; in the End, the Hair was found to hide a Mark in the Object $10\frac{1}{4}$ Feet below the Mark, as at P, ſo that P M = $10\frac{1}{4}$ Feet.

28. This done, the Condenſer was applied, and one Atmoſphere injected into the Priſm, which was applied to the Teleſcope, as before, and letting out the Air, the Object was ſeen to deſcend through the ſame Space of $10\frac{1}{4}$ Feet. Now ſince the Radius P I = 2588, and P M = 10,25, we ſhall find the Angle P I M = 68″, the half of which gives 34″ for the Angle Q D I, which

Sun X Y, which is, perhaps, one of the moſt ſurprizing and agreeable Spectacles of Nature.

taken from the Q D K or Q B D (= 32° = half the Angle of the Priſm) gives the Angle K D I or L D S = 31° 59′ 26″, and ſo the Sine of the Angle of Incidence in *Vacuo* (32°) is to the Sine of the Angle of Refraction into Air (31° 59′ 26″) as 1000000 to 999736. (See Mr. *Hawkſbee*'s own Figures in *Plate* 35. *Fig.* 7, 8, 9.)

29. In order to underſtand the Difference between the true and apparent Places of Objects, ſeen through a Medium of different Denſity from the Air, let the Scheme be conſtructed as in the Figure, where the Sines of Incidence and Refraction are H L and G I; and theſe are in a given Ratio of A to B, that is, H L : G I :: A : B; but becauſe of parallel Lines N Q K C, we have H L = N K; therefore N K : I G :: A : B :: N C : I C = H C; but N C : H C :: C E : C M, becauſe P E is parallel to K D, therefore C E : C M :: A : B. Plate XXXIV. Fig. 9.

30. Now ſince the Ray E C coming from an Object at E is refracted in the Air into the Ray H C; if H C be continued to F, the apparent Place of the Object will be in the refracted Ray at M in the Perpendicular E P, and projected to F on the horizontal Plane O R, but the Point M will always be the viſible Place of the Image; therefore when the Angle C E O is indefinitely ſmall, or the Point C coincides with O, the Lines C E and C M will become O E and O M; and in that caſe, O E : O M :: A : B :: 4 : 3, in Water. Whence 'tis evident that the apparent place of an Object immerſed in Water, and view'd in the Perpendicular, will be at $\frac{3}{4}$ of the Depth of Water.

31. But if the Medium be Glaſs, then O E : O M :: 3 : 2, or more nearly as 17 to 11; ſo that $\frac{11}{17}$ O E = O M, or the apparent Place of an Object ſeen through a Medium of Glaſs, will be at the Diſtance of $\frac{11}{17}$ of

THE ſeveral Sorts of Rays, after they are refracted, appear in their own proper

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the Thickneſs of the Glaſs O E. In Diamond, it would be at the Depth of $\frac{2}{5}$ of the Thickneſs, and ſo on for all the other Bodies mentioned in the foregoing Table.

32. On the other Hand, as the Point C recedes from the Point O, the Angle C E O which is equal to the Angle of Incidence E C D, becomes greater, and therefore alſo the Angle of Refraction H C K, or the refracted Ray H C will have a greater Inclination to the horizontal Line A B, and therefore alſo C F; on which Account 'tis evident the Diſtance of the apparent Place of the Object, *viz.* the Line O M, will decreaſe, and of Courſe the Object will ſeem to riſe in the Perpendicular. And when the Angle C E O is ſo great, that the Ray C H is refracted parallel to the Horizon or becomes coincident with A C, then C M will become C O, and the Object at the Bottom at E will appear on the Surface of the Medium at O.

33. In this Caſe, if the Medium be Water, we have C E : C O :: 4 : 3, whence we ſhall find O E = 2,65 nearly; therefore in any Veſſel, whoſe Width is 2 C O = 6, and Depth O E = 2,65, when fill'd with Water, any Object placed at the Bottom, when viewed in the Perpendicular, will appear raiſed from E to M, $\frac{1}{4}$ of the Depth; and as the Eye recedes from the Perpendicular to the horizontal Line A C, the Object will appear to riſe from M to the Surface of the Fluid at O; all which may be confirmed by pouring Water into a common Tea-Diſh, or Baſon, and viewing the Flower, *&c.* painted at the Bottom.

34. Hence appears the Reaſon why a ſtrait Stick, as N C E, when placed with one Part C E in Water, will always appear crooked, *viz.* in the Form N C M, the Part C E being raiſed by Refraction into the apparent Situation C M; and the Part under Water will always appear ſhorter, for E C will be contracted into C M. All which is known by common Experience.

Colours in Order as follows, *viz.* Thoſe which are leaſt refracted, or fall neareſt the

35. Alſo, ſince E M, the Difference between the true and apparent Place of Objects, ſeen through a Medium, is always greater in Proportion to the Depth O E, and the Obliquity of the Rays refracted to the Eye, it will follow, that any circular Body immerſed in Water, in a Poſition perpendicular or inclining to the Horizon, will ſuffer a greater Refraction of Rays from the lower Parts, than from thoſe above; and conſequently the lowermoſt Semicircle will put on the Appearance of a Semi-ellipſis; and alſo the upper one, but not ſo much ſo, the Refraction being leſs than below. The Conſequence of which is, that the Circle thus viewed in the Medium will appear elliptical, as having its vertical Diameter ſhortened by the Refraction; whereas the horizontal Diameter will remain of the ſame Length, being only raiſed apparently above its real Situation, whence the Reaſon of the Figure of the *horizontal Sun and Moon* above-mentioned.

36. From what has been ſaid, 'tis eaſy to underſtand, that when the Ray E C in the Medium is refracted into the Air nearly parallel or coincident with the Horizon A C, in which Caſe (if the Medium be Water) the Line C E = A C being Radius, we have the following Analogy; As 4 is to 3, ſo is Radius A C or C E to the Sine of the Angle of Refraction C O or D E; which Angle is therefore nearly 48°; I ſay, 'tis eaſy to underſtand, that if the Ray of Light E C fall on the Surface of the Medium with a greater Obliquity than what is here ſpecified, that is, ſo as to make the Angle E C D greater than 48°, the Ray will be wholly reflected back again to the lower Surface, and none will go out into the Air at either Surface of the Medium.

37. Again, if the Medium be Glaſs, ſince the Sines of Incidence and Refraction in that Caſe are as 11 to 17, the Angle E C D will be about 41°, when the refracted Ray C H becomes coincident with the horizontal Line A C; and therefore when the Angle is greater,

Perpendicular P E, are *Red*, and make the red Part of the *Spectrum* at R; the next are the *Orange* at O, the *Yellow* at Y, the *Green* at G, the *Blue* at B, the *Indigo* at I, and the *Violet* at V: And theſe Seven are all the *original ſimple Colours in Nature*; and of which, by various Mixtures, all others are compounded, in the common Refractions and Reflections from natural Bodies. (CXVIII.)

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the Light will be wholly reflected from one Surface of the Glaſs to the other; and never let out into the Air; whence it follows, that though the Particles of Matter in Bodies be in themſelves tranſparent, yet if they are ſo diſpoſed one among another as to reflect the Light very obliquely, 'tis plain, the Light in ſuch a caſe will be loſt by various Reflections within the Body, and thus prove a Cauſe of the Body's Opacity.

(CXVIII.) 1. This different Refrangibility of the Sun's Light proceeds from hence, that the Particles of Light are of different Degrees of Magnitude; for if any Power act upon a Body, ſo as to give it a particular Determination or Direction of Motion, that Determination or Direction of the Body's Motion will always be the ſame, while the Energy of the Power and the Quantity of Matter remain the ſame, and will be variable in Proportion as either of theſe is ſo.

2. But the refracting Power of the Medium will be always the ſame while it is homogeneous or all of one Sort of Matter, therefore when a Ray of Light paſſes through a Subſtance of Water, Glaſs, Cryſtal, &c. and a different Direction of Motion is thereby communicated to different Parts of the Ray, it follows, that the Particles which conſtitute thoſe Rays, which have a different Direction, muſt be among themſelves unequal in Quantity

SINCE a Lens does, in the manner of a Priſm, more or leſs ſeparate the Rays of

of Matter, and conſequently in Bulk; and ſince the Quantity of Motion is in the Ratio of the Bulk and Velocity, (in this Caſe) 'tis plain, the greater the Velocity is, the leſs will be the Bulk; and therefore thoſe Rays of Light which ſuffer the greateſt Refraction are leſs in Bulk or Magnitude than others which are not ſo much refracted, the greater Particles being not ſo much ſubject to the Power of the Glaſs; as a large Needle is not ſo eaſily moved by a Loadſtone, nor at ſo great a Diſtance.

Pl. 48. Fig. 1.

3. This being the Caſe, 'tis eaſy to be underſtood, that when a Beam of Light, as H F, is let into a dark Room, through a Hole in the Window-Shutter, and is made to fall on a Priſm A C B at F, it will be attracted by the Surface of the Glaſs at F in a perpendicular Direction, and cauſe the ſeveral Particles to deviate from their right-lined Courſe to T, (which they before had) and decline towards the Perpendicular *a b*, that is, towards the Parts F *a* within the Glaſs; which Deviation or Refraction will be greater in Proportion as the Particles of Light are ſmaller.

4. Hence the ſeveral Particles of Light will proceed from the Side A C to the Side D C in different Directions; where, when they arrive, and go out again into the Air, they will be again affected by the ſame attracting Power of the Glaſs, which will here produce the ſame Effect as before, that is, it will cauſe each Sort of Ray to incline towards the Side of the Glaſs, and conſequently to be refracted from the Directions they ſeverally had in the Glaſs, and from the Perpendicular P E.

5. Thus thoſe Rays, whoſe Particles are largeſt, will deviate leaſt from the Perpendicular, and will therefore go to R, and make the loweſt Part of the colour'd *Spectrum*, and theſe will appear of a *Red Colour*. The Particles next leſs in Magnitude will be ſomewhat more refracted, and will go to O, and be of an *Orange Co-*

Light paſſing through it, it follows, that all the ſeveral Sorts of Rays will have

lour; the next Size leſs will be ſtill more refracted, and appear *Yellow* at Y, and thus the Refraction will proceed in the *Green* at G, the *Blue* at B, the *Indigo* at I, and the *Violet-coloured Rays* at V; which as they are moſt refracted, are thereby proved to be the leaſt of all in Magnitude.

6. I ſhall now proceed to ſhew, ſince the Sun's Light is variouſly refrangible, what the particular Degree of Refraction is which every Species of Rays undergoes, and the Sines of thoſe Angles reſpectively. In order to this it muſt be conſidered, that the Sine of Incidence is the ſame in all; and that when the Incidence is ſuch as that the Ray F K, upon the firſt Refraction, ſhall paſs in the Direction parallel to the upper Side of the Priſm G H, the Refractions made at each Side of the Priſm are equal, and equal to the refracting Angle of the Priſm G D H; all which is evident from what was demonſtrated in *Annot.* CXVI. Plate XXXVI. Fig. [illegible]. 5.

7. Alſo it is known by Experience, that when the Priſm A B C is held with its Axis perpendicular to the Sun-Beam, and then turned round upon its Axis, the Image or coloured *Spectrum* will firſt deſcend to a certain Limit, where it will become ſtationary, and then aſcend to the ſame Place as at firſt; whence it appears plain, that ſince the Altitude of the Image above the Place where the Sun-Beam would fall, were the Priſm away, is owing to the Sum of the Refractions made at each Side of the Priſm, while the Image deſcends the Sum of theſe Refractions decreaſe, and when the Image aſcends the ſaid Sum muſt increaſe.

8. Conſequently, ſince the Image falls twice upon the ſame Place in one Rotation of the Priſm, there are two Poſitions of the Priſm wherein the Sum of the Refractions at its Sides are equal; and theſe happen when the Angles of the incident Beam H D L and O D L are ſuch as will cauſe the refracted Parts D G and D F to be equally inclined to the Sides of the Priſm, but con- Fig. 9.

their proper *Focus*'s, or be convened to ſo many different Points in the Axis of the

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trary Ways; that is, ſo as to make the Angle D G B = B D F, and G D B = D F B, and therefore the Triangles D B G and D B F equi-angular. For in the Poſition of the Ray H D G the Refractions at the Angles D and G are reſpectively equal to the Angles E and D in the other Situation of the Ray O D E; and therefore the Sum of the Refractions on each Side in each Caſe muſt be equal, and cauſe the Image to appear twice in the ſame Place.

9. While the unequal Refractions at each Side the Priſm, at D and G, or D and F, are approaching towards Equality, the refracted Ray D G or D F is continually approximating to the Situation D E; where when it arrives, the Angles at D and E being then equal, the Refractions at each Side will be equal alſo, and the Image in that Caſe be brought to its Limit or loweſt Site. Then R D will be the incident Ray, and E P the emergent one.

10. Produce R D and E P till they interſect each at I, and any horizontal Line in M and N; then let the Angle R M N be the Altitude of the Sun, and P N M that of the *Spectrum* at P; which Angles are eaſily meaſured with a Quadrant. Their Sum is equal to the external Angle P I M, which is again equal to the two internal Angles of Refraction IDQ and IEQ; and, by what has been now ſhewn, IDQ = IEQ = QDK; wherefore Q D K = D B K = $\frac{1}{2}\overline{N+M}$. Hence $\frac{1}{2}\overline{N+M}$ + I D Q = I D K or R D L, the Angle of Incidence.

11. We ſhall give Sir *Iſaac Newton*'s Example in this Affair. The refracting Angle of his Priſm was A B C = 62° 30′, the Half of which is 31° 15′, whoſe Sine is 5188, the Radius being 10000. When the *Spectrum* was in its Limit, or ſtationary, he obſerved with a Quadrant the Angle P N M of a mean refrangible Ray E P, that is, of one that went to the Middle of the colour'd Image at P; and by adding this to the Angle R M N of the Sun's Altitude taken at the ſame Time, he ob-

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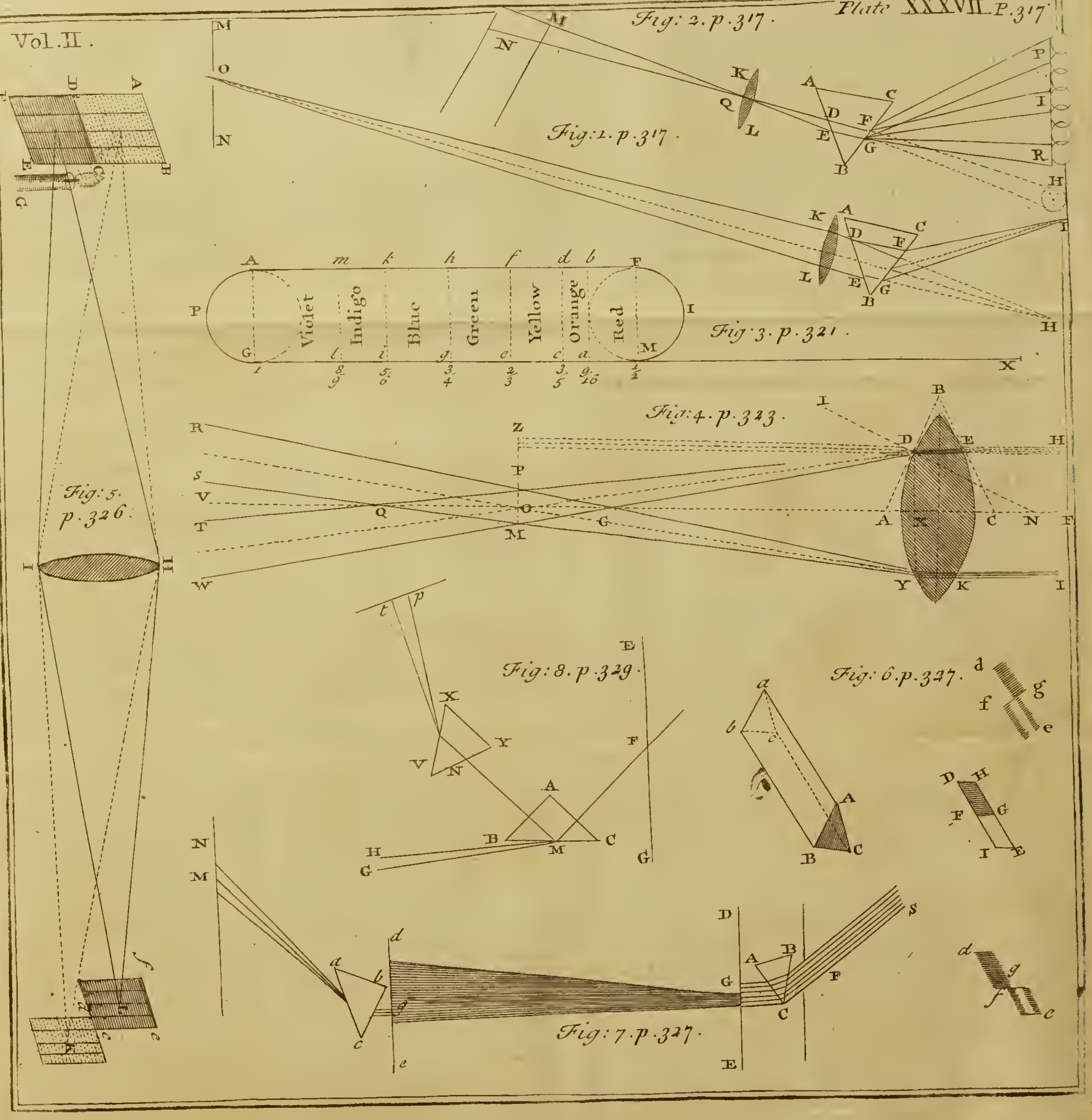

Fig: 1. p. 317.

Fig: 2. p. 317.

Fig: 3. p. 321.

Fig: 4. p. 323.

Fig: 5. p. 326.

Fig: 6. p. 327.

Fig: 7. p. 327.

Fig: 8. p. 329.

Lens, and not all to one Point only, as is neceſſary for a perfect and uniform Repre-

tained the Angle P I M to be 44° 40′; whoſe Half 22° 20′, added to Half the Angle of Refraction 31° 15′, makes the Angle of Incidence R D L = 53° 35′, whoſe Sine is 8047. The Sine of Incidence, therefore, is to the Sine of Refraction of a mean refrangible Ray, or that of yellow Light, as 8047 to 5188, which is as 31 to 20. (See the Table, *Annot.* CXVII. 24.)

12. If there were but one Sort of Light, it then would be equally refracted, and the Image of the Sun would not then be long, but round; and if the Rays were firſt received by a Convex Lens, they would all paſs to its Focus, and there repreſent the Sun's Image very diſtinctly in a circular Spot, which Image would ſubtend the ſame Angle at the Lens as the Sun itſelf does, or half a Degree, at a Mean. All this will be demonſtrated hereafter.

13. If theſe Rays, after having paſs'd through the Lens, were received by a Priſm, ſince the Sum of Refractions at the Sides of the Priſm are equal, (as we have ſhewn they are when the Image is ſtationary, *Art.* 8.) the Rays will have the ſame Inclination to each other after Refraction through the Priſm as before; whence the Angle is not changed, but gives the Image of the Sun ſtill equal to 30′. But to illuſtrate this, let M N be the Section of the Window-Shutter in a dark Room, in which, through a Hole O, a Pencil of Rays K O L is tranſmitted to the Lens K L; which would converge them to a Focus at H, were they not intercepted by the Interpoſition of the Priſm A B C, by which means they are refracted to I. And ſince the Sum of the Refractions at E and D is equal to that at F and G, the Angle F I G will be equal to the Angle F H G; and if the Sun were but a Point, its Image at H and I would be a Point alſo.

Plate XXXVII. Fig. 1.

14. But ſince the Sun has the apparent Magnitude of 30′, let the Angle M Q N be the Angle under which the Sun appears; that is, let M Q be a Ray coming

Fig. 2.

ſentation of the Image of any Object: For the Red Rays proceeding from the Object

from the upper Limb of the Sun, and N Q another from the lower Limb. Theſe croſſing each other in the Center of the Lens K L, at Q, make the Angle D Q E = M Q N; nor is this Angle altered by the Refractions through the Priſm, as being equal on each Side; therefore the Image at I will be ſubtended under an Angle of 30 Minutes.

15. And ſince this will be the Caſe of every Sort of Rays contained in the Sun's Light, if that which we have been conſidering be a mean refrangible Ray, then the leaſt refrangible Rays will form an Image in like manner at R, the moſt refrangible Rays another at P, and the intermediate Rays their ſeveral Images reſpectively. So that the coloured *Spectrum* P R conſiſts of as many circular Areas as there are different Sorts of Rays; and is every where of an equal Breadth, *viz.* half a Degree.

16. Now 'tis evident, that if the Sun be ſuppoſed a Point, each of thoſe Circles, being the Images of the Sun and ſimilar to it, muſt alſo be contracted into a Point, and ſo the coloured *Spectrum* P I R would in that Caſe have no Breadth; and its Length would decreaſe at each End by the Semidiameter of the Circle P and R, and therefore would ſubtend an Angle of 30′ leſs than it now does.

17. In order to determine the Angles of Refraction of the leaſt and moſt refrangible Rays, we muſt firſt determine the Angle P I R, which the Image P R ſubtends at the Diſtance it is formed from the Priſm A B C. The Sun being ſuppoſed a Point, let S D be the Incident Ray, which continue out to V; and let L D K be perpendicular to the Side A B in the Point of Incidence D. The Ray S D at its firſt Refraction is diffuſed through the Space G D F within the Priſm; D G is the leaſt refrangible Ray, D F the greateſt, and D E (parallel to A C) the mean refrangible Ray. The Sine of the Angle of mean Refraction E D K to that

Plate XXXVI. Fig. 10.

will be converged to a *Focus* at a greater Diſtance from the Lens, than the Indigo

of Incidence I D K has been already ſhewn to be as 5188 to 8047, or as 20 to 31.

18. We are now to find the Quantity of the Angles G D K and F D K. Since each Ray will ſuffer the ſame Degree of Refraction at the ſecond Surface as at the firſt very nearly; let the refracted Rays F P, E T, G R, be produced, and they will interſect each other in the Point I, making I F, I E, I G ſeverally very nearly equal to D I, and therefore the Angles I F D = I D F, and I G D = I D G; therefore P I V = 2 F D V, and R I V = 2 G D V. Hence P I V — R I V = P I R = 2 F D V — 2 G D V; conſequently, $\frac{1}{2}$ P I R = F D V — G D V = F D G.

19. The Angle P I R is diſcovered by meaſuring the Length of the Image P R, and its Diſtance from the Priſm A B C. This Sir *Iſaac Newton* has done with great Exactneſs. The refracting Angle of his Priſm was A B C = 62° 30′, the Diſtance of the *Spectrum* 18$\frac{1}{2}$ Feet, the Length 9$\frac{3}{4}$ or 10 Inches, the Breadth 2$\frac{1}{3}$ Inches. This ſubducted from the Length leaves 7$\frac{3}{4}$ for the Length of the Image were the Sun but a Point, and therefore ſubtends the Angle which the moſt and leaſt refrangible Rays P F and R G do contain with one another after their Emergence from the Priſm.

20. But at the Diſtance of 18,5 Feet, the Length 7$\frac{3}{4}$ Inches is the Chord of an Arch equal to 2 0′ 7″ = P I R; therefore $\frac{1}{2}$ P I R = F D G = 1 0′ 3$\frac{1}{2}$″; whence E D G (= $\frac{1}{2}$ F D G) = 0° 30′ 2″ = F D E. But the Angle E D K = D B K = 31° 15′ 0″. Wherefore E D K + E D G = 31 45′ 2″ = G D K, the Angle of Refraction of the leaſt refrangible Rays; and E D K — E D F = 30° 44′ 58″ = E D K, the Angle of Refraction of the moſt refrangible Rays.

21. The natural Sine of 31° 45′ 2″ is 5262, (as *per* Table, *Annot.* XLVI) alſo the Sine of 30 44′ 58″ is 5112. The common Sine of Incidence being I D K or S D L = 53 35′, and Sine 8047; this compared

or Violet Rays; and ſo the Image will be coloured and confuſed in every Point be-

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with the Sines of Refraction of the moſt, mean, and leaſt refrangible Rays will ſtand as follows.

The Sine of Incidence is to the Sine of	The moſt refrangible Rays F D, as 8047 to 5112.
	The mean refrangible Rays E D, as 8047 to 5188.
	The leaſt refrangible Rays G D, as 8047 to 5262.

22. I have hitherto conſidered the Refraction made out of Air into Glaſs, after the common Way. But as Sir *Iſaac Newton* has proceeded in a contrary Method, and ſtated the Proportions of the Sines of Refraction (as they are out of Glaſs into Air) to the common Sine of Incidence in Glaſs, I ſhall for the future follow his Steps; and therefore ſuppoſing a Beam of common Light within the Priſm, as D E, ſhall conſider its Refraction into the Air at the Side B C in the Point E. The common Sine of the Angle of Incidence K E D or I E L, = 31° 15′, was found to be 5188; and the Angle P E R, = 1° 0′ 3½″, the ſame as before. Alſo the Angle of the mean refrangible Rays T E L being 53° 35′, we have the Angle of the leaſt refrangible Rays R E L = 53° 4′ 58″, and the Angle of the moſt refrangible Rays P E L = 54° 5′ 2″. The Sines of theſe Angles are 7995 and 8099; the Sine of Incidence, therefore, and of Refraction into Air, in the leaſt and moſt refrangible Rays, are in the leaſt round Numbers as 50 to 77 and 78.

Plate XXXVI. Fig. 4. 11

23. Now if you ſubduct the common Sine of Incidence 50 from the Sines of Refraction 77 and 78, the Remainders 27 and 28 ſhew, that in ſmall Refractions, the Refractions of the leaſt refrangible Rays is to that of the moſt refrangible as 27 to 28 very nearly; and that the Difference of the Refractions of the leaſt and moſt refrangible Rays is about the 27½ Part of the Refraction of the mean refrangible Rays.

tween thoſe Extremes, except juſt in the middle Point, where the ſeveral Sorts of

24. Now in order to determne the Refrangibility of the ſeveral intermediate Rays of Light, Sir *Iſaac* took the following Method. He cauſed the *Spectrum* to be well defined, and delineated upon Paper its Perimeter, as FAPGMT; this he held in ſuch a Manner, that the *Spectrum* might fall upon and exactly agree with the delineated Figure; this done, an Aſſiſtant drew the Lines *a b*, *c d*, *ef*, &c. across the Figure very nicely upon the Confines of the ſeveral Colours, that is, of the Red M *a b* F, of the Orange *a b c d*, of the Yellow *c d e f*, and ſo of the reſt; which Operation being divers times repeated, he found the Obſervations agreed very well, and that the Diviſions made by the Croſs Lines were thoſe of a *Muſical Chord.* Plate XXXVII. Fig. 3.

25. That is, if G M be produced to X, ſo that it be G M = M X, then the Line X M = $\frac{1}{2}$ X G will be the *Octave.* The Line *a* X : X G :: 9 : 16; therefore *a* X will be the *Leſſer Seventh.* The Line *c* X will be $\frac{3}{5}$ of X G, and therefore the *Sixth Greater.* *e* X will be $\frac{2}{3}$ of X G, which is the *Fifth.* *g* X is $\frac{3}{4}$ of X G, a *Fourth.* *i* X is $\frac{5}{6}$ of X G, a *Third Leſſer.* *l* X is $\frac{8}{9}$ of X G, the *Second Greater.* So nicely has Nature obſerved an harmonical Diſtribution of Colours in the *Solar Spectrum.*

26. If then the Difference between the Sines of 77 and 78 be in like manner divided, that is, as the Line M G is divided, we ſhall have the Sines of Refraction in the ſeveral Red Rays extend from M to *a*, or from 77 to $77\frac{1}{8}$; thoſe of the Orange Colour from *a* to *c*, or from $77\frac{1}{8}$ to $77\frac{1}{5}$; thoſe of the Yellow from $77\frac{1}{5}$ at *c*, to $77\frac{1}{3}$ at *e*; thoſe of the Green from $77\frac{1}{3}$ to $77\frac{1}{2}$ at *g*; thoſe of the Blue from $77\frac{1}{2}$ to $77\frac{2}{3}$ at *i*; thoſe of the Indigo from $77\frac{2}{3}$ to $77\frac{7}{9}$ at *l*; and from thence the Violet to 78 at G.

27. This Diſcovery of the *Harmonic Proportion* of Colours in the Sun's Light has ſuggeſted the curious Hint or Idea of a *Viſual Muſic* by means of an *Ocular*

Rays all interſect each other, and exhibit the Image tolerably diſtinct and colourleſs. To this different Refrangibility of the Rays is owing the Imperfection of the common *refracting* Teleſcope, as will be but too eaſy to experiment.

HENCE alſo Objects of any of the ſimple Colours, though contiguous to each other, yet, if viewed through a Priſm, appear ſeparated, and at a diſtance from one another: And thoſe Objects will have their Images formed by a Lens at very different Diſtances in its Axis, eſpecially in Experiments of the deepeſt *Red*, and *Violet*, or *Blue Colours*; as a Card painted half with *Carmine*, and half with *Ultramarine*, made deeper with a little *Indigo*. (CXIX.)

Harpſichord, which ſhall entertain the Eye with the Succeſſion of harmonic Colours, as the common Harpſichord does the Ear with muſical Sounds. Yea, ſome have carried this Matter ſo far, as actually to attempt the making of ſuch a Harpſichord, with full Aſſurance of being able to play Tunes to the Eyes. It were greatly to be wiſhed this Chromatic Muſic could be made as effectual to give Pleaſure to our Eyes, as common Muſic does to the Ears. We ſhould then have Harmony the Subject of two of our Senſes: And who can tell but we may have *muſical Eyes*, as well as *muſical Ears*, could they be exerciſed by proper Objects? Nay, who can tell what may be the Conſequence of this Diſcovery in regard of our other Senſes in the Ages to come? They who would be farther amuſed with this chimerical Subject may conſult *Voltaire*'s Philoſophy and *Algarotti* on Light and Colours.

Sir *Iſaac Newton* found, by a very curious and convincing Experiment, that the Rays

(CXIX.) 1. Let D E K I be a double Convex Lens; N its Center, and N D the Radius of Convexity at D; F V its Axis, and H E a Beam of the Sun's Light incident on the Lens parallel to its Axis in the Point F. Let A B C be a Priſm touching the Lens in the Points E and D, and it is evident the Law and Manner of Refraction of the Beam at E will be the ſame, whether we conſider it as made through the ſolid Glaſs Priſm A B C, or through the Lens D E K, becauſe the Point of Incidence E is the ſame in or common to them both. Plate XXXVII. Fig. 4.

2. The Beam being refracted at D, it is plain the Refraction will be there alſo made into the Air in the ſame Manner from the Lens as from the Priſm, ſuppoſing them to touch in the Point D. Let N D be continued to L, then will L D be perpendicular to the Lens in D; and the Refraction being made into the Air, (a rarer Medium) the refracted Rays will tend towards the Axis, and meet it ſooner or later as they are more or leſs refrangible. Thus the moſt refrangible Rays D W will cut the Axis in G, the leaſt refrangible Rays D T in Q, and the mean refrangible Rays in O; and the others in the intermediate Space between O and G, and O and Q. The ſame is to be underſtood of the Beam I K on the other Side the Axis.

3. Hence we ſee, that in the Axis of the Lens the Images of an Object will be formed in ſeveral Parts from G to Q; by which means the Object will appear *Red* at G, *Violet-colour'd* at Q, and of other Hues in the Parts between. Nor are we to underſtand that ſeven Images only are formed by the ſeven Sorts of Rays; but each particular Sort of Ray, according to the Intenſity of the Colour, from the ſtrongeſt to the fainteſt Part, conſiſts of an indefinite Number of differently refrangible Rays, each of which will form an Image of the Object in its proper Focus: And therefore we may conceive as many Images formed in the Space from G to Q, as there are Points in the Line GQ.

of

of Light were as variouſly reflexible as refrangible; and that thoſe which were moſt

4. The Object ſeen by ſuch a Refraction of Rays, in ſuch an Infinity of Images, muſt neceſſarily appear very indiſtinct, confuſed, coloured, and obſcure; and the Object-Glaſs of every common Dioptric Teleſcope being of this Sort, is the Occaſion why they will not bear an Eye Glaſs of ſo deep a Charge, or ſo ſhort a focal Diſtance as is requiſite for great Degrees of magnifying. This put Sir *Iſaac* upon inventing another Sort of Teleſcope by Reflection, of which we ſhall ſpeak largely hereafter.

5. Suppoſe D E parallel to the Axis of the Lens, and produced to Z; then is Z D L = E D N the Angle of Incidence, and P D L, O D L, M D L, the Angles of Refraction in the leaſt, mean, and moſt refrangible Rays; and conſequently the Angles Z D P, Z D O, and Z D M will ſhew the Quantity of Deviation or reſpective Refraction of thoſe Rays from the firſt Direction E Z. Whence Z P : Z M :: 27 : 28; and Z P : Z O :: 27 : $27\frac{1}{2}$. Alſo $PM = \frac{1}{27\frac{1}{2}} ZO$, the whole Refraction of the mean refrangible Rays.

6. But the Angle Z D P = D Q X, and Z D O = D O X, and Z D M = D G X. Now the Sines of the Angles D Q X, and D O X or D O Q, are as their oppoſite Sides O D and Q D, that is, nearly as X O and X Q. For the ſame Reaſon the Sines of the Angles D O X and D G X are nearly as G X and O X. Wherefore Z P, Z O, Z M, are as G X, O X, and Q X; whence P M : Z M :: Q G : Q X; therefore $GQ = \frac{1}{28} QX$. But Q O is nearly equal to O G, when Q X is very great; and Q O : Q X :: P M : D Y :: 1 : 5', becauſe Q X = 28 G Q, or 56 Q O.

7. Or thus more accurately, without regard to the focal Diſtance Q X or O X. Let I, L, G, be as the Sines of Incidence, and of the leaſt and greateſt Refraction, or as the Numbers 50, 77, 78; (See *Annotat.*

or leaſt refrangible were alſo moſt or leaſt reflexible: And farther, that Rays of Light

CXVII. 22.) then will $ZP = L - I$, $ZM = G - I$, and $PM = G - L$; whence $PM : ZP :: G - L : L - I$; and doubling the Conſequents, we have $PM : 2ZP (= DY - PM) :: G - L : 2L - 2I$. Then conjointly, $PM : DY :: G - L : G + L - 2I :: 78 - 77 : 78 + 77 - 100 :: 1 : 55$; or $PM = \frac{1}{55} DY$, the Aperture of the Glaſs.

8. From hence it appears, that the Ratio between PM and DY is conſtant, or always the ſame, whatever be the focal Diſtance of the Lens. It is alſo very evident, that PM is the Diameter of a Circle, in which will be a Mixture of every Sort of Rays, from the leaſt to the moſt refrangible. This Circle therefore is that in which the Light is *white*, or not tinctured with the Colour of any particular Sort of Rays; for the Rays being here promiſcuouſly thrown together, the Light compounded of them muſt be nearly the ſame with that of the Beam before Refraction.

9. By the ſame Rule we may find the Diameter of the leaſt Circle that receives the Rays of any ſingle Colour, or of any contiguous Colours. Thus all the *Yellow* is contained in a Circle whoſe Diameter is a 409th Part of the Breadth of the Aperture of the Glaſs, (which we ſuppoſe a *Plano-Convex*, becauſe of DE parallel to the Axis) for in this Caſe $G = 77\frac{1}{3}$, $L = 77\frac{1}{5}$, and $I = 50$. (See *Annot.* CXVII. 26.) Whence by the Analogy we have $PM : DY :: G - L : G + L - 2I :: 0{,}133 : 54{,}533 :: 1 : 409$. Thus for two contiguous Colours, the *Orange* and *Yellow*; the Sines on each Side being $77\frac{1}{8}$, $77\frac{1}{5}$, give the Diameter of the Circle in which both theſe coloured Rays are contain'd, a 260th Part of DY.

10. From hence it is plain, that when the Sun's Rays are received upon a large and very convex Lens, the conic Superficies of the converging Rays DPMY will conſiſt of the the Red-colour'd Rays; and if received on a white Paper, held perpendicular to the Axis, the

were

were not reflected by impinging on the ſolid Parts or Corpuſcles of Bodies, and

Circumference of the circular Section or Area will be remarkably tinged with a reddiſh Colour inclining to Orange, by the leaſt refrangible Rays D P and Y M. On the contrary, the diverging Rays will, in the conic Surface R P M W, have all the Violet and Indigo Rays N P K and O M W, and will therefore exhibit ſuch a colour'd Circle about the Light received on the Paper held any where in that Cone of Rays.

11. Since G is the Focus of Violet Rays, that is the Place where any Body of a Violet Colour will be ſeen diſtinctly, becauſe the Rays of that Kind, paſſing from that Point to the Lens, will after Refraction paſs parallel to the Eye; which is a Condition abſolutely neceſſary to diſtinct Viſion, as will appear hereafter. For the ſame Reaſon Q will be the Focus or Place where Objects of a red Colour will be moſt diſtinctly ſeen. Whence it appears, that in viewing Objects through Glaſſes (as *Spectacles* for Inſtance) the Diſtance of the Glaſs from the Object will be variable according to its different Colour.

Plate XXXVII. Fig. 5. 12. Hence a various-colour'd Object A B E F will have its Image formed in Parts by the Lens H I. Thus ſuppoſe A B C D be a Red Part, and D C E F a deep Blue; if this Object be well illumined, and black Threads or Silks laid acroſs thoſe Colours, they will appear diſtinctly in their reſpective Focus's, *viz.* the red Part A B C D will have its Image diſtinctly formed at K, and the blue Part at L; the former will be repreſented by *a b c d*, the latter by *d c e f*; and theſe Images will be at very different Diſtances from the Lens. Thus if the Lens H I be of 3 Feet *focal Diſtance*, and the Object be placed at the Diſtance of 6 Feet from it, the Images on the other Side, at the Diſtance of 6 Feet, will be formed *one Inch and a half* from each other; that is, the Red at K will be 1½ Inch beyond the Blue at L.

rebounding from thence like a *Tennis-Ball*, but from ſome other Principle depending

13. Another Conſequence of this different Refrangibility of the Rays of Light is, that if two Objects of different Colours, as *Red* and *Blue*, be view'd thro' a *Priſm*, they will be refracted to different Heights; and though they were contiguous before, or Parts of *one and the ſame Object*, yet will they appear ſeparate, or as two *diſtinct and diſtant Objects*. Thus ſuppoſe D H E I be an Object whoſe Part D G is intenſely *blue*, and the other Part F E intenſely *red*; if this be viewed by a Priſm B A C *b a c*, with the refracting Angle or Edge A *a* upwards and parallel to the Horizon and the Sides D I and H E of the Object, the Image of this Object will appear at d e, with the *blue* Part d g refracted higher than the *red* f e. On the contrary, if the refracting Angle of the Priſm be turned downwards, the Image will be refracted downwards to *d e*, the *blue* Part lower to *d g*, and the *red* higher at *f e*. Plate XXXVII. Fig. 6.

14. We alſo ſee the Reaſon why Objects appear differently coloured when the Eye is held near the Priſm, as at D, to view them, *viz.* becauſe the Rays of every Colour are there ſo very near together, that they can be all received by the Pupil of the Eye, and will therefore paint the Image in all its proper Colours on the *Retina*. Whereas if the Eye be removed to a greater Diſtance from the Priſm, as to *a*, *b*, *c*; there, becauſe the Rays ſpread through ſo wide a Space, but few can enter the Pupil, perhaps only one particular Sort, and then the Object will appear of that particular Colour only; as *Violet-colour'd* at *a*, *Green* or *Yellow* at *b*, and *Red* at *c*.

15. The Rays of the Sun's Light, once refracted, undergo no farther Refraction by a ſecond Priſm, and of courſe exhibit no other Colours: For let an Hole be made at *g* in the Board *d e*, on which the coloured *Spectrum* is made in the dark Room, by Rays which come thro' a Hole G in a Board D E placed juſt before the Priſm A B C; by turning the Priſm A B C ſlowly about its Axis, the Image will be made to move up and down Fig. 7.

on the Size of the Particles of Light, and the Thickneſs or Denſity of the Particles of the Body reflecting it, which are all of them, in the moſt opake Bodies, tranſparent in themſelves, as is eaſy to be ſhewn in the thin *Lamellæ* or Plates, of which an Oyſter-Shell doth conſiſt.

IT will be thought very ſtrange to aſſert, that a rare Medium is more impervious to the Rays of Light than a denſer one; and yet nothing is more certain, or eaſier proved by Experiment: For Example, a Beam of Light is much more copiouſly reflected from

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on the Board *d e*, by which means each colour'd Ray will paſs ſingly through the Hole *g* ſucceſſively; and if theſe Rays be refracted a ſecond time through the Priſm *a b c* placed juſt behind the Hole *g*, they will go from thence to the oppoſite Wall at M or N, and there appear juſt as before in their proper ſimple Colour; the *Blue* will appear *Blue*, the *Red* will be ſtill *Red*, and the *Violet* the ſame *Violet* as before.

16. But though the Rays are not any farther refrangible by the ſecond Priſm *a b c*, yet it appears that thoſe Rays which were leaſt and moſt refrangible by the firſt Priſm are likewiſe ſo by the ſecond; for the Boards D E and *d e* being fixed, cauſe the Incidence of Light on the ſecond Priſm to be always the ſame: Yet by moving the firſt Priſm A B C about its Axis, the *Red* Light would go by a ſecond Refraction to M, but the *Violet* Light would go higher to N. Which plainly and undeniably ſhews that ſome Sort of Rays will always be more refracted, and are therefore more refrangible than others. And hence this deciſive Experiment has gain'd the Title of *Experimentum Crucis*.

the ſecond Surface of a Piece of Glaſs when contiguous to the *Air*, than when it touches *Water*; and ſtill more, if contiguous to *Water*, than when it is contiguous to *Glaſs*; in which Caſe the Rays are totally tranſmitted.

HENCE, wonderful as it may ſeem, 'tis neceſſary, in order that a Body may be tranſparent, that its Subſtance ſhould be very denſe, and its Pores very ſmall; and that Opacity reſults chiefly from the Largeneſs of the Pores of a Body, occaſioned by its Particles touching in but very few Points: Becauſe, if the Pores of ſuch a Body be fill'd with a Subſtance nearly of the ſame Denſity, it becomes in ſome Degree tranſparent, as *Paper wetted with Water or Oil*: And on the contrary, *Water blown up into ſmall Bubbles* has its Denſity diminiſhed, and its Poroſity increaſed, and thus becomes opake (CXX.)

(CXX.) 1. The ſame great Author of the Doctrine of the *different Refrangibility* of the Sun's Rays, (as delivered in the laſt *Annotation*) found alſo by other Experiments, that they were in the ſame Manner *differently reflexible*; or that thoſe Rays which were leaſt and moſt refrangible were alſo leaſt and moſt reflexible. This he proved in the following Manner.

2. From a Hole F in the Window-Shutter E G, a Beam of the Sun's Light F M paſs'd to the Baſe B C of a Priſm A B C, whoſe Angles B and C were equal

Plate XXXVII. Fig. 8.

IF the Object-Glaſs of a large Teleſcope be laid with its convex Surface on a plain

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and half right ones, and the Angle A a right one. The Light was firſt refracted at M into the diverging Beam M G H, of which M G was the leaſt refrangible Part, and M H that which was moſt ſo. M N is the Light reflected from the Baſe through the other Side to a ſecond Priſm V X Y, by which the reflected Beam is refracted to *t* and *p*; N *t* being the leſs, and N *p* the more refracted Part.

3. When the firſt Priſm A B C is turned about its Axis according to the Order of the Letters A B C, the Rays M H emerge more and more obliquely out of the Priſm, till at length they become reflected towards N. And it was evidently obſerved, that as the Priſm A B C was ſlowly moved about its Axis, all the Rays from M H to M G became ſucceſſively reflected towards N.

4. The Conſequence of this was, that the Violet Colour *p* received an Addition to its Strength and Brightneſs upon the firſt Reflection of the Rays M H, beyond any of the other Colours towards *t*; but as the Priſm A B C continued its Motion, and the other Ray between H and G became reflected, ſo the other Colours from *p* to *t* became more intenſe and vivid, one after another, by the new Acceſſion of Light to the Beam M N.

5. In this Experiment no Notice has been taken of any Refraction made at the Sides of the firſt Priſm A B C, becauſe the Experiment was made in ſuch Circumſtances that the Beam F M enters it perpendicularly at the firſt Side A C, and goes out ſo at the ſecond A B; and therefore can ſuffer no Refraction, or ſo little that the Angles of Incidence at the Baſe are not ſenſibly altered by it. In order to this, the Angle F M C ſhould be about 45 Degrees; and then a ſmall Motion of the Priſm, to make the Angle F M C = 49°, will cauſe the Beam F M to begin its Refraction. Or if the Angles B and C were each of them 41°, the Sun-Beam F M making an Angle F M C = 49° will begin

Glaſs, the Light falling on the thin Portion or Plate of Air contained between

to be reflected at the ſame Time that it contains a Right Angle with the Side A C (by *Annot.* CXVII. 37.)

6. The Reaſon of the different Reflexibility of the Rays of Light is the ſame as was before aſſigned for their different Refrangibility, *viz.* the different Sizes or Magnitudes of the ſeveral Rays; for when the refracted Beam M G H approaches very near the Baſe of the Priſm B C, the attracting Power of the ſaid Baſe will ſooner affect the Particles of a leſſer Size than thoſe of a larger, even though they were at an equal Diſtance from it; and therefore the moſt refrangible Rays M H will be firſt within the reflexive Power of the Surface B C, on account of the greater Tenuity of its Particles, as well as on account of its greater Proximity than the other Rays M G; on both which Accounts therefore the Ray H M will be firſt and moſt eaſily refracted.

7. Now though in Refraction the Sine of the Angle of Incidence is different from the Sine of the Angle of Refraction, on account of a ſmaller Particle being attracted more out of its Way towards the Perpendicular than a larger one, whereby a Separation of the Rays is produced; yet becauſe in Reflections every Particle whether great or ſmall muſt neceſſarily be reflected under an Angle equal to that of Incidence, it follows, that all the Rays after Reflection will have the ſame Inclination to each other as before, and ſo no Separation can be made among them, and conſequently no different colour'd Light will be produced by a total Reflection of the Sun's Rays.

8. What has been ſaid of the Manner in which Light is reflected is in the groſs only, and true but in part; for though in Reflections the Angle of Incidence be ever equal to the Angle of Reflection, yet the Reflection of the Particles of Light is not made by their impinging on the ſolid or impervious Parts of Bodies, as is commonly believed. This our great Author proves by the following Reaſons.

the Glaſſes will be, at ſeveral Diſtances from the Centre, alternately tranſmitted

9. *Firſt*, That in the Paſſage of Light out of Glaſs into Air, there is a Reflection as ſtrong as in its Paſſage out of Air into Glaſs, or rather a little ſtronger, and by many Degrees ſtronger than in its Paſſage out of Glaſs into Water. And it ſeems not probable that Air ſhould have more ſtrongly reflecting Parts than Water or Glaſs: But if that ſhould be ſuppoſed, it will avail nothing; for the Reflection is as ſtrong or ſtronger when the Air is drawn away from the Glaſs by an Air-Pump, as when it is adjacent to it.

10. *Secondly*, If Light in its Paſſage out of Glaſs into Air be incident more obliquely than at an Angle of 40 or 41 Degrees, it is wholly reflected; if leſs obliquely, it is in a great Meaſure tranſmitted. Now it is not to be imagined that Light at one Degree of Obliquity ſhould meet with Pores enough in the Air to tranſmit the greater Part of it, and at another Degree of Obliquity ſhould meet with nothing but Parts to reflect it wholly; eſpecially conſidering, that in its Paſſage out of Air into Glaſs, how oblique ſoever be its Incidence, it finds Pores enough in the Glaſs to tranſmit a great Part of it.

11. If any Man ſuppoſe that it is not reflected by the Air, but by the outmoſt ſuperficial Parts of the Glaſs, there is ſtill the ſame Difficulty; beſides that ſuch a Suppoſition is unintelligible, and will alſo appear to be falſe by applying Water behind ſome Part of the Glaſs inſtead of Air: For ſo in a convenient Obliquity of the Rays, as of 45 or 46 Degrees, (at which they are all reflected where the Air is adjacent to the Glaſs) they ſhall be in great meaſure tranſmitted where the Water is adjacent to it; which argues that their Reflection or Tranſmiſſion depends on the Conſtitution of the Air and Water behind the Glaſs, and not on the ſtriking of the Rays on the Parts of the Glaſs.

12. *Thirdly*, If the Colours made by a Priſm placed at the Entrance of a Beam of Light into a darken'd

and reflected. In the Centre of the Lens, where it touches the Glaſs, it will be tranſ-

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Room be ſucceſſively caſt upon a ſecond Priſm placed at a greater Diſtance from the former, in ſuch a manner that they are all alike incident upon it, (as they will be when trajected through the Holes G and *g* in the two Boards mention'd in *Art.* 15. of the laſt Note) the ſecond Priſm may be ſo inclined to the incident Rays, that thoſe which are of a *Blue* Colour ſhall be all reflected by it, and yet thoſe of a *Red* Colour pretty copiouſly tranſmitted. Now if the Reflection be cauſed by the Parts of Air or Glaſs, I would aſk why, at the ſame Obliquity of Incidence, the *Blue* ſhould wholly impinge on thoſe Parts ſo as to be all reflected, and yet the *Red* find Pores enough to be in a great meaſure tranſmitted?

13. *Fourthly*, Where two Glaſſes touch one another, there is no ſenſible Reflection, (as will be ſhewn *Annot.* CXXI. 7.) yet I ſee no Reaſon why the Rays ſhould not impinge on the Parts of Glaſs as much when contiguous to other Glaſs, as when contiguous to Air.

14. *Fifthly*, When the Top of a Water-Bubble, (as will be ſhewn *Annot.* CXXI. 24.) by the continual ſubſiding and exhaling of the Water, grows very thin, there is ſuch a little and almoſt inſenſible Quantity of Light reflected back from it, that it appears intenſely black; whereas round about the black Spot, where the Water is thicker, the Reflection is ſo ſtrong as to make the Water ſeem very white. Nor is it only at the leaſt Thickneſs of thin Plates or Bubbles that there is no manifeſt Reflection, but at many other Thickneſſes continually greater and greater. And yet in the Superficies of the thinned Body, where it is of any one Thickneſs, and the Rays are tranſmitted, there are as many Parts for them to impinge on, as where it is of any other Thickneſs where the Rays are reflected.

15. *Sixthly*, If Reflection were cauſed by the Parts of reflecting Bodies, it would be impoſſible for thin Plates or Bubbles at one and the ſame Place to reflect the Rays of one Colour, and tranſmit thoſe of another, as

mitted, and ſo cauſe a *dark Spot* to appear: At a ſmall Diſtance from thence, all a-

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is known by Experiment they do: For it is not to be imagined, that at one Place the Rays, which, for Inſtance, exhibit a *Blue* Colour, ſhould have the Fortune to daſh upon the *Parts*, and thoſe which exhibit a *Red* to hit upon the *Pores* of the Body; and then at another Place, where the Body is a little thicker or a little thinner, that (on the contrary) the *Blue* ſhould hit upon its *Pores*, and the *Red* upon its *Parts*.

16. *Seventhly*, and laſtly, Were the Rays of Light reflected by impinging on the ſolid Parts of Bodies, their Reflections from poliſhed Bodies could not be ſo regular as they are: For in poliſhing Glaſs with *Sand*, *Putty*, or *Tripoli*, it is not to be imagined that thoſe Subſtances can, by grating and fretting the Glaſs, bring all its leaſt Particles to an accurate Poliſh, ſo that all their Surfaces ſhall be truly plain or truly ſpherical, and look all the ſame Way, ſo as together to compoſe one even Surface. The ſmaller the Particles of thoſe Subſtances are, the ſmaller will be the Scratches by which they continually fret and wear away the Glaſs until it be poliſhed; but be they never ſo ſmall, they can wear away the Glaſs no otherwiſe than by grating and ſcratching it, and breaking the Protuberances, and therefore poliſh it no otherwiſe than by bringing its Roughneſs to a very fine Grain, ſo that the Scratches and Frettings of the Surface become too ſmall to be viſible: And therefore if Light were reflected by impinging on the ſolid Parts of Glaſs, it would be ſcattered as much by the moſt poliſhed Glaſs as by the rougheſt. So then it remains a Problem, *How Glaſs poliſhed by fretting Subſtances can reflect Light ſo regularly as it does?*

17. And this Problem is ſcarce otherwiſe to be ſolved than by ſaying, *That the Reflection of the Ray is effected, not by a ſingle Point of the reflecting Body, but by ſome Power of the Body which is evenly diffuſed over all its Surface, and by which it acts upon the Ray, without immediate Contact.* For that the Parts of Bodies do act

round, the Light will be reflected in various-colour'd Rings: In the next Diſtance

upon Light at a Diſtance, has been already obſerved, and may be ſeen more at large in the Third Part of our Author's admirable Treatiſe of *Optics*.

18. Now (continues Sir *Iſaac*) if Light be reflected, not by impinging on the ſolid Parts of Bodies, but by ſome other Principle, it is probable that as many of its Rays as impinge on the ſolid Parts of Bodies are not reflected, but ſtifled and loſt in the Bodies; for otherwiſe we muſt allow two Sorts of Reflections. Should all the Rays be reflected which impinge on the internal Parts of clear Water or Cryſtal, thoſe Subſtances would rather have a cloudy Colour than a clear Tranſparency.

19. Concerning this Power, by which Light is reflected and refracted, Sir *Iſaac* underſtands it to be of an attractive and repulſive Nature; for he reaſons thus: Since Metals diſſolved in Acids attract but a ſmall Quantity of the Acid, their attractive Force can reach to but a ſmall Diſtance from them. And as in *Algebra*, where Affirmative Quantities vaniſh and ceaſe, there Negative ones begin; ſo in *Mechanics*, where Attraction ceaſes, there a repulſive Virtue ought to take place.

20. And that there is ſuch a Virtue ſeems to follow, (1.) From the Reflections and Inflections of Light, as before obſerved. (2.) From the Emiſſion of Light; the Ray, ſo ſoon as it is ſhaken off from the ſhining Body by the vibrating Motion of the Parts of the Body, and gets beyond the Reach of Attraction, being driven away with exceeding great Velocity. For that Force which is ſufficient to turn it back in Reflection may be ſufficient to emit it. (3.) It ſeems alſo to follow from the Production of Air and Vapours; the Particles when they are ſhaken off from Bodies by Heat or Fermentation, ſo ſoon as they are beyond the Reach of the Attraction of the Body, receding from it, and from one another, with great Strength, and keeping at a diſtance, ſo as ſometimes to take up a Million of times more

it will be tranſmitted, and in the next to that reflected; and ſo on alternately to a conſiderable Diſtance from the central Spot. If we take the Diſtances as the Numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, *&c.* then at the Diſtance 0, 2, 4, 6, 8, 10, *&c.* the Light will be tranſmitted; and at the Diſtances 1, 3, 5, 7, 9, *&c.* it will be reflected in coloured Rings: And this alternate Diſpoſition of Light to be reflected and tranſmitted, Sir *Iſaac* calls the *Fits of eaſy Reflection*, and *Fits of eaſy Tranſmiſſion* (CXXI.)

Space than they did before in the Form of a denſe Body.

21. To this repulſive Power he aſcribes the *Reflection* of Rays, and to the attractive Power the *Refraction*; as has been before deſcribed. But how the Light is partly reflected and partly refracted at the Surfaces of Bodies, and what Phænomena do thence ariſe, we ſhall ſhew from the ſame illuſtrious Author in the following *Annotation.*

(CXXI.) 1. Concerning the particular Manner in which Light is reflected from natural Bodies, whether it be by a repulſive Power before it arrives at the Surface, or by an undulating Virtue every where diffuſed over the Surface, and cauſing a Reflection by the riſing Wave, and a Tranſmiſſion by the ſubſiding Wave; or laſtly, whether the Reflection be occaſioned by the Vibrations of the Parts of Bodies, or the Mediums next the reflecting or refracting Surfaces, it will not be worth while here to ſpend Time in examining, ſince Sir *Iſaac Newton* has confeſs'd himſelf unable to determine the *Modus agendi*, which nature makes uſe of in this Affair.

2. Nor is his Doctrine of the *Fits of eaſy Reflection and eaſy Tranſmiſſion* to be eſteemed a meer Hypotheſis,

RPJCB

Vol. II. Plate XXXVIII

P. 337.

Fig: 1. p. 338.

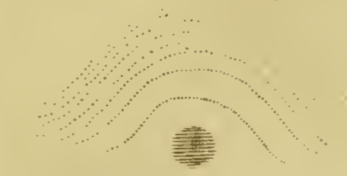
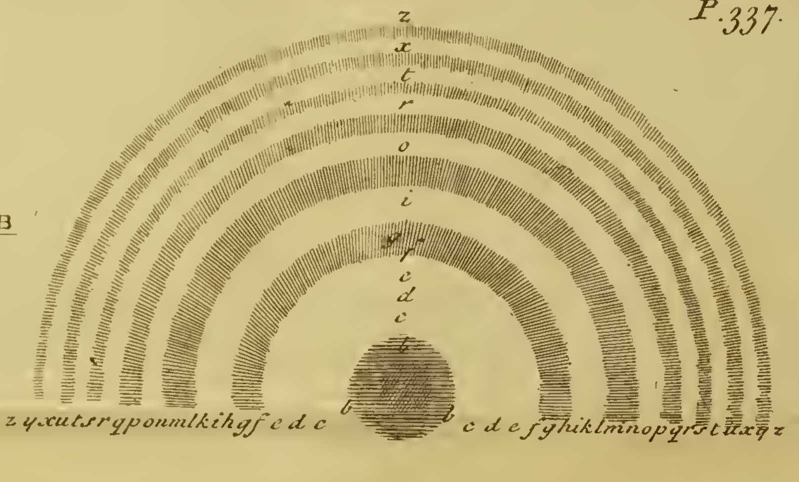


Fig: 2. p. 339.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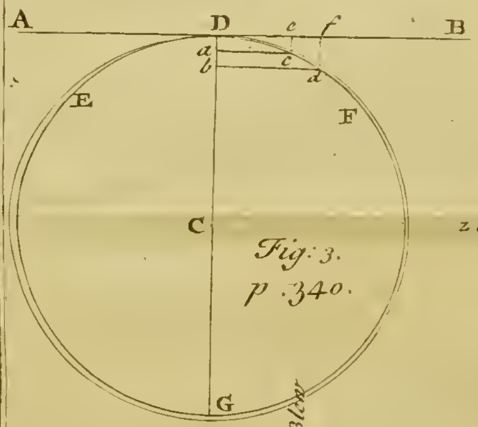


Fig: 3. p. 340.

Fig: 4. p. 343.

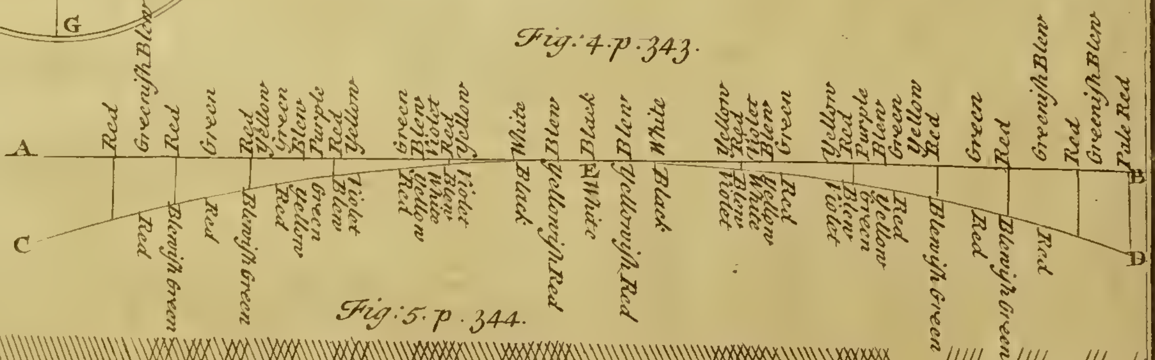
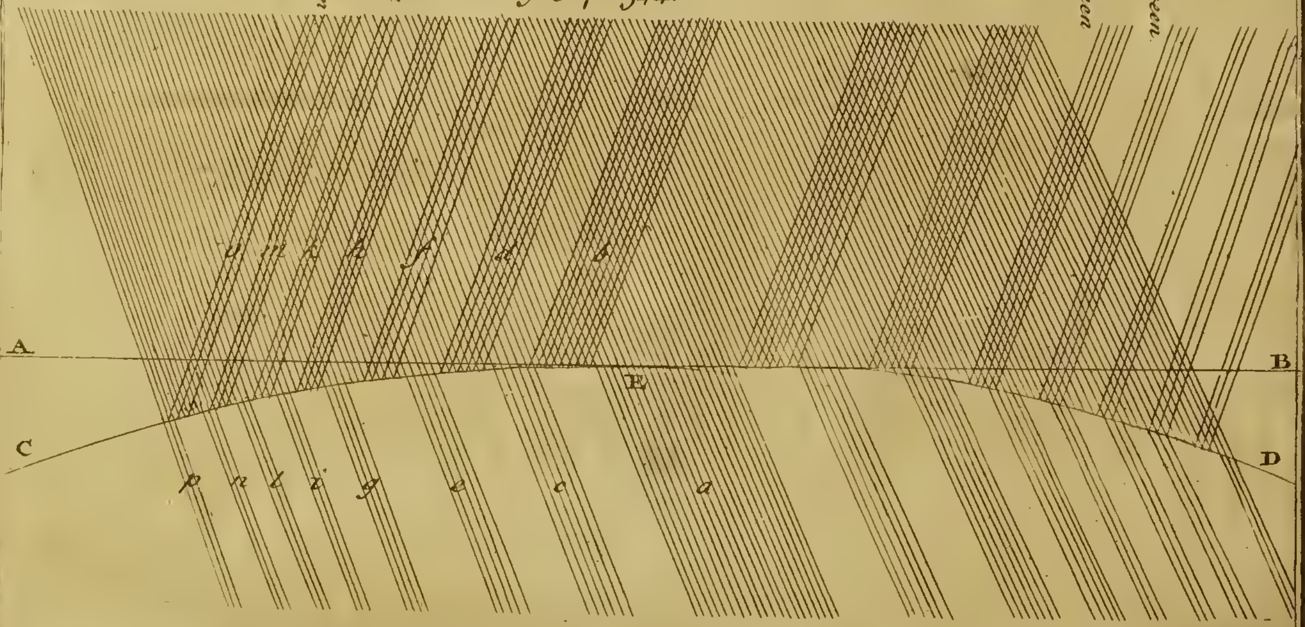


Fig: 5. p. 344.



As Light, falling upon this thin Plate of Air between the Glaſſes, is variouſly diſ-

or ſo much *clogg'd with Suppoſitions*, as to be diſſonant from that Simplicity, Uniformity and Regularity, with which Nature is every where obſerved to act; ſince nothing can be more certain than that Light is at one Diſtance reflected, at another refracted, and that this is by a continual Alternation at exceeding ſmall Intervals through the Subſtance of various *Media* or Bodies; and the Experiments which he made were many, and moſt convincing Proofs of the Thing.

3. And his Vibrations in the Parts of Bodies, and the elaſtic Medium which every where ſurrounds them, ariſing from thence, is very conſonant to the Proceſs of Nature, in propagating Sounds by the Undulations of the Air ariſing from the Vibration of the Parts of Bodies agitated by Percuſſion. Nature in each Caſe ſeems very conſiſtent with her ſelf, and to act with a wonderful Uniformity, and equal Simplicity. Nor can I ſee any Reaſon to hope for (much leſs to promiſe) a Solution of this Phænomenon from the ambigeneous Principle of Attraction, whoſe Action is well known to be always the ſame to a certain Diſtance or Limit one Way, and beyond that as conſtantly the reverſe; ſuch a Circumſtance little favours the Prediction of an eaſy and ſimple Solution. See *Rowning*'s *Compendious Syſtem*, Part III. pag. 167.

4. I ſhall therefore proceed to give an Idea of one of the moſt beautiful, delicate, and importing Diſcoveries that was ever made; and that as nearly as may be after the Manner, and in the Words of the Author, by the Experiments which he made, and his Obſervations and Reaſonings thereupon.

5. The firſt Experiment he mentions is the Compreſſion of two Priſms hard together, whoſe Sides were a little convex, by which Means they touched by a ſmall Part of their Surfaces, and contain'd every where elſe a thin Plate of Air, as it may be properly call'd, whoſe Thickneſs did every where gradually increaſe

poſed to be *reflected* or *tranſmitted*, according to the ſeveral Degrees of Thickneſs;

from the touching Parts. He obſerved the Place where they touched became abſolutely tranſparent, as if they had there been one continued Piece of Glaſs.

6. For when the Light fell ſo obliquely on the Plate of Air between the Priſms as to be all reflected, it ſeem'd in that Place of Contact to be wholly tranſmitted, inſomuch, that when looked upon, it appeared like a black or dark Spot, by reaſon that little or no ſenſible Light was reflected from thence, as from other Places.

7. When he looked through the Priſms, this Place of Contact ſeemed (as it were) a Hole in the Plate of Air, and through this Hole Objects that were beyond might be ſeen diſtinctly, which could not be ſeen through other Parts of the Glaſſes where the Air was interjacent. By harder Compreſſion, the Spot was dilated by the yielding inwards of the Parts of the Glaſſes.

8. When the Plate of Air, by turning the Priſms about their common Axis, became ſo little inclined to the incident Rays, that ſome of them began to be tranſmitted, there aroſe in it many ſlender colour'd Arches, which at firſt were ſhaped almoſt like the Conchoid, as in *Fig.* 1. and by continuing the Motion of the Priſms, theſe Arches increaſed and bended more and more about the ſaid tranſparent Spot, till they were compleated into Circles or Rings encompaſſing it; and afterwards continually grew more and more contracted.

Plate XXXVIII. Fig. 1.

9. Theſe Arches and Rings became tinged with various Colours, as the Motion of the Priſms was continued, being at firſt of a *Violet* and *Blue*; afterwards of a *White*, *Blue*, *Violet*; *Black*, *Red*, *Orange*, *Yellow*, *White*, *Blue*, *Violet*, &c. After this, the colour'd Rings contracted, and became only *black* and *white:* The Priſms being farther moved about, the Colours all began to emerge out of the Whiteneſs, and in a contrary Order to what they had before.

10. But to obſerve more nicely the Order of the Colours which aroſe out of the white Circles, as the Rays became leſs and leſs inclined to the Plate

ſo when it falls on the Surface of natural Bodies, it is as variouſly reflected from the

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of Air, Sir *Iſaac Newton* made uſe of two Object-Glaſſes, one a Plano-Convex, and the other a Double-Convex, of the ſame Sphericity on both Sides, of 51 Foot focal Diſtance; and upon this he laid the plane Side of the other, preſſing them ſlowly together to make the Colours ſucceſſively emerge in the Middle of the Circles, and then ſlowly lifted the upper Glaſs from the lower to make them ſucceſſively vaniſh again in the ſame Place.

11. Upon Compreſſion of the Glaſſes, various Colours would emerge and ſpread into concentric Circles or Rings of different Breadths and Tints encompaſſing the central Spot. Their Form, when the Glaſſes were moſt compreſſed, is delineated in the 2d Figure, where *a* is the central black Spot, and the Circuits of Colours from thence outwards as follows. Plate XXXVIII. Fig. 2.

1. { *b*, Blue. *c*, White. *d*, Yellow. *e*, Red.
2. { *f*, Violet. *g*, Blue. *h*, Green. *i*, Yellow. *k*, Red.
3. { *l*, Purple. *m*, Blue. *n*, Green. *o*, Yellow. *p*, Red.
4. { *q*, Green. *r*, Red.
5. { *s*, Greeniſh Blue. *t*, Red.
6. { *u*, Greeniſh Blue. *x*, Pale Red.
7. { *y*, Greeniſh Blue. *z*, Reddiſh White.

12. To determine the Thickneſs of the Plate of Air, where each of the Colours was produced, he meaſured the Diameter of the firſt ſix Rings at the moſt lucid Part of their Orbits, and ſquaring them found thoſe Squares to be in the Arithmetical Progreſſion of the odd Numbers 1, 3, 5, 7, 9, 11; and ſince one of thoſe Glaſſes was plane, and the other ſpherical, their Intervals at thoſe Rings muſt be in the ſame Progreſſion. Alſo he meaſured the Diameters of the dark or faint Rings between the more lucid Colours, and found their

Pores of Air of different Thickneſſes in thoſe Bodies; and according to the diffe-

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Squares to be in the Arithmetical Progreſſion of the even Numbers 2, 4, 6, 8, 10, 12.

13. All this follows from the Nature of the Circle; for let the Circle E F G be the Section of the Sphere whoſe Convexity is equal to that of the Double-Convex above-mentioned, and the Line A B a Section of the plane Surface of the Plano-Convex touching the other in the Point D; then ſuppoſing De, Df, the Semidiameters of two Rings, the Thickneſs of the Air between the Glaſſes at thoſe Rings will be ec and fd, which are equal to Da and Db reſpectively. If therefore, as uſual, we put $DG = a$, $Da = x$, $Db = X$, $ac = (De =) y$, and $bd = (Df =) Y$; then by the Property of the Circle we have $yy = ax - xx$, and $YY = aX - XX$; and therefore $y^2 : Y^2 :: ax - xx : aX - XX :: x : \frac{a - X}{a - x} \times X$. But when a or Dg is very great with reſpect to x and X, or Da, Db, then $\frac{a - x}{a - X} = 1$ nearly; conſequently, in the preſent Caſe $y^2 : Y^2 :: x : X$; or the Squares of the Semidiameters of the Rings De, Df, are as the Intervals ec, fd, or Thickneſſes of the Plates of Air in thoſe Places; and therefore the Squares of the whole Diameters are in the ſame Ratio.

Plate XXXVIII. Fig. 3.

14. Sir *Iſaac* meaſured the Diameter of the 5th dark Circle, (ſuppoſe $2Df$) and found it equal to $\frac{1}{5}$ of an Inch; but then viewing it through a Glaſs $\frac{1}{6}$ of an Inch thick, and nearly in the Perpendicular, it muſt by Refraction appear diminiſhed nearly in the Proportion of 79 to 80; ſo that, As $78 : 80 :: \frac{1}{5} : \frac{16}{79} = 2Df$ the real Diameter between the Glaſſes. Whence $Df = \frac{8}{79}$, and in this Experiment $DG = 182$ Inches, we have

rent Texture of Bodies, and Magnitude of the Particles of Light, it will be either

$DG : bf (= Df) :: bf : Db = fd$; or, in Numbers, As $182 : \frac{8}{79} :: \frac{8}{79} : \frac{32}{567931}$ or $\frac{100}{1774784} = fd$; and ſince the Thickneſs of the Air at the 5th Ring is to that at the firſt as 10 to 2, or 5 to 1, (by *Art.* 12.) therefore $\frac{1}{5}$ of $\frac{100}{1774784} = \frac{1}{88739}$ Part of an Inch, for the Thickneſs of the Air at the firſt dark Ring.

15. By another Object-Glaſs (of a Sphere whoſe Diameter $DG = 184$ Inches) he found the Dimenſion or Thickneſs of Air at the ſame dark Circle to be $\frac{1}{88850}$ Part of an Inch: But the Eye in both theſe Obſervations was not quite perpendicularly over the Glaſs, and the Rays were inclined to the Glaſs in an Angle of 4 Degrees; therefore (as *per* next Article) had the Rays been perpendicular to the Glaſſes, the Thickneſs of the Air at theſe Rings would have been leſs, and that in Proportion of the Radius 10000 to the Secant of 4 Degrees 10024. The Thickneſſes found diminiſhed in this Ratio will be $\frac{1}{88952}$ and $\frac{1}{89063}$, or in the neareſt round Numbers $\frac{1}{89000}$ Part of an Inch. Now half of this, *viz.* $\frac{1}{178000}$, is the Thickneſs of the firſt colour'd Ring; and of the reſt as follows, $\frac{3}{178000}$, $\frac{5}{178000}$, $\frac{7}{178000}$, &c. And $\frac{2}{178000}$, $\frac{4}{178000}$, $\frac{6}{178000}$, &c. are the Thickneſſes at the ſeveral dark Rings.

16. The Rings were obſerved to be leaſt when the Eye was held perpendicularly over the Glaſſes in the Axis of the Rings; and when they were view'd obliquely, they became bigger continually, ſwelling as the Eye

tranſmitted wholly or in part; and that which is reflected will be *all of one Sort of*

was removed farther from the Axis. And by meaſuring the Diameters of the ſame Circle at ſeveral Obliquities of the Eye, and by ſome other Methods, Sir *Iſaac* found its Diameter, and conſequently the Thickneſs of the Air at its Periphery in all thoſe Obliquities, to be very nearly in the Proportions expreſſed in the following Table; where the firſt Column expreſſes the Angles of Incidence which the Rays of Light make with the Perpendicular in the Glaſs; the ſecond Column expreſſes the Angle of Refraction into the Plate of Air; the third Column ſhews the Diameter of any coloured Ring at thoſe Obliquities expreſſed in Parts, of which ten conſtitute the Diameter when the Rays are perpendicular; and the fourth Column ſhews the Thickneſs of the Air at the Periphery of that Ring expreſſed in Parts, of which the Diameter conſiſts of ten alſo when the Rays are perpendicular.

17.

Angle of Incidence on the Plate of Air.		*Angle of Refraction into the Pl. of Air.*		*Diameter of the Ring.*	*Thickneſs of the Air.*
Deg.	Min.	Deg.	Min.		
00	00	00	00	10	10
06	26	10	00	$10\frac{1}{13}$	$10\frac{2}{13}$
12	45	20	00	$10\frac{1}{3}$	$10\frac{2}{3}$
18	49	30	00	$10\frac{3}{4}$	$11\frac{1}{2}$
24	30	40	00	$11\frac{2}{5}$	13
29	37	50	00	$12\frac{1}{2}$	$15\frac{1}{2}$
33	58	60	00	14	20
35	47	65	00	$15\frac{1}{2}$	$23\frac{1}{4}$
37	19	70	00	$16\frac{4}{5}$	$28\frac{1}{4}$
38	33	75	00	$19\frac{1}{4}$	37
39	27	80	00	$22\frac{6}{7}$	$52\frac{1}{4}$
40	00	85	00	29	$84\frac{1}{12}$
40	11	90	00	35	$122\frac{1}{2}$

18. By looking through the two contiguous Object-Glaſſes or Priſms, it was obſerved that the Rings of Colours appeared as well by tranſmitted as by reflected

Rays, or of *ſeveral Sorts promiſcuouſly and unequally*, or of *all Sorts equally*. Whence

Light. The central Spot now became white and tranſparent. The Order of the Colours was *Yellowiſh Red*; *Black*, *Violet*, *Blue*, *White*, *Yellow*, *Red*; *Violet*, *Blue*, *Green*, *Yellow*, *Red*, &c. as they are written in the 4th Figure below, thoſe above being the Colours by Reflection; A B and C D being the Surfaces of the Glaſſes contiguous at E, with Lines between ſhewing the Intervals or Thickneſſes of Air in Arithmetical Progreſſion. Where comparing the Colours, you obſerve that White is oppoſite to Black, Red to Blue, Yellow to Violet, Green to Red, *&c.* in reflected and refracted Light: But the Colours by refracted Light were very faint and dilated, except when viewed very obliquely, for then they became pretty vivid.

Plate XXXVIII. Fig. 4.

19. By wetting the Glaſſes round their Edges, the Water crept in ſlowly between them, and the Circles thereby became leſs, and the Colours more faint. Their Diameters being meaſured were found in Proportion to thoſe of the Rings made in Air as 7 to 8, and therefore the Thickneſs of Air at like Circles as $7 \times 7 = 49$ to $8 \times 8 = 64$, or as 3 to 4 very nearly, which is the Ratio of the Sines of Incidence and Refraction out of Water into Air. And this perhaps (ſays Sir *Iſaac*) may be a general Rule for any other Medium interceding the Glaſſes more or leſs denſe than Water.

20. The coloured Rings made in Air became much more diſtinct, and viſible to a far greater Number, when viewed in a dark Room by the Reflection of the colour'd Light of the Priſm. The Rings made by Reflection of *Red* Light were manifeſtly bigger than thoſe made by the *Blue* and *Violet*; and it was very pleaſant to ſee them gradually ſwell and contract according as the Colour of the Light was changed. The Motion was quickeſt in the *Red*, and ſloweſt in the *Violet*; and by an Eſtimation made of the Diameters of the Rings, the Thickneſſes of Air in the Places where the Rings are made by the Limits of the ſeven Colours, *Red*, *Orange*,

it will follow, (1.) If the Light be wholly tranſmitted, the Body will appear *black*,

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Yellow, *Green*, *Blue*, *Indigo*, *Violet*, ſucceſſively in Order, were to one another as the Cube Roots of the Squares of the 8 Lengths of a Chord which ſound the Notes of an *Octave*, that is, of the Numbers 1, $\frac{8}{9}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{9}{16}$, $\frac{1}{2}$.

21. Theſe Rings were not of various Colours, as thoſe made in the open Air, but appeared all over of that Priſmatic Colour only with which it was illumin'd; and by throwing the colour'd Light directly on the Glaſſes, that which fell on the dark Spaces between the Rings was tranſmitted through the Glaſſes without any Variation of Colour. This appeared by placing a white Paper behind, on which the Rings were painted of the ſame Colour as thoſe by reflected Light, and of the Bigneſs of their immediate Spaces.

22. Hence the Origin of theſe Rings is manifeſt; namely, that the Air between the Glaſſes, according to its various Thickneſs, is diſpoſed in ſome Places to reflect, in others to tranſmit the Light of any one Colour; and in the ſame Place to reflect that of one Colour, where it tranſmits that of another; in the Manner as you ſee repreſented in the 5th Figure: Where A B, C D, are the Glaſſes, as before; and *a*, *c*, *e*, *g*, *i*, *l*, *n*, *p*, the Parts of the Beam tranſmitted; and *b*, *d*, *f*, *h*, *k*, *m*, *o*, the Parts of the Beam reflected, making the colour'd Rings. Plate XXXVIII. Fig. 5.

23. The Squares of the Diameters of theſe Rings made by any Priſmatic Colour, and conſequently the Thickneſſes of the Air at each, were in Arithmetical Progreſſion, as in the Rings of common Light; and the Dimenſion of the Rings made by Yellow Light the ſame as ſpecified in *Article* 14. Theſe Obſervations were made with a rarer thin Medium terminated by a denſer, *viz.* Air and Water between Glaſſes. In thoſe which follow are ſet down the Phænomena of a denſer Medium thinn'd within a rarer, as Plates of *Muſcovy*

which is the Abſence of all colour'd Light. (2.) If the Light reflected from Bodies be

Glaſs, Bubbles of Water, *&c.* bounded on all Sides with Air.

24. In the Experiment made with a Bubble of Soap-Water covered by clear Glaſs, and expoſed to the white Light of the Sky, it was obſerved, that as the Bubble grew thinner by the continual ſubſiding of the Water, it exhibited Rings of Colours ſlowly diluting, till they overſpread the whole Bubble, and vaniſhed at the Bottom ſucceſſively. The Bubble was black at Top, and this central Spot was ſurrounded with Rings of the ſame Colours, and in the ſame Order as thoſe of Air in *Art.* 11, but much more extended and lively.

25. As the Thickneſs of the aqueous Shell diminiſhed, the Colours of the ſeveral Rings by Dilatation were ſucceeded by others in Order from the Red to the Purple. Thus the Red of the ſecond Ring from the Top (or ſixth from the Bottom) was at firſt a fair and lively Scarlet, then became of a brighter Colour, being very pure and briſk, and the beſt of all the Reds. Then after followed a lively Orange, which was ſucceeded by the beſt of Yellows, which ſoon changed into a greeniſh Yellow, and then into a greeniſh Blue. Afterwards a very good Blue, of an azure Tint, appeared; which was ſucceeded by an intenſe and deep Violet. And ſo it happened in all the other Orders of Colours, only not in ſo regular and perfect a Manner, the Colours in them being more compounded and leſs diſtinct.

26. Theſe Rings of Colours, viewed in various Poſitions of the Eye, were found to dilate according as the Obliquity of the Eye increaſed, but not ſo much as thoſe of Air in *Art.* 16. For by the Table, *Art.* 17, it appears they expanded to a Part where the Thickneſs of the Air was to that where they appeared when viewed perpendicularly, as $122\frac{1}{2}$ to 10, or more than 12 to 1; whereas Sir *Iſaac* found, by meaſuring the Thickneſs of the Bubble at the ſeveral Rings, as they appeared at the ſeveral Degrees of Obliquity mentioned

all of one Sort, that Body will appear all of one Colour, which will be moſt ſimple and

in the Table below, that the greateſt was to the leaſt only as 15½ to 10; which Increaſe is but about a 24th Part of the former in Air.

27. The Angles of Incidence on the Water, and the Refraction into the Water, are ſhewn in the two firſt Columns, and in the third the Thickneſſes of the aqueous Shell correſponding thereto.

Incidence on the Water.		*Refraction into the Water.*		*Thickneſs of the Shell.*
Deg.	Min.	Deg.	Min.	
00	00	00	00	10
15	00	11	11	10¼
30	00	22	01	10⅘
45	00	32	02	11⅘
60	00	40	30	13
75	00	46	25	14½
90	00	48	35	15⅕

28. The Sines of theſe Angles out of Water into Air are aſſumed as 3 to 4; and Sir *Iſaac* has collected (with a prodigious Sagacity) that *the Thickneſs of the Plate of Air or Shell of Water, requiſite to exhibit one and the ſame Colour at ſeveral Obliquities of the Eye, is proportional to the Secant of an Angle whoſe Sine is the firſt of* 106 *mean Proportionals between the Sines of Incidence and Refraction.*

29. As in *Art.* 18, ſo here the Bubble by tranſmitted Light appeared of a contrary Colour to that which it exhibited by Reflection: Thus that Part which looked *Red* by reflected Light looked *Blue* by refracted, and the Part which was *Blue* by reflected Light was *Red* by Rays tranſmitted. Theſe Rings appear much more numerous, and more dilated, when viewed through a Priſm than to the naked Eye; and by means of the Priſm ſeveral Rings may be diſcovered between the

intenſely

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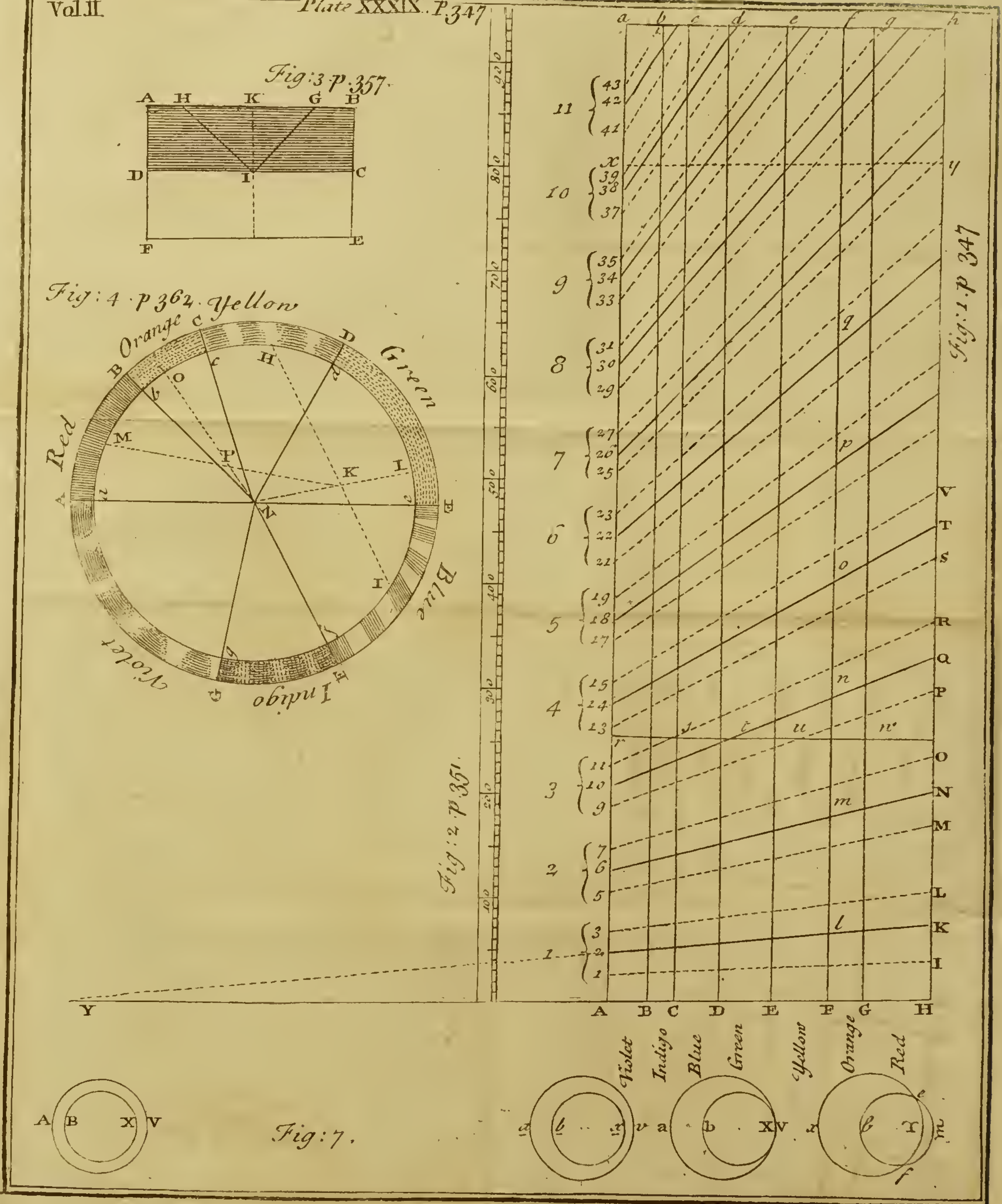
Plate XXXIX. P.347



intenſely deep. (3.) If the Rays are promiſcuouſly reflected, but one Sort more

Glaſſes or in the Bubble, when none appear to the bare Eye.

30. The colour'd Rings now deſcribed appear alſo in thin Pieces of *Muſcovy* Glaſs; which when they were wetted on the Side oppoſite to the Eye exhibited ſtill the ſame Colours, but more languid and faint. Whence, and by *Art.* 19. it evidently appears, that the Thickneſs of a Plate requiſite to produce any Colour depends only on the Denſity of the Plate, and not on that of the ambient Medium. And upon the Whole, if the Plate be denſer than the ambient Medium, it exhibits more briſk and lively Colours than that which is ſo much rarer.

31. The Colours which ariſe on poliſh'd Steel being heated are of the ſame Kind with thoſe in the Rings of the Bubble, emerging one after another from Red to Blue or Purple ſucceſſively; and like the others will change in being view'd at different Obliquities of the Eye, but not in ſo great a Degree.

32. That we may be able to ſhew how the Colours in the ſeveral Rings are produced, we ſhall a little illuſtrate Sir *Iſaac*'s Invention for that Purpoſe. In order to this, Let there be taken, in any Right Line Y H, the Lengths Y A, Y B, Y C, Y D, Y E, Y F, Y G, Y H, in Proportion to each other as the Cube Roots of the Squares of the Numbers $\frac{1}{2}$, $\frac{9}{16}$, $\frac{3}{5}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{8}{9}$, 1; that is, in the Proportion of the Numbers 6300, 6814, 7114, 7631, 8255, 8855, 9243, 10000. See *Article* 20.

Plate XXXIX. Fig. 1.

33. In the Points A, B, C, D, E, F, G, H, erect the Perpendiculars A *a*, B *b*, C *c*, *&c.* by whoſe Intervals the Extent of the Colours wrought by them will be repreſented. For if at the Thickneſs Y A the Violet Colour begins, and the Indigo at B, the Extent A B will repreſent the Breadth of the Violet; and ſo of the reſt.

than the reſt, the Body will appear of the Colour proper to that Sort of Ray, but it

34. Then let the Line A *a* be divided into equal Parts, and number'd as in the Figure to 43; and through thoſe Diviſions from Y draw the Lines 1 I, 2 K, 3 L, 5 M, 6 N, 7 O, *&c.* Then will the Parts A 2, A 6, A 10, A 14, *&c.* be in Proportion to the odd Numbers 1, 3, 5, 7, 9, 11, *&c.* or as the Thickneſſes of the Air at the ſeveral Rings. See *Art.* 12.

35. Therefore ſince A 2 repreſents the Thickneſs of any thin tranſparent Body, at which the Violet of the firſt Order or Ring is moſt copiouſly reflected; then will H K repreſent its Thickneſs where the Red of that Order is moſt copiouſly reflected: Becauſe, in the ſimilar Triangles A Y 2 and H Y K, we have Y A : Y H :: A 2 : H K. But Y A and Y H are as the Thickneſſes of the Plate of Air at theſe Colours, and therefore alſo A 2 and H K. See *Art.* 32.

36. Again; becauſe (by *Art.* 12.) A 6 is the Thickneſs where the Violet of the 2d Ring is moſt copiouſly reflected, and (by *Art.* 20.) the Ratio of the Thickneſs of the Air where Violet and Red are reflected is the ſame as of Y A to Y H; therefore ſince Y A : Y H :: A 6 : H N, the Line H N will repreſent the Thickneſs of the Plate where the Red of the ſecond Order is reflected moſt copiouſly. Thus alſo A 10 and H Q will repreſent the ſame for the Violet and Red of the third Order, and ſo on.

37. And the Thickneſſes at which the intermediate Colours will be reflected moſt copiouſly will be defined by the Diſtance of the Line A H from the intermediate Parts of the Line 2 K, 6 N, 10 Q, *&c.* againſt which the Names of the Colours are written; which is eaſy to underſtand.

38. But farther to define the Latitude or Breadth of the Colours in each Ring, let A 1 denote the leaſt Thickneſs, and A 3 the greateſt, at which the extreme Violet in the firſt Series or Ring is reflected; then ſhall H I and H L be the like Limits for the extreme Red,

will be not ſo pure and ſtrong as before. (4.) If three or four Sorts of Rays are

and the intermediate Colours will be limited by the intermediate Parts of the Lines 1 I and 3 L, againſt which the Names of thoſe Colours ſtand; and ſo on. *Note*, The ſame Latitude is aſſign'd to every Series of Colours, A H L 3, 5 M O 7, 9 P R 11, *&c.* becauſe the Difference of the Breadths of the Rings in the Plates of Air and Water were inſenſible to the Eye in the Experiment.

39. From hence it is eaſy to obſerve, that the Spaces A 1 I H, 3 5 M L, 7 9 P O, *&c.* are thoſe at which the Rays are tranſmitted, and the dark Circles appear. And therefore we may know from this Scheme what Colour muſt be exhibited (in the open Air) at any Thickneſs of a tranſparent thin Body: For if a Ruler be applied parallel to A H, at the Diſtance from it by which the Thickneſs of the Body is repreſented, the alternate Spaces 1 I L 3, 5 M O 7, *&c.* which it croſſes, will denote the reflected original Colours, of which the Colour exhibited in the open Air is compounded.

40. Thus, for Example, if it be required to find what is the Conſtitution or component Colours of the Green of the third Order or Series, apply the Ruler as you ſee at *r s t u w*, (parallel to A H) and by its Paſſage through ſome of the Blue at *s*, and Yellow at *u*, as well as through the Green at *t*, you may conclude that the Green exhibited at that Thickneſs of the Body is principally conſtituted of original Green, with a Mixture of ſome Blue and Yellow.

41. By this means alſo you may know how the Colours from the Gentre of the Rings outward ought to ſucceed in the Order as they have been deſcribed in *Art.* 11. For if you move the Ruler gradually from A H through all the Diſtances, having paſs'd over the firſt Space A 1, which denotes little or no Reflection to be made by thinneſt Subſtances, it will firſt arrive at 1 the Violet, and then quickly at the Blue and Green, which together with the Violet compound Blue; and then at

promiſcuouſly reflected more than the reſt, the Colour of the Body will be Mixed or

the Yellow and Red, by whoſe farther Addition that Blue is converted into Whiteneſs, which continues during the Tranſit of the Ruler from I to 3; and after that, by the ſucceſſive Deficience of its component Colours, turns firſt to compound Yellow, and that to Red, which ceaſes at L. Thus are the Colours of the firſt Series generated.

42. Then begin the Colours of the ſecond Series, which ſucceed in Order during the Tranſit of the Edge of the Ruler from 5 to O, and are more lively than before, becauſe more expanded and ſevered: And here, becauſe the Ruler arrives to and paſſes over the Point 7 before it comes to M, there cannot be a Reflection of all the Colours at the ſame Time, and therefore no Whiteneſs between the Blue and Yellow, as before; but there will be a Reflection of original Green, with Yellow and Orange on one Side, and Blue and Indigo on the other, which together make a compound Green. The Violet will here firſt appear at 5, before it comes to be reflected with Indigo and Blue.

43. So the Colours of the third Series happen in Order; firſt the Violet at 9, which as it interferes with the Red of the ſecond Order, is thereby inclined to a reddiſh Purple. Then the Blue and Green, which here are leſs mixed with other Colours, and conſequently are more lively than before, eſpecially the Green. Then follows the Yellow, ſome of which towards the Green is diſtinct and good, but that Part towards the ſucceeding Red, as alſo that Red, is mixed with the Violet and Blue of the fourth Order; whereby various Degrees of Red, very much inclining to Purple, are compounded.

44. Hence the Violet and Blue, which ſhould ſucceed and begin the fourth Series, being mixed with and hidden in the Red of the third Order, there ſucceeds a Green, which at firſt is much inclined to Blue, but ſoon becomes a good Green, being the only unmixed and lively Colour of this fourth Order: For as it verges

Compound, inclining to the Tint of the moſt predominant Colour. (5.) When all

towards the Yellow, it begins to interfere with the Colours of the fifth Series, by whoſe Mixture the ſucceeding Yellow and Red are very much diluted and made dirty, eſpecially the Yellow, which being the weaker Colour is ſcarce able to ſhew itſelf; ſo that this Order conſiſts of Green and Red only.

45. After this, by paſſing the Edge of the Ruler along parallel to A H, it will cut the Colours of the ſecond, third, and fourth Series at once; which will ſhew thoſe Colours become more and more intermixed, till after three or four more Revolutions (in which the Red and Blue predominate by turns, making the fifth, ſixth, and ſeventh Rings) all Sorts of Colours are in all Places pretty equally mixed, and compound an even Whiteneſs. Thus the Line *xy* paſſing through the Red of the 7th Series, the Yellow and Green of the 8th, the Blue of the 9th, and the Purple of the 10th, ſhews Whiteneſs at the Thickneſs A *x* or H 4 muſt neceſſarily reſult from the Mixture of ſo many original Colours.

46. Since (by *Art.* 20, 21.) the Rays of one Colour are tranſmitted where thoſe of another Colour are reflected, the Reaſon of the colour'd Rings made by tranſmitted Light is from hence manifeſt; becauſe what has been ſaid with reſpect to the Colours made by Reflection from the Spaces 1 L, 5 O, 9 R, *&c.* is equally applicable to account for the Colours made by Refraction through the Spaces A I, 3 M, 7 P, 11 S, *&c.*

47. Not only the Order and Species, but alſo the preciſe Thickneſs of the Plate at which any of thoſe Colours are exhibited in Parts of an Inch, may be obtain'd as follows. Since (by *Art.* 14, 15, and 23.) we have the Thickneſs of the Plate where Yellow Light is reflected already meaſured, *viz.* F $l = \frac{1}{178000}$, F $m =$ Plate
$\frac{3}{178000}$, F $n = \frac{5}{178000}$, $Fo = \frac{7}{178000}$, *&c.* and ſince XXXIX.
$\frac{1}{178000} = 0{,}0000056$, or 56 Parts of Ten Million of Fig. 2.
an Inch; if the Scale of equal Parts be conſtructed ſuch

Sorts of Rays are equally reflected from Bodies, thoſe Bodies appear *white*, or of

of which $Fl = 56$, it is plain any other Thickneſs of Air may be immediately meaſured thereon by means of a Pair of Compaſſes, or by a parallel Ruler. Thus $Gw = 0{,}0000254$; $A2 = 0{,}0000040$; $HK = 0{,}0000065$; $A6 = 0{,}0000119$; $HN = 0{,}0000194$. And thus any other Thickneſs for any propoſed Colour or Series is evident almoſt by Inſpection, to the Ten Millionth Part of an Inch.

48. Since by *Art.* 19. it appears, that the Thickneſſes of Air and Water, exhibiting the ſame Colour, are as 4 to 3; if the Thickneſſes in Air are known for the ſeveral Rings, you'll have the Thickneſs of the Bubble of courſe where the ſeveral Colours appear; and thus the Table in *Art.* 27. was made. Alſo hence the Thickneſſes of thin Plates of Glaſs producing the Rings of Colours will be known, being to thoſe of Air as 20 to 31, *viz.* in the Proportion of the Sines of Incidence to Refraction out of Glaſs into Air for Yellow Light; and the Difference of the Proportion of the Sines for the other Rays is not conſiderable.

49. Theſe are the Meaſures nearly, which Sir *Iſaac* has expreſs'd in the following Table, where the Numbers are ſo many Millionth Parts of an Inch for the Thickneſſes of the Plates of Air, Water, and Glaſs, which exhibit the various Colours of the ſeveral Orders.

		Air.	*Water.*	*Glaſs.*
Colours of the Firſt Order.	Very Black,	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{3}$
	Black,	1	$1\frac{1}{2}$	$1\frac{2}{7}$
	Blue,	$2\frac{2}{5}$	$1\frac{4}{5}$	$1\frac{1}{2}$
	White,	$5\frac{1}{4}$	$3\frac{7}{8}$	$3\frac{2}{5}$
	Yellow,	$7\frac{1}{9}$	$5\frac{1}{3}$	$4\frac{3}{5}$
	Orange	8	6	$5\frac{1}{6}$
	Red,	9	$6\frac{3}{4}$	$5\frac{4}{5}$

the Colour of the Sun's Light. (6.) Where there is no Light at all incident on Bodies,

		Air.	*Water.*	*Glaſs.*
Of the Second Order.	Violet,	$11\frac{1}{6}$	$8\frac{3}{8}$	$7\frac{1}{5}$
	Indigo,	$12\frac{5}{6}$	$9\frac{5}{8}$	$8\frac{2}{11}$
	Blue,	14	$10\frac{1}{2}$	9
	Green,	$15\frac{1}{8}$	$11\frac{1}{3}$	$9\frac{5}{7}$
	Yellow,	$10\frac{2}{7}$	$12\frac{1}{5}$	$10\frac{2}{5}$
	Orange,	$17\frac{2}{9}$	13	$11\frac{1}{9}$
	Bright Red,	$18\frac{1}{3}$	$13\frac{3}{4}$	$11\frac{5}{6}$
	Scarlet,	$19\frac{2}{3}$	$14\frac{3}{4}$	$12\frac{2}{3}$
Of the Third Order.	Purple,	21	$15\frac{3}{4}$	$13\frac{1}{2}$
	Indigo,	$22\frac{1}{10}$	$16\frac{4}{7}$	$14\frac{1}{4}$
	Blue,	$23\frac{2}{5}$	$17\frac{1}{2}$	$15\frac{1}{10}$
	Green,	$25\frac{1}{5}$	$18\frac{9}{10}$	$16\frac{1}{4}$
	Yellow,	$27\frac{1}{7}$	$20\frac{1}{3}$	$17\frac{1}{2}$
	Red,	29	$21\frac{3}{4}$	$18\frac{5}{7}$
	Blueiſh Red.	32	24	$20\frac{2}{3}$
Of the Fourth Order.	Green,	$35\frac{2}{7}$	$26\frac{1}{2}$	$22\frac{3}{4}$
	Blue,	$40\frac{1}{3}$	$30\frac{1}{4}$	26
Of the Fifth Order.	Greeniſh Blue,	46	$34\frac{1}{2}$	$29\frac{2}{3}$
	Red,	$52\frac{1}{2}$	$39\frac{3}{8}$	34
Of the Sixth Order.	Greeniſh Blue,	$58\frac{3}{4}$	44	38
	Red,	65	$48\frac{3}{4}$	42
Of the Seventh Order.	Greeniſh Blue,	71	$53\frac{1}{4}$	$45\frac{4}{5}$
	Ruddy White,	77	$57\frac{3}{4}$	$49\frac{2}{3}$

50. Theſe are the principal Phænomena of thin Plates or Bubbles, which follow from the Properties of Light by a mathematical Way of Reaſoning; whence it follows, that the colorific Diſpoſition of Rays is connate with them, and immutable, there being always a conſtant Relation between Colours and the Refrangibility and Reflexibility of the Rays. In this reſpect the Science of Colours becomes a Speculation as truly Mathematical as any other Part of *Optics*; and conſiſts of two

thoſe Bodies can have no Colour, which is a Property of the Rays of Light only (CXXII.)

Parts, one *Theoretical*, which delivers the Properties of Light, and the Principles on which the various Phænomena of Colours depend. This Part we have hitherto been treating of: The other is *Practical*, and conſiſts in applying theſe Principles to account for the permanent Colours of Natural Bodies; to which we ſhall now proceed in the following Note.

(CXXII.) 1. As I here intend to deliver the whole *Newtonian* Doctrine of Colours, it will be neceſſary to begin and proceed with the Definitions and Precautions which Sir *Iſaac Newton* himſelf has made uſe of, and which are as follow.

2. His general Poſition is, *That if the Sun's Light conſiſted but of one Sort of Rays, there would be but one Colour in the whole World; nor would it be poſſible to produce any new Colour by Reflections and Refractions; and by Conſequence that the Variety of Colours depends upon the Compoſition of Light.* All which is evident from the Subject of the foregoing *Annotations* on the Properties and Phænomena of Light by Reflection and Refraction.

3. His Definition of Light is as follows: The Light whoſe Rays are all alike refrangible he calls *Simple, Homogeneal*, and *Similar*; and that whoſe Rays are ſome more refrangible than others he calls *Compound, Heterogeneal*, and *Diſſimilar*.

4. The Colours of *Homogeneal Lights* he calls *Primary, Homogeneal*, and *Simple*; and thoſe of *Heterogeneal Lights* he calls *Heterogeneal* and *Compound*, becauſe theſe are all compounded of the Colours of Homogeneal Lights; as hath been in part already, and will be farther ſhewn in the Sequel of this *Annotation*.

5. The Homogeneal Light and Rays which appear *Red*, or rather make Objects appear ſo, he calls *Rubrific* or *Red-making* Rays; thoſe which give Objects a *Yellow, Green, Blue*, or *Violet* Colour, he calls *Yellow-making*,

LET BNFG be a ſpherical Drop of falling Rain, and AN a Ray of the Sun

Plate XLVIII. Fig. 2.

Green-making, *Blue-making*, *Violet-making* Rays; and ſo of the reſt. And therefore whenever he ſpeaks of Light and Rays as colour'd, or endued with Colours, he would be underſtood to ſpeak not philoſophically and properly, but groſsly, and according to the vulgar Notion of common People.

6. For the Rays, to ſpeak properly, are not colour'd; in them there is nothing but a certain Diſpoſition and Power to excite a Senſation of this or that Colour. For as Sound in a Bell or muſical String is nothing but a tremulous Motion, and in the Air nothing but that Motion propagated from the Object in aerial Undulations; and in the *Senſorium* 'tis a Senſe of Motion under the Notion of Sound: So Colours in the Object are nothing but a Diſpoſition to reflect this or that Sort of Rays more copiouſly than the reſt; in the Rays they are nothing but their Diſpoſition to propagate this or that Motion to the *Senſorium* by the Optic Nerve; and in the *Senſorium* they are Senſations or Ideas of thoſe Motions under the Forms or Notions of Colours.

7. Every Ray of Light in its Paſſage through any refracting Surface is put into a certain tranſient Conſtitution or State, which in the Progreſs of the Ray returns at equal Intervals, and diſpoſes the Ray at every Return to be eaſily tranſmitted through the next refracting Surface, and between the Returns to be eaſily reflected by it. This is manifeſt from *Art.* 21, 22, of the laſt *Note*. Theſe Returns of the Diſpoſition of any Ray to be reflected he calls its *Fits of eaſy Reflection*, and thoſe of its Diſpoſition to be tranſmitted its *Fits of eaſy Tranſmiſſion*; and the Space it paſſes between every Return he calls the *Interval of the Fits*.

8. This Alternation of its Fits depends on both the Surfaces of every thin Plate or Particle, becauſe it depends on its Thickneſs; and alſo becauſe, if either Surface be wetted, the Colours cauſed both by Reflection and Refraction grow faint, which ſhews it to be affected

falling upon it in the Point N, which Ray ſuppoſe refracted to F, from thence re-

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at both. It is therefore performed at the ſecond Surface, for if it were performed at the firſt, it could not depend on the ſecond; and it is influenced by ſome Action or Diſpoſition propagated from the firſt to the ſecond, becauſe otherwiſe at the ſecond it could not depend on the firſt.

9. This Action or Diſpoſition, in its Propagation, intermits and returns at different Intervals in different Sorts of Rays, emerging in equal Angles out of any refracting Surface into the ſame Medium. Thus in the Experiment of *Art.* 20. and 21. of the laſt *Annotation*, 'tis plain, the Violet Ray being in a Fit of eaſy Tranſmiſſion at its Incidence on the Plate of Air, was again in that Fit at the fartheſt Surface, in paſſing through a leſs Space than that which the Red paſs'd through in the Interval of its Fits; for thoſe Spaces were as the Thickneſſes of the Glaſſes, and conſequently the Intervals of theſe Fits were as the Numbers 63, 68, 71, 76, 82½, 88½, 92½, 100, for the Rays reſpectively from Violet to Red. See *Art.* 32. of the laſt *Annotation.*

10. Hence when a Ray of Light falls upon the Surface of a Body, if it be in a Fit of eaſy Reflection, it ſhall be reflected; if in a Fit of eaſy Tranſmiſſion, it ſhall be tranſmitted: and thus all thick tranſparent Subſtances are found to reflect one Part of the Light which is incident upon them, and to refract the reſt.

11. The leaſt Parts of almoſt all Natural Bodies are in ſome meaſure tranſparent. This is well known to thoſe who are converſant in Experiments with the common and ſolar Microſcopes: As alſo by the Solution of denſe and opake Bodies in Menſtruums; for then their Particles being ſo minutely divided become tranſparent. And therefore, conſidering the inconceivable Smallneſs of the Particles of Light, even in Compariſon of the ſmalleſt Parts of Natural Bodies, we may conceive them as always incident on the Surface of tranſparent Subſtances.

flected to G, and there again refracted in the Direction G R to the Eye of a Specta-

12. I have obſerved (briefly) before, that thoſe Superficies of tranſparent Bodies reflect the greateſt Quantity of Light, which have the greateſt refracting Power. Thus Glaſs produces a total Reflection of Light, at a leſs Angle of Incidence on the Air than Water; for in Glaſs that Angle is but 40° 10′, but in Water it is 48° 35′. Thus alſo Diamond, whoſe refractive Power to that of Glaſs is as 34 to 26 nearly, is found to reflect a much greater Quantity of Light than Glaſs.

Plate XXXIX. Fig. 3.

13. Hence 'tis obvious, there can be no Reflection at the Confines of equally refracting Mediums. For let H I be a ſingle Ray of Light paſſing out of a denſer Medium A C into a rarer D E; in this Caſe there will be a certain Limit or Angle of Incidence H I K, in which the Ray will be reflected into I G. If the Medium A C be ſuppoſed to have its Denſity decreaſing, then the Limit or Angle H I K will be continually increaſing; or, which is all one, the Ray H I muſt have a greater Obliquity than H I K that it may be reflected. Therefore when the Denſity of the Medium A C becomes equal to that of D E, the Angle H I K will become equal to D I K; and ſo no Ray inclined to the Perpendicular K I can in that Caſe poſſibly be reflected.

14. Hence the Reaſon why uniform pellucid Mediums, as Water, Glaſs, Cryſtal, *&c.* have no ſenſible Reflection but in their external Superficies, where they are adjacent to other Mediums of different Denſities, is becauſe all their continuous Parts have one and the ſame Degree of Denſity.

15. Hence alſo it is, that ſince in common Subſtances there are many Spaces, Pores, or Interſtices, either empty or repleniſhed with Mediums of other Denſities, various Reflections muſt be made in the Confines of theſe differently refracting Mediums; and thus the Bodies become variouſly coloured and opake in different Degrees. As for Example, Water between the tinging Corpuſcles wherewith Liquor is impreg-

tor; and let IG be perpendicular to the Point G: Then will the Beam, by its Refraction at G, be ſeparated into its ſe-

nated; Air between the aqueous Globules which conſtitute the Clouds or Miſt; and Water, Air, and perhaps other ſubtil Media between the Parts of ſolid Bodies, give them their proper Colours and Degrees of Opacity, by a confuſed and promiſcuous Reflection and Refraction of Light.

16. The Parts of Bodies and their Interſtices muſt not be leſs than of ſome definite Bigneſs to render them opake and colour'd: For, as was ſaid before, the opakeſt Bodies, if their Parts be ſufficiently attenuated by Solution, become tranſparent. Thus the Top of the Water-Bubble being very thin made no ſenſible Reflection, and therefore exhibited no Colours; but, tranſmitting the Light, appeared black. Hence it is that Water, Salt, Glaſs, Stones, *&c.* having their Parts and Interſtices too ſmall to cauſe Reflections, become tranſparent and colourleſs.

17. The tranſparent Parts of Bodies, according to their ſeveral Sizes, reflect Rays of one Colour, and tranſmit thoſe of another, on the ſame Grounds that thin Plates or Bubbles did the ſame; and this is undoubtedly the Ground and Reaſon of all their Colour. For if ſuch a thin Plate ſhould be broke into ſeveral Fragments, or ſlit into Threads of the ſame Thickneſs, they would all appear of the ſame Colour; and by conſequence, an Heap of thoſe Threads or Fragments would conſtitute a Maſs or Powder of the ſame Colour the Plate exhibited before it was broken; and the Parts of all Natural Bodies, being like ſo many Fragments of a Plate, muſt on the ſame Grounds exhibit the ſame Colours.

18. And that they do ſo will appear by the Affinity of their Properties. The finely colour'd Feathers of ſome Birds, as of Peacocks Tails, do in the very ſame Part of the Feather appear of ſeveral Colours in ſeveral Poſitions of the Eye, in the ſame manner that thin Plates were found to do in *Articles* 16 and 26 of the

veral Sorts of Rays, which will paint their reſpective Colours in that Part of the Drop;

laſt *Annotation*; and therefore their Colours ariſe from the thin tranſparent Parts of the Feathers, that is, from the Tenuity of the very fine Hairs or *Capillamenta*, which grow out of the Sides of the groſſer Parts or lateral Branches of thoſe Feathers.

19. And hence it is, that the Webs of ſome Spiders being ſpun very fine have appeared coloured; and that the coloured Fibres of ſome Silks, by varying the Poſition of the Eye, do vary their Colours.

20. Another Circumſtance in which they agree is, that the Colours of Silks, Cloaths, and other Subſtances, which Water or Oil can intimately penetrate, become more faint and obſcure by being immerſed into thoſe Liquors, and recover their Vigour and Vivacity again by being dried, in the ſame Manner as was obſerved of thin Bodies in *Art.* 19. and 30. of the laſt *Annotation*.

21. A third Circumſtance, in which Natural Bodies agree in their colorific Quality with thin Plates, is, that they reflect one Colour and tranſmit another. Thus Leaf-Gold looks Yellow by reflected Light, and of a blueiſh Green by the tranſmitted Light. Alſo an Infuſion of *Lignum Nephriticum* reflects the Blue and Indigo Rays, and therefore by Reflection appears of a deep Mazarine Blue; whereas by refracted Light it appears of a deep Red. And the ſame Thing is obſervable in ſeveral Sorts of painted Glaſſes.

22. Again; as thin Plates and Bubbles exhibit different Colours in different Thickneſſes, ſo the Parts of Natural Bodies are obſerved to undergo a Change of Colour in ſome Degree from Trituration, and a Comminution of their Parts. Thus ſome Powders which Painters uſe, by being elaborately and finely ground, have their Colours a little changed. Thus Mercury by ſeveral Chymical Operations has its Parts ſo altered as to look Red in one Caſe, Yellow in another, and White in a third. Thus Copper in the Maſs appears Red, but

of which that next the Perpendicular I G will be *red*, as being leaſt refracted, and

having its Parts attenuated by Solution in acid Mediums, appears intenſely Blue. Hence the Production and Changes of Colours by the various Mixture of tranſparent Liquors. Thus Clouds receive their different compound and beautiful Hues and Tints from the different Sizes of the aqueous Globules of which they conſiſt.

23. The Sizes of the Particles of Bodies, on which their Colours depend, are indicated by thoſe Colours: Thus the leaſt Particles of Light exhibit the *Violet-Colour*, and the leaſt Thickneſs of the Plate of Air or Water exhibited the ſame Colour in the ſeveral Rings. Again: The largeſt Particles of Light exhibit a *Red Colour*, and Red is produced by Reflection and Refraction in the thickeſt Part of the Plate in each Ring; and the intermediate Colours, Blue, Green, Yellow, are produced from Particles of a larger Size in Order.

24. The Magnitude of the Particles of colour'd Bodies may be pretty nearly conjectur'd by the Colours they exhibit: For 'tis pretty certain they exhibit the ſame Colours with the Plate of equal Thickneſs, provided they have the ſame refractive Denſity; and ſince their Parts ſeem for the moſt part to have much the ſame Denſity with Water or Glaſs, as by many Circumſtances is obvious to collect, we need only have Recourſe to the Table or Scale (in *Art.* 47, 48, 49, of the laſt *Annotation*) by which the Thickneſs of Water or Glaſs exhibiting the ſame Colour is ſhewn.

25. Thus if it be deſired to know the Diameter of a Corpuſcle, which being of equal Denſity with Glaſs ſhall reflect Green of the third Order; then in the ſaid Table you ſee under *Glaſs*, and oppoſite to *Green* of that Order, the Number $16\frac{1}{4}$, which ſhews the Corpuſcle to be $\frac{16\frac{1}{4}}{1000000}$ Parts of an Inch. But here the Difficulty is to know of what Order the Colour of any Body is: But for this Purpoſe we may be aſſiſted by

the reſt in Order above it. Now it is found by Computation, that the greateſt Angle

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viewing the Scheme of the ſeveral Orders of Colours; and by laying the parallel Ruler acroſs them ſeverally, you will obſerve thoſe which are leaſt compounded with others in every Order, and conſequently are moſt vivid and intenſe.

26. Thus *Scarlets* and other *Reds*, *Oranges*, and *Yellows*, if they appear pure and intenſe, you may conclude they are of the Second Order. Good *Greens* may be of the Fourth Order, but the beſt are of the Third. *Blues* and *Purples* may be of the Second or Third Order, but the beſt and leaſt compounded are of the Third. *Whiteneſs*, if moſt intenſe and luminous, is that of the Firſt Order; if leſs ſtrong and bright, it is that ariſes from the Mixture of the Colours of ſeveral Orders.

27. The Reds therefore of Carmine, Cinnabar, Vermilion, of ſome Roſes, Pinks, Peonies, *&c.* are of the Second Order. The Green of all Vegetables is of the Third Order; Ultramarine is a Blue of the Third Order, Biſe a Blue of the Second Order, and the Azure Colour of the Sky ſeems to be of the Firſt Order. Gold is a Yellow of the Second Order. The Whiteneſs of Paper, Linen, Froth, Snow, Silver, *&c.* is of the Firſt Order. Concerning all which ſee more in Sir *Iſaac Newton*'s *Optics*, p. 230—237.

28. It has been obſerved, (See *Art.* 41. of the laſt *Annotation*) that *Whiteneſs* ariſes from a promiſcuous Reflection of all the Colours together; and this is proved by ſeveral Experiments. Thus the Colour of the Sun's Light is White inclining a little to Yellow, as being a Compoſition of all the different colour'd Rays, among which the Yellow being the brighteſt is moſt predominant. Thus alſo the Rays when ſeparated by a Priſm, if received by a broad convex Lens of a large focal Diſtance, will all be thrown together in a ſmall round Spot in the Focus, and appear of a white Colour. Thus alſo a Powder compounded of Orpiment, Purple, Biſe

S *e* O,

Pl. 48, Fig. 3. S *e* O, or E O P, (drawing O P parallel to S E) under which the moſt refrangible Rays

and Verdigreaſe, in proper Proportion, appear'd of a perfect Whiteneſs in the Beams of the Sun.

29. But the moſt curious Experiment for Proof of this is as follows. Let any circular Area be divided on its Periphery into ſuch Parts A B, B C, C D, D E, E F, F G, and G A, as are proportional to the Differences of the Lengths of the Muſical Strings in an Octave, or the Numbers $\frac{1}{2}, \frac{9}{16}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{8}{9}, 1$; then ſtriking a Circle *a b c d e f g* at a ſmall Diſtance from the Periphery, the ſeveral Diviſions of this *Annulus* or Ring are to be laid over with the primary Colours proper to each, that is, Red from A to B, Orange from B to C, and the reſt in Order as they are wrote in the Figure. Then making all the internal Space very black, let this Area with its painted Ring be whirl'd or ſpun round in the manner of a Top, and the Ring will appear very white, eſpecially in the Sun-Beams: For in this Caſe all the Colours are blended together in the View, and muſt therefore exhibit Whiteneſs. Plate XXXIX. Fig. 4.

30. On the other hand, *Blackneſs* is the Abſence of all Colours; for it was obſerved, that in the Middle or Centre of the Rings of Colours, both in the Plates of Air and Water, there was a *black Spot*, which was occaſion'd by a Tranſmiſſion of all the Light in that Part, and conſequently by a total Deficiency of Colour.

31. But this happen'd in that Part of the Plate of Air, and Water Bubble, where it was thinneſt; and hence we are taught that the Corpuſcles of black Bodies are leſs than any of thoſe which exhibit Colours. Hence we ſee the Reaſon why Fire, and the more ſubtil Diſſolver Putrefaction, by attenuating the Particles of Bodies turn them black: Why a Razor, while ſetting, turns the Oil upon the Hone black: Why a ſmall Quantity of a black Subſtance will tinge ſo great a Quantity of any other ſo intenſely: Why black Subſtances ſooneſt of all others do become hot in the Sun's Light and burn: Why being ſoft, and ſtroked hard with the Hand,

can come to the Eye of a Spectator at O, is 40 Deg. 17 Minutes; and that the greateſt

they ſcintillate, or emit Sparks of Light in the dark: Why a black Cloth will, if wet, dry ſooner than a white one: Why moſt Blacks are a little inclined to a blueiſh Colour: With various other Phænomena of this Kind.

32. From what has been ſaid, the *Newtonian* Method of compounding and decompounding Colours may be eaſily underſtood, if we only firſt premiſe, that the Colour reſulting from a Mixture of any primary Colours is an Effect in which each primary Colour has a Share in Proportion to its Quantity; therefore this compound Colour is analogous to the *Common Centre of Gravity* between two Powers acting againſt each other: For as this Centre of Gravity will always be neareſt to the greateſt Power, ſo the Hue of the compound Colour will always approach neareſt the Complexion of that primary Colour which was largeſt in the Mixture.

33. Therefore to know what Colour will reſult from a Mixture of two Parts Yellow, and three Parts Blue; from the Middle of the Yellow Arch at H to the Middle of the Blue at I draw the Line H I, and divide it into five equal Parts, three of which ſet from the Point H, or two from the Point I, will give the Point K, through which if you draw the Line N L, it will point out the Colour of the Mixture at L, which is Green; but becauſe the Point L is ſo much nearer the Blue than the Yellow, it will be a blueiſh Green.

34. Again: If it be required to know what Colour the Mixture ſhall be of that has two Parts Yellow, three of Blue, and five of Red; then ſince we have already determined the Point K for the two firſt Quantities, which are five Parts; alſo ſince there are five Parts of Red, if we draw the Line M K, and divide it into two equal Parts in P, and through P draw the Line N O, this, as it falls upon the Orange, but near the Red, ſhews the Compound will be of an Orange

Angle F O P, under which the leaſt refrangible Rays come to the Eye at O, is 42 Deg.

Colour inclining to Red. And thus you proceed in other Caſes.

35. It muſt be farther obſerved, that the Colour will be leſs or more broken or imperfect as the Point of Interſection K or P falls nearer to or farther from the Circumference towards the Centre N, where White is repreſented: That is, the farther the Point K is ſituated from L towards N, the leſs pure and intenſe, or the more broken and mottley, the Green Colour will be.

36. Hence, if it be required to find (on the other hand) what Colours muſt be taken, and in what Quantity, to exhibit by their Mixture the broken blueiſh Green at K, let the Line H *i* be any how drawn through K, and it will ſhew that if you take ſuch Quantities of Yellow and Blue as are in Proportion to I K and K L, they will when mixed produce the given Green at K. Alſo the Line L N, paſſing through the ſame Point K, ſhews that a Quantity of pure Green and White, in the Proportion of N K, L K, will in the Mixture produce the ſame Green Tint at K as required.

37. What has been ſaid relates to Theory, and to the Colours of the Sun's Light; and therefore in Practice we muſt not expect ſo great Accuracy on ſeveral Accounts; as, (1.) Becauſe the Powders made uſe of in artificial Mixtures have different Powers of reflecting Light: Thus lighter Materials reflect more, and darker ones leſs; and therefore their Quantities muſt be in Proportion. (2.) Different Bodies, being mixed, operate upon each other; and thereby, either by attenuating the Parts, or by incraſſating them, produce Colours quite different from what we might expect from a Mixture of Bodies or Particles which do not affect or act one upon another. (3.) Becauſe all artificial Colours are in themſelves more or leſs compounded, and therefore cannot produce the Effects of pure, unmixed, and primary Colours. Yet notwithſtanding theſe Excep-

2 Minutes.

3

2 Minutes. And ſo all the Particles of Water within the Difference of thoſe two

tions, this Theory, when well conſidered and underſtood, will be of the greateſt Service to Painters.

38. Laſtly, I ſhall apply this Theory to explain and account for ſeveral other Phænomena of Colours. Thus in examining *Mineral Waters*, it is uſual, in order to diſcover whether the Salts contained in them are of an *Acid*, *Alcaline*, or *Neutral* Sort, to mix *Syrup of Violets* with them; becauſe then, if there be an *Acid*, it will change the Syrup Red by attenuating its Parts; ſo that if the Syrup be a Purple of the Third Order, the Acid will change it to a Red of the Second Order, the Particles which reflect that Colour being of the Size next leſs.

39. Again: If an *Alkali* abound in the Water, the Mixture will turn Green; for the *Alcali* by incraſſating the Particles will increaſe their Size to thoſe of the Green of the Third Order; therefore the Syrup, and conſequently the Mixture will appear of that Colour. But if there be neither an *Acid* nor an *Alcali* in the Water, it will neither turn Green nor Red.

40. Hence alſo it is, that when even the Fume or ſubtil Vapour of a ſtrong Acid, as *Aqua fortis*, reaches a Green Cloth, it changes to a Blue, becauſe that in the ſame Order reſults from the next leſs Size of Particles. If the Acid be dropped on the Cloth in Subſtance, it acts more violently in attenuating the Particles, and thereby produces a Yellow of the next preceding Order, whoſe Particles are leſs than the aforeſaid Blue. And after a like Manner may this Theory be extended, to account for other Phænomena of the ſame Kind.

41. To conclude: Since any Object becomes viſible when it ſubtends an Angle of *one Minute*, and alſo becauſe Objects are diſtinctly viewed in the Focus of a Lens; therefore ſuppoſing the Focus of a Lens were $\frac{1}{30}$ of an Inch, (as they have been made thus ſmall) it will be found by Calculation, that an Object in the Focus of ſuch a Lens ſubtending an Angle of one Minute, will be equal to 0,0000097 Parts of an Inch in

Angles E F will exhibit ſeverally the various Colours of the Priſm, and conſtitute the *interior Bow* in the Cloud.

If the Beam go not out of the *Drop* at G, but is reflected (a ſecond time) to H, and is there refracted in the Direction H S, making the Angle S Y A with the incident Ray A N, it will paint on the Part H the ſeveral Colours of Light, but in an inverſe Order to the former, and more faint, by reaſon of the Rays loſt by the *ſecond Reflection.* It has been found alſo, that the leaſt Angle S G O, or G O P, under which the leaſt refrangible Rays can come to the Eye at O, after two Reflections and two Refractions, is 50 Deg. 57 Minutes; and the leaſt Angle H O P, under which the

Length. Therefore the Diameter of a Particle leſs than the Diameter of any colour'd Particle (except thoſe of the Firſt Order) will be viſible in the Focus of ſuch a Lens: And therefore the Particles of all colour'd Bodies would become viſible by ſuch a Lens, were it not that Particles equally thick appear of the ſame Colour, and all ſo very ſmall are tranſparent; whence, though they are big enough to be viſible, yet we may want a Difference of Colour, and ſome other Means, to render them diſtinct, and capable of being view'd ſeparately from each other. Sir *Iſaac Newton* thinks the Diſcovery of thoſe Corpuſcles by the Microſcope will be the utmoſt Improvement of this Science: For it ſeems impoſſible to ſee the more ſecret and noble Works of Nature within the Corpuſcles, by reaſon of their Tranſparency.

RPJC

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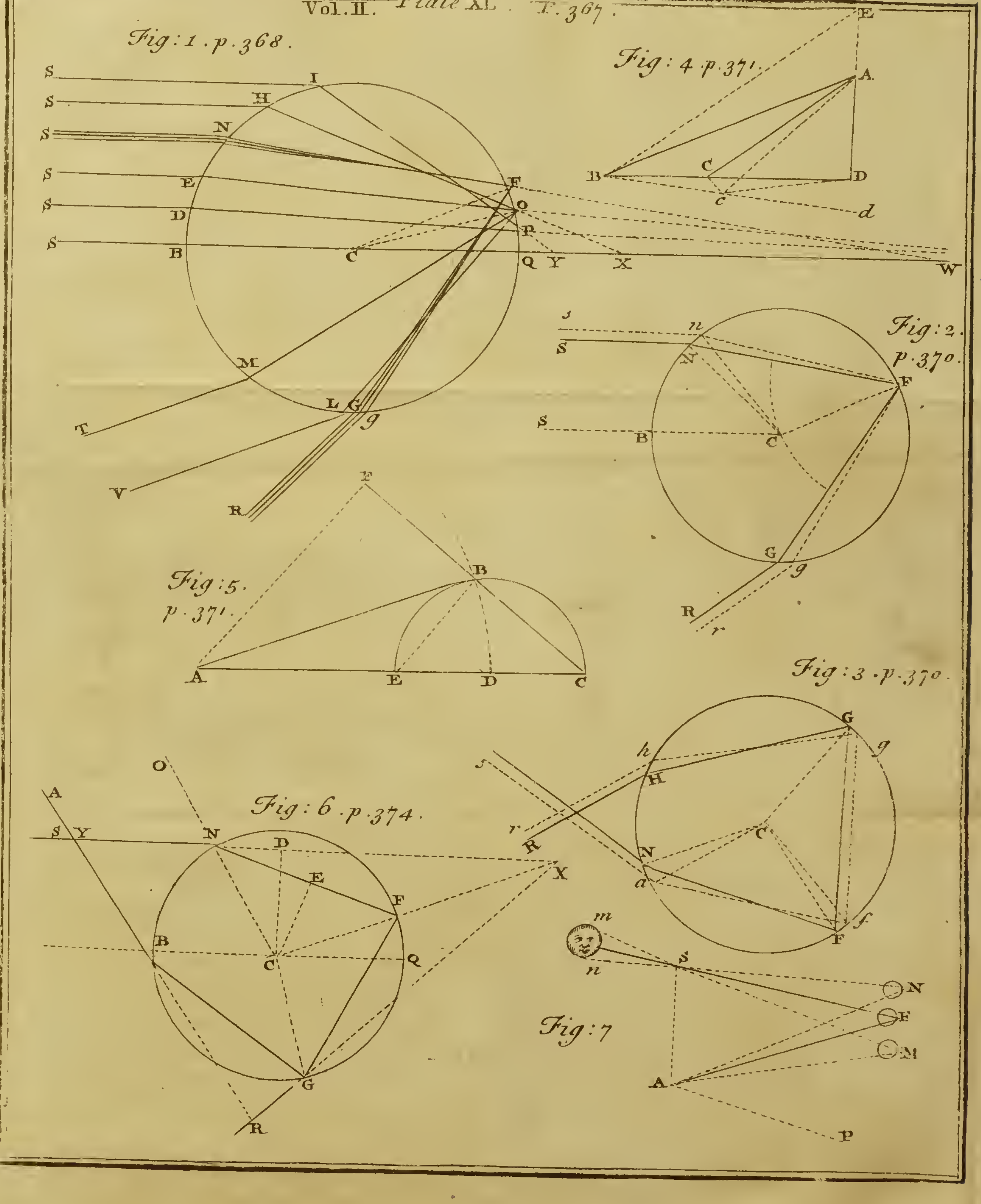

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Fig: 5. p. 371.

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Fig: 7

moſt refrangible Rays can come to the Eye in this Caſe, is 54 Deg. 7 Minutes. Whence all the Colours of the *exterior Bow* will be formed in the Drops from G to H, which is the Breadth of this Bow, *viz.* 3 Deg. 10 Minutes; whereas the Breadth of the other, *viz.* EF, is but 1 Deg. 45 Minutes, and the Diſtance between the Bows, *viz.* FG, is 8 Deg. 55 Minutes. And ſuch would be the Meaſures of the Bows, were the Sun but a *Point*; but ſince his Body ſubtends an Angle of half a Degree, it is evident, by ſo much each Bow will be increaſed, and their Diſtance diminiſhed (CXXIII.)

(CXXIII.) 1. Having explain'd the Doctrine of the different *Refrangibility* of the Rays of Light, and the *Theory of Colours* conſequent thereupon, it will now be eaſy to explain and underſtand the natural Cauſe of the *Rainbow*, which is wholly owing to the above mention'd Property of Light. For though it was, by long Obſervation, known to proceed from the Sun's ſhining upon the falling Drops of Rain; and even before Sir *Iſaac Newton*'s Time it was diſcover'd to be the Effect of the Sun's Light ſeveral times refracted and reflected in the aqueous Globules; firſt of all by *Antonius de Dominis*, Archbiſhop of *Spalàto*, in a Book publiſhed in the Year 1611, and after him by *Deſcartes*: Yet no one could ever account for the Diverſity of Colours, and their inverſe Order in the two Bows, or give a direct Method of Calculation, before Sir *Iſaac Newton*.

2. To apprehend rightly the different Affections of this remarkable Phænomenon, we muſt attend to the following Particulars. *Firſt*, That though each Bow

HALO's are formed by Rays of Light coming to the Eye after two Refractions

be occaſioned by the refracted and reflected Light of the Sun falling on the Drops of Rain, yet neither of them is produced by any Rays falling on any Part of the Drop indifferently, but by thoſe only which fall on the Surface of the Drop B L Q G in or about the Point N, as the Ray A N; thoſe which fall nearer to B, or farther towards L, being unconcerned in this Production.

Plate XLVIII. Fig. 2.

3. *Secondly*, The internal Bow is produced by two Refractions and one Reflection. The firſt Refraction is of the incident Rays extremely near A N, by which they proceed from N to one common Point or Focus at F, from whence they are reflected to G, and are there a ſecond time refracted towards R, and produce the various Colours of the ſaid Bow.

Pl. XL. Fig. 1.

4. *Thirdly*, There is a Neceſſity that ſeveral Rays ſhould be refracted together to the Point F, that being reflected together from thence to G they may there go out parallel, and ſo come in Quantity ſufficient to excite the Senſation of Colours in a ſtrong and lively Manner. Now thoſe Rays, and thoſe only, which are incident on the Globule about the Point N, can do this, as will appear from what follows: For,

5. *Fourthly*, The Point F makes the Arch Q F a *Maximum*, or the Diſtance Q F from the Axis of the Drop S Q is greater than any other Diſtance from whence any other Rays nearer to the Axis, as S D, S E, or farther from it, as S H, S I, are reflected; becauſe thoſe which are nearer after the firſt Refraction tend to Points in the Axis produced more remote than that to which the Ray S N tends; and therefore as their Diſtance from the Axis increaſes, ſo likewiſe will the Diſtances of their Points of Reflection Q P, Q O, till the Ray becomes S N; after which the Rays more remote from the Axis, as S H, S I, are refracted towards the Points X Y, which are nearer and nearer to the Axis; and this occaſions the Points of Reflection on

through *Drops of Rain*, or *ſpherical Hail-ſtones*; which Light ought to be ſtrongeſt

the fartheſt Side of the Drop to decreaſe again from F towards Q.

6. *Fifthly*, Hence it will neceſſarily happen, that ſome Rays above and below the Ray S N will fall upon the ſame Point as O or P; on the fartheſt Side; and for that Reaſon they will be ſo reflected from thence as to go out of the Drop by Refraction parallel to each other. Thus let S E below, and S H above the Ray S N be refracted both to one Point O; from hence they will be reflected to M and L, and will there emerge parallel, 'tis true, but alone; being diveſted of their intermediate Rays S N, which going to a different Point F will be reflected in a different Direction to G, and emerge on one Side, and not between thoſe Rays, as when they were incident on the Drop. All which is evident from the Figure.

7. *Sixthly*, As this will be the Caſe of all the Rays which are not indefinitely near to S N, it is plain, that being deprived of the intermediate Rays, their Denſity will be ſo far diminiſhed, as to render them ineffectual for exciting the Senſation of Colours; and they are therefore called *Inefficacious Rays*, in Contra-diſtinction to thoſe which enter the Drop near S N, and which, having the ſame Point F of Reflection, are not ſcatter'd like the others, but emerge together at G, ſo as to conſtitute a Beam G R of the ſame Denſity with the incident Beam S N, and therefore capable of exhibiting a vivid Appearance of Colours, and for this Reaſon are called *Efficacious Rays*.

8. Theſe Things premiſed, we ſhall now ſhew the Mathematical Principles on which the Calculations relating to this Phænomenon depend, according to Dr. *Halley*'s moſt elegant and eaſy Conſtructions a little explained and facilitated by Dr. *Morgan*, late Biſhop of *Ely*. Let S N, *s n*, be two of the efficacious Rays incident upon a Drop of Rain; theſe when refracted to the ſame Point F, and thence reflected to G, *g*, will

Pl. 40
Fig. 2.

at the Diſtance of about 26 Degrees from the Sun or Moon, or ſomewhat leſs, if the

have the Parts within the Drop on one Side, N F, *n* F, equal to thoſe on the other Side, F G, F *g*, from the Nature of the Circle and the Angles of Incidence C F N, C F *n*, being equal to the Angles of Reflection C F G, C F *g*. Since the Parts within the Drop are equal and alike ſituated, they will alſo be ſo with it; and therefore as the incident Rays S N, S *n*, are ſuppoſed parallel, the emergent Rays G R, *g r*, will be ſo too.

Pl. XL. Fig. 2. 9. From C the Centre draw the Radii C N, C *n*, C F; then is $CNF = CFN$ the Angle of Refraction, and the ſmall Arch N *n* is the naſcent Increment of the Angle of Incidence B C N; and as it meaſures the Angle at the Centre N C *n*, it is double of the Angle at the Circumference N F *n*, which is the naſcent Increment of the Angle of Refraction N F C.

Fig. 3. 10. Again: Let the Ray S N enter the lower Part of the Drop, and be twice reflected within the Drop at F and G; then is the Ray $NF =$ the Ray F G, and the Arch $NF =$ to the Arch F G. Draw *fg* parallel to F G, and it will be the reflected Part of ſome Ray *s n*, whoſe Obliquity to the Drop is ſuch as obliges it to croſs the Ray N F in its Refraction, as it muſt do if it be a little more oblique than S N, (by *Art.* 6.) Then alſo will the Part $nf = fg$, and the Arch $nf = fg$, and the ſmall Arch $Ff = Gg$.

11. Therefore, $2Ff = (Ff + Gg =$ the Arch $FG -$ the Arch $fg =$ the Arch $NF -$ the Arch nf $=) Nn - Ff$; conſequently $Nn = 3Ff$. That is, the naſcent Increment of the Angle of Incidence is equal to *three times* that of the Angle of Refraction. After a like Manner you proceed to ſhew, that after 3, 4, 5, &c. Reflections, the Increment of the Angle of Incidence will be 4, 5, 6, &c. times greater than that of the Angle of Refraction.

12. Hence, in order to find the Angle of Incidence of an efficacious Ray, after any given Number of Re-

ſaid Hail-ſtones be a little flatted, as often they are. Theſe *Halo*'s, if the Hail be

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flections, we are to find an Angle whoſe naſcent Increment has the ſame Ratio to the Increment of its correſponding Angle of Refraction, generated in the ſame Time, as the given Number of Reflections (n) increaſed by Unity has to Unity; that is, in the Ratio of $n+1$ to 1. Now thoſe Increments are as the Tangents of the reſpective Angles directly; as is thus demonſtrated.

13. Let ACD, ABD, be the Angles of Incidence and Refraction propoſed; and if we ſuppoſe the Line AC to move about the Point A in the Plane of thoſe Angles, the Extremity thereof C will deſcribe the circular Arch Cc; and when AC is arrived to the Situation Ac, the Line BD will be thereby removed into the Situation Bd. Draw cD; then is the Angle $ACD = ABC + CAB$, and the Angle $Acd = ABc + cAB$. Wherefore the Exceſs of Acd above ACD, or the Increment of ACD, is equal to both the Angles CBc and CAc. But ſince the Angle AcC differs infinitely little from a Right one, a Circle deſcribed on the Diameter AC ſhall paſs through the Points D and c; and therefore the Angles CAc, CDc, (inſiſting on the ſame Arch Cc of the ſaid Circle) will be equal. Wherefore the Increment of the Angle ACD is equal to $CBc + CDc = Dcd$. But the naſcent Angles Dcd and DBc are as their Sines, that is, as their oppoſite Sides BD and $Dc = DC$, becauſe of the Angle CDc infinitely ſmall. But $BD : CD :: DE : DA$ (the Line BE being parallel to AC) :: Tangent of the Angle ($EBD =$) ACD : Tangent of the Angle ABD. Therefore the Increment Dcd of the Angle ACD is to the Increment CBc of the Angle ABD (generated in the ſame Time) as the Tangent of the former to the Tangent of the latter directly. Pl. XL. Fig. 4.

14. Hence, having given the Ratio of the Sine of Incidence I, to the Sine of Refraction R, we may find the Angles of Incidence and Refraction of an efficacious Fig. 5.

duly figur'd, will be colour'd, and muſt then appear *red* within by the leaſt re-

Ray, after any given Number (n) of Reflections, thus: In any Right Line A C, let there be taken $AC : AD :: I : R$; and again, $AC : AE :: n+1 : 1$. Upon the Diameter E C deſcribe the Semicircle E B C; and on the Centre A with the Radius A D deſcribe the Arch D B, interſecting the Circle in B. Draw A B and B C; then let fall the Perpendicular A F on C B continued out to F. So ſhall A B F and A C F be the Angles of Incidence and Refraction required.

15. For drawing B E parallel to A F, the Triangles A C F and E C B are ſimilar. Now the Sine of the Angle A B C or A B F is to the Sine of A C B as A C to $AB = AD$, that is, as I to R; therefore if A B F be the Angle of Incidence, A C F will be the Angle of Refraction. Moreover, the naſcent Increment of A B F is to that of A C B (generated in the ſame Time) as C F to B F, (by *Art.* 13.) that is, as C A to A E, (by ſimilar Triangles) that is, as $n+1$ to 1 by Conſtruction. The Ratio therefore of the naſcent Increment of the Angle of Incidence A B F, to that of the Angle of Refraction A C B, is that which is required in the Angles of Incidence and Refraction of an efficacious Ray, after a given Number of Reflections, (in *Art.* 12.) Conſequently the Angles A B F and A C F are thoſe required. *Q. E. D.*

16. From this Conſtruction we eaſily deduce Sir *Iſaac Newton*'s Rule for finding the Angle of Incidence A B F in *p.* 148, 149. of his *Optics*, thus. We had $AC : AB :: I : R$, whence $AC = \frac{I}{R} \times AB$. Alſo $CF : BF :: n+1 : 1$; therefore $CF = \overline{n+1} \times BF$, or (putting $n+1 = m$) $CF = m \times BF$; and becauſe of the Right Angle at F, it is $AC^2 - CF^2 = AB^2 - BF^2$, that is, $\frac{II}{RR} AB^2 - m^2 FB^2 = AB^2 - BF^2$;

frangible Rays, and *blue* without by the moſt refrangible ones.

and therefore $m^2 FB^2 - FB^2 = \frac{II}{RR} AB^2 - AB^2$;

and conſequently $\frac{FB}{AB} = \sqrt{\frac{II - RR}{m^2 RR - R}}$.

17. Hence, becauſe in the firſt Bow the Ray emerges after one Reflection, we have $n = 1$, $m = 2$, $m^2 = 4$, $m^2 - 1 = 3$; therefore $\sqrt{3RR} : \sqrt{II - RR} :: AB : BF ::$ Radius : Co-Sine of the Angle of Incidence. In the ſecond Bow, where there are two Reflections, $m^2 - 1 = 8$; whence $\sqrt{8RR} : \sqrt{II - RR} : AB : BF$. In the third Bow, after three Reflections, $m^2 - 1 = 15$; and $\sqrt{15RR} : \sqrt{II - RR} : AB : BF$; and ſo on for any given Number of Reflections.

18. To find the Values of I and R, it muſt be remembered, that the Ratio of the Sines of Incidence and Refraction was ſhewn to be conſtant, (in *Annot.* XCVII. 13.) and therefore their Exceſſes in divers Sorts of Mediums are alſo in a given Ratio. Thus it was ſhewn, that in the leaſt refrangible Rays I : R :: 50 : 77, out of Glaſs into Air; the Exceſs of R above I is here 27. If the Refraction be made out of Rain-Water into Air, then it is I : R :: 3 : 4 very nearly for the leaſt refrangible Rays; the Exceſs here is 4 — 3 = 1. Wherefore ſay, As 1 : 27 :: 3 : 81 :: 4 : 108. Whence it appears, that the Sines I and R out of Water into Air are as 81 to 108, in the leaſt refrangible Rays: And if to the leſſer Sine you add the given Differences between thoſe Sines out of Glaſs into Air for all the other Sorts of Rays, *viz.* $27\frac{1}{8}$, $27\frac{1}{5}$, $27\frac{1}{3}$, $27\frac{1}{2}$, $27\frac{2}{3}$, $27\frac{7}{9}$, 28; we ſhall have the ſeveral Values of R for thoſe Rays, *viz.* $108\frac{1}{8}$, $108\frac{1}{5}$, $108\frac{1}{3}$, $108\frac{1}{2}$, $108\frac{2}{3}$, $108\frac{7}{9}$, 109.

19. But ſince the Refraction here is not out of Water into Air, but the contrary, we ſhall have the Values of I and R interchanged; or they will ſtand for the ſeveral Sorts of Rays as below.

THE Reaſon why there is always a determinate Angle for exhibiting the *Bows*,

For the Red,	$I:R::108$	$:81$	Extreme.
For the Orange,	$I:R::108\frac{1}{8}$	$:81$	Beginning.
For the Yellow,	$I:R::108\frac{1}{5}$	$:81$	Beg.
For the Green,	$I:R::108\frac{1}{3}$	$:81$	Beg.
For the Blue,	$I:R::108\frac{1}{2}$	$:81$	Beg.
For the Indigo,	$I:R::108\frac{2}{3}$	$:81$	Beg.
For the Violet,	$I:R::108\frac{7}{9}$	$:81$	Beg.
For Violet,	$I:R::109$	$:81$	Extreme.

20. Wherefore in the leaſt refrangible Rays, ſince $I = 108$, $H = 11664$; alſo $R = 81$, and $RR = 6\ 61$, and $I^2 - R^2 = 5103$; $\sqrt{3RR} = 140{,}3$, and $\sqrt{I^2 - R^2} = 71{,}4$. Therefore ſay, (by *Art.* 17.)

As	$\sqrt{3RR}$	$= 140{,}3$	$= 2{,}147045$
Is to	$\sqrt{I^2 - R^2}$	$= 71{,}4$	$= 1{,}853913$
So is Radius		$= 90°\ 00'$	$= 10{,}000000$
To the Co-Sine of the Angle of Incidence	BAF	$= 30°\ 37'$	$= 9{,}706868$

21. Hence the Angle of Incidence A B F is 59° 23′, in the Red or leaſt refrangible Rays. Wherefore in the Drop of Rain whoſe Axis is S Q, if we make the Arch $BN = 59°\ 23'$, we ſhall have S N the leaſt refrangible Ray. Having given the Angle of Incidence, and the Ratio of I to R, we have alſo given the Angle of Refraction: For ſay,

As	$I = 108$	$= 2{,}033424$
Is to	$R = 81$	$= 1{,}908485$
So is the Sine of Incidence	$59°\ 23'$	$= 9{,}934798$
To the Sine of the Angle of Ref.	$40°\ 12'$	$= 9{,}809859$

Pl. XL. Fig. 6. 22. Therefore, making the Angle $CNF = 40°\ 12'$, N F will be the refracted Ray; which at F is reflected into F G, and at G emerges in G R. Produce the incident and emergent Rays S N and R G till they interſect each other at X; and as C F biſects the Angle N F G, ſo when produced it will biſect the Angle S X R,

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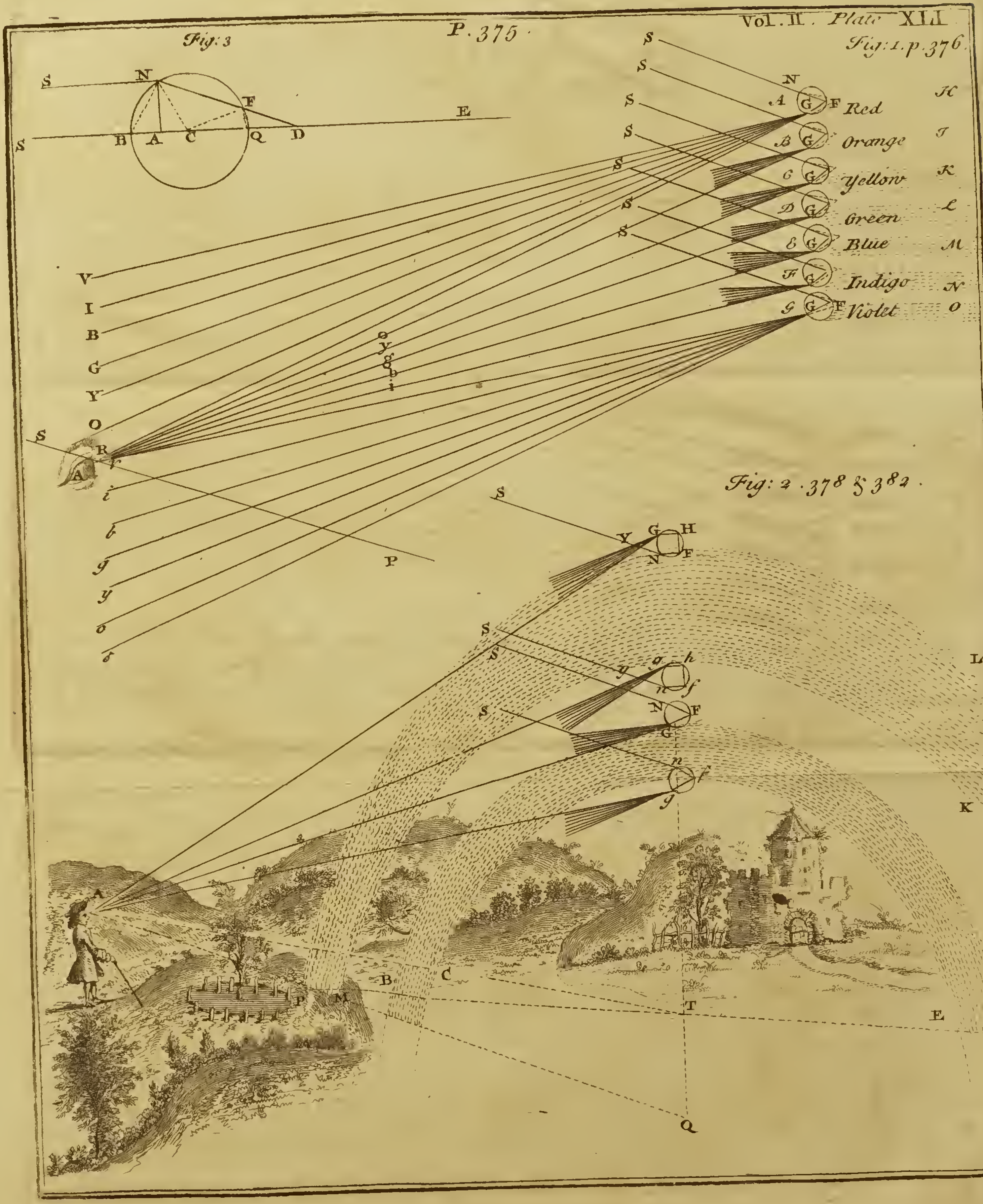

Fig: 3

Fig: 1. p. 376.

Fig: 2. 378 & 382.

or *Halo's*, is becauſe there is but one particular Point N in all the Part of the Drop

Then $CFN = CXN + FNX$, but $FNX = CNG - CNF$ or CFN; therefore $CFN = CXN + CNX - CFN$; that is, $2CFN - CNX = CXN$. Or $80^\circ 24' - 59^\circ 23' = 21^\circ 01' = CXN$; therefore $2CXN = SXR = 40^\circ 02'$, which is the Meaſure of the Angle that the incident and emergent Rays, which are the leaſt refrangible, contain with each other.

23. If inſtead of the Ratio 108 to 81, we take that of 109 to 81, we ſhall find the Values of $\sqrt{3RR}$ and $\sqrt{I - R^2}$ ſuch as will give the Angle of Incidence BCN, or the Arch $BN = 58^\circ 40'$, and the Angle $SXR = 40^\circ 17'$, which will be the Caſe for the moſt refrangible, or extreme Violet Rays.

24. If the Ray be twice reflected, *viz.* at F and G, as in the Production of the exterior Bow, and emerges at H in the Direction H A interſecting the incident Ray S N in Y; then we may find the Angle A Y S, which thoſe Rays contain with each other, thus. Produce A H, till it meets G X produced in R; then in the Triangle H G R, the external Angle $HGX = HRG + GHR$ But becauſe of equal Angles of Reflection at F and G, it is $GHR = FGX$; therefore $HGX - FGX = HGF = HRG = 2CGF$ or CNF. And (in *Art.* 22.) we had $SXR = 4CNF - 2CNX$; therefore in the Triangle Y X R we have the two internal Angles $R + X = 6CNF - 2CNX =$ the external Angle at Y, *viz.* A Y N.

25. In this Caſe to find the Angles of Incidence and Refraction, we have $\sqrt{8RR} : \sqrt{I^2 - R^2}$ :: Radius : the Co-Sine of the Angle of Incidence; whence the ſaid Angle of Incidence will be found $71^\circ 50' = CNX$. And as 108 : 81 :: Sine of $71^\circ 50'$: Sine of $45^\circ 27' = CNF$ the Angle of Refraction; therefore $\overline{45^\circ 27'} \times 6 - 2 \times \overline{71^\circ 50'} = 129^\circ 02' = AYN$, and therefore its Complement $AYS = 50^\circ 58'$ the Angle required, for the leaſt refrangible Rays.

between B and L, where the Rays AN can enter, ſo that after a ſecond Refraction at

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26. But for the moſt refrangible Rays, where I : R :: 109 : 81, we have the Angle of Incidence 71° 26′, and the Angle of Refraction 44° 47′; and therefore the Angle A Y S = 54° 10′. After this Manner you proceed to calculate the ſame Angles after three, four, or more Reflections; but becauſe the Beam in being ſo often reflected loſes ſo many of its Rays, that the remaining refracted Part is in general too faint to excite the Idea of Colours, we paſs it by, and proceed to apply what has been ſaid to account for the *Phænomena* of the Bows, which ſo ſtrongly ſtrike the Eye; the principal whereof here follow.

27. The *Firſt* is, *That each is variegated with all the Priſmatic Colours.* This is a neceſſary Conſequence of the different Refrangibility of the Rays refracted and reflected in Drops of falling Rain. Let A be ſuch a Drop, S N a Ray entering it at N, which is refracted to F, from thence reflected to G, where, as it emerges, it is refracted into all the ſeveral Sorts of Rays of which it is compoſed, *viz.* G R the leaſt refrangible or *Red-making* Ray, G O the *Orange*, G Y the *Yellow*, G G the *Green*, G B the *Blue*, G I the *Indigo*, and G V the *Violet* or moſt refrangible Ray.

Pl. XLI. Fig. 1.

28. Now we have ſhewn (*Art.* 22, 23.) that the Angle S F R is to the Angle S F V as 42° 02′ to 40° 17′; the Difference whereof is the Angle V G R = 1° 45′. Through this Angle all the original Rays are diffuſed; and though the Angle be ſmall, yet at a great Diſtance it ſpreads to a conſiderable Width; and therefore by coming from the Drop *A* to the Eye of a Spectator at A, they will be ſufficiently ſeparated, and fall upon the Eye ſingly, each Sort of Rays by themſelves alone.

29. Hence, were there only one Drop *A*, the Eye at A would ſee only one Colour in that Drop, *viz.* the *Red*, by the leaſt refrangible Ray G R; the others, G O, G Y, &c. being refracted above it, as is evident enough in the Figure. If now we ſuppoſe this Drop to

F for *Halo's*, or Reflection at F and G for the *Bows*, there can enough go out toge-

deſcend to the Situation *B*, then would the Orange-making Ray G O fall upon the Eye continuing in A, and then the Drop would exhibit an Orange Colour. If after this it ſhould ſink down to *C*, the Yellow-making Ray G Y would enter the Eye at A, and excite the Idea of Yellow in the Drop at G. And ſo continually, if we ſuppoſe the Drop to ſucceed to the ſeveral Situations *D*, *E*, *F*, *G*, the other more refrangible Rays G G, G B, G F, G V, will fall upon the Eye ſucceſſively, and raiſe the Senſation of their proper Colours, Green, Blue, Indigo, and Violet when the Drop is at *G*.

30. The Truth of this may be eaſily proved by Experiment, by ſuſpending a Glaſs Globe fill'd with Water in the Sunſhine, and viewing it in ſuch a Poſition that the Rays S N which fall upon it may emerge to the Eye at A, under the ſeveral Angles from S F R to S F V; which may be eaſily effected by letting the Globe deſcend from *A* to *G* by a String going over a Pulley. And this was the famous Experiment of *Antonius de Dominis* and *Des Cartes*, who by this means confirmed the Truth of their Doctrine of the Rainbow, which had been demonſtrated mathematically. The ſame Thing may be alſo ſhewn, if the Globe be at Reſt at *A*, and the Eye be raiſed from R to V.

31. If now, inſtead of depreſſing the Drop from *A* to *G*, we ſuppoſe a Drop placed in each Point *A*, *B*, *C*, *D*, *E*, *F*, *G*; then theſe will ſeverally ſend an original Ray to the Eye, according to their Situations in reſpect of it. Thus the Drop in *A* will refract the Red-making Ray G R; the Drop *B* will refract the Orange G o; the Drop *C* the Yellow G y; and ſo the other Drops *D*, *E*, *F*, *G*, will by the Rays G g, G b, G i, G v, excite the ſeveral Colours, Green, Blue, Indigo, Violet, all at the ſame Time; and therefore all that Part of the Rain from *A* to *G* will appear variouſly coloured, as is repreſented in the Scheme.

ther at G or H, to form a ſtrong and diſtinct Image of the Sun; which Rays,

31. Now let S P be a Line drawn through the Spectator's Eye at A, parallel to the Sun's Rays S N, and conceive the ſeveral Rays G R turning about the Line A P as an Axis, and always under the ſame invariable Angle G R P; 'tis evident, the Extremity of each Ray would in the Cloud or Rain deſcribe a Circle which would be the Baſe of a Cone whoſe Axis is A P, and its Vertex A; and for the ſame Reaſon that the Drop *A* excites the Senſation of Red, every Drop in the Circle deſcribed by the Extremity of the Ray G R will excite the ſame Senſation; thus will a red circular Arch *A H* be formed as far as the Rain extends. Next to that the Ray *B* A, by revolving, deſcribes the Arch of an Orange Colour, as *B I*; the Ray *C* A will in like manner trace out the Yellow Circumference, as *C K*; and ſo of all the reſt, as repreſented in the *Figure*.

32. Hence the *Second Phænomenon*, *viz.* the *circular Form*, is accounted for; and alſo the *Third*, which is *the Breadth of the Bow*; for that will be equal to the Pl. XLI. Fig. 2. Angle *A* R G = R G V = 1° 45′, where the Ray, as here, emerges after one Reflection. Theſe Particulars are repreſented more compleatly in the *Figure*, where B G D is the red Circumference formed by the Rotation of the Ray A G, that can firſt come to the Eye at A; and C *g* E is the Violet Arch formed by the leaſt refrangible Ray *g* A; after which the Rays are all refracted below the Eye. And thus by the intermediate Rays and Colours the whole interior Bow is produced.

33. The *Fourth Phænomenon* is the *Appearance of Two Bows*. This follows from hence, that after an efficacious Ray of Light S N, entering a Drop of Rain, has been twice reflected on the fartheſt Side at F and H, it will emerge refracted into all its ſimple or conſtituent Rays at G upon the upper Side of the Drop, ſo as to make with the incident Ray the Angle G Y N or S Y A = 54° 10′, if that Ray be the Violet Sort, or moſt refrangible, (by *Art.* 26.) but if it be of the red or leaſt refrangible

Sort, then the ſaid Angle is but $50^\circ.58' = SyA$ (by *Art.* 25.)

34. Therefore all thoſe Drops which are ſo ſituated around the Eye, that their moſt refrangible Rays ſhall fall upon it, muſt with thoſe Rays make an Angle with the Line A P paſſing through the Eye parallel to the Sun's Rays, *viz.* the Angle G A P, equal to the Angle S Y A, or $GAP = 54^\circ\ 10'$. Theſe Rays therefore will every where exhibit a Violet Colour in the Arch P G L. For the ſame Reaſon thoſe Drops whoſe leaſt refrangible Rays fall upon the Eye at A, make the Angle $gAP = 50^\circ\ 58'$; and ſo the Ray A *g*, revolving about the Axis A Q, will deſcribe the circular Arch M *g* K, which will exhibit the deepeſt Red; and all the Drops between G and *g* will paint the ſeveral other colour'd Peripheries, all which together will compleat the exterior Bow.

35. The *Fifth Phænomenon* is the *greater Breadth of the exterior Bow.* Thus, if from 54° 10′ we ſubduct 50° 58′, we ſhall have $3^\circ\ 12' = Gg =$ the Width of the outer Bow; which therefore is almoſt twice as wide as the interior Bow.

36. The *Sixth Phænomenon* is the *Diſtance between the two Bows*, which is thus determined: From the Angle which the leaſt refrangible Ray in the upper Bow makes with the Axis A P, *viz.* 50° 58′, ſubtract the Angle 42° 02′ which the moſt refrangible Rays make therewith in the lower Bow, and the Remainder $80^\circ\ 56' = gAF$ is the Arch of Diſtance between the Bows.

37. The *Seventh Phænomenon* is the *inverſe Order of the Colours in the two Bows.* This follows from the contrary Parts of the Drop on which the Ray is incident, and from whence it emerges and is refracted. Thus becauſe the Rays S N enter the upper Part of the Drop and emerge from the lower, 'tis evident the Rays refracted in this Caſe (*viz.* in the interior Bow) will have a Situation quite the reverſe of thoſe which enter on the lower Part of the Drop, and are refracted from the upper, as in the exterior Bow, whoſe Colours are *Violet*, *Indigo*, *Blue*, *Green*, *Yellow*, *Orange*, and *Red*; whilſt thoſe of the other are *Red*, *Orange*, *Yellow*, *Green*, *Blue*, *Indigo*,

Indigo, and *Violet*; counting from the upper Parts downwards in both.

38. The *Eighth Phænomenon* is the *Faintneſs of the exterior Bow in Compariſon of the interior one.* This is the Conſequence of the Rays being twice reflected within the Drops which form the outer Bow. They who make the Experiment in a dark Chamber may wonder when they obſerve how large a Part of the Beam (that enters the Globule at N) goes out at F, that there ſhould be enough in the remaining Part F G to exhibit the Colours ſo ſtrong and vivid in the firſt Bow as they appear; but then conſidering how much of this reſidual Ray is refracted at G, 'tis rather Wonder how the very ſmall Part reflected to H ſhould there when refracted be in Quantity ſufficient to excite any diſtinct Ideas of Colours at all.

39. The *Ninth Phænomenon* is, *that ſometimes more than two Bows appear*; as in a very black Cloud I have myſelf obſerved *four*, and a faint Appearance of a *fifth*: But this happens rarely. Now theſe ſpurious Bows, as I may call them, cannot be formed in the Manner as the two principal Bows are, that is, by *Refraction* after a *third*, *fourth*, *fifth*, &c. *Reflection*; for the Beam is by much too weak to exhibit Colours by Refraction, even after the *third* Reflection only, much leſs would it after a *fourth* or *fifth*. Beſides, though after a third and fourth Reflection of the Rays they ſhould be ſuppoſed capable of ſhewing their Colours, yet the Bows made thereby would not appear at the ſame Time with the other two, nor in the ſame Part of the Heavens, but in the Rain between us and the Sun, and muſt be viewed by the Spectator's Face turned towards the Sun, and not from it, as in the other Caſe.

40. To account for the Appearance of theſe colour'd Rings within the interior primary Bow, we ſhall here tranſcribe what the learned Dr. *Pemberton* has wrote upon the Subject. He obſerves, that Sir *Iſaac Newton* takes notice, that in Glaſs which is poliſh'd and quickſilver'd there is an irregular Refraction made, whereby ſome ſmall Quantity of Light is ſcattered from the principal reflected Beam. If we allow the ſame Thing to happen in the Reflexion by which the Rainbow is cauſed,

RPJCB

Vol. II. Plate XLII. P. 381.

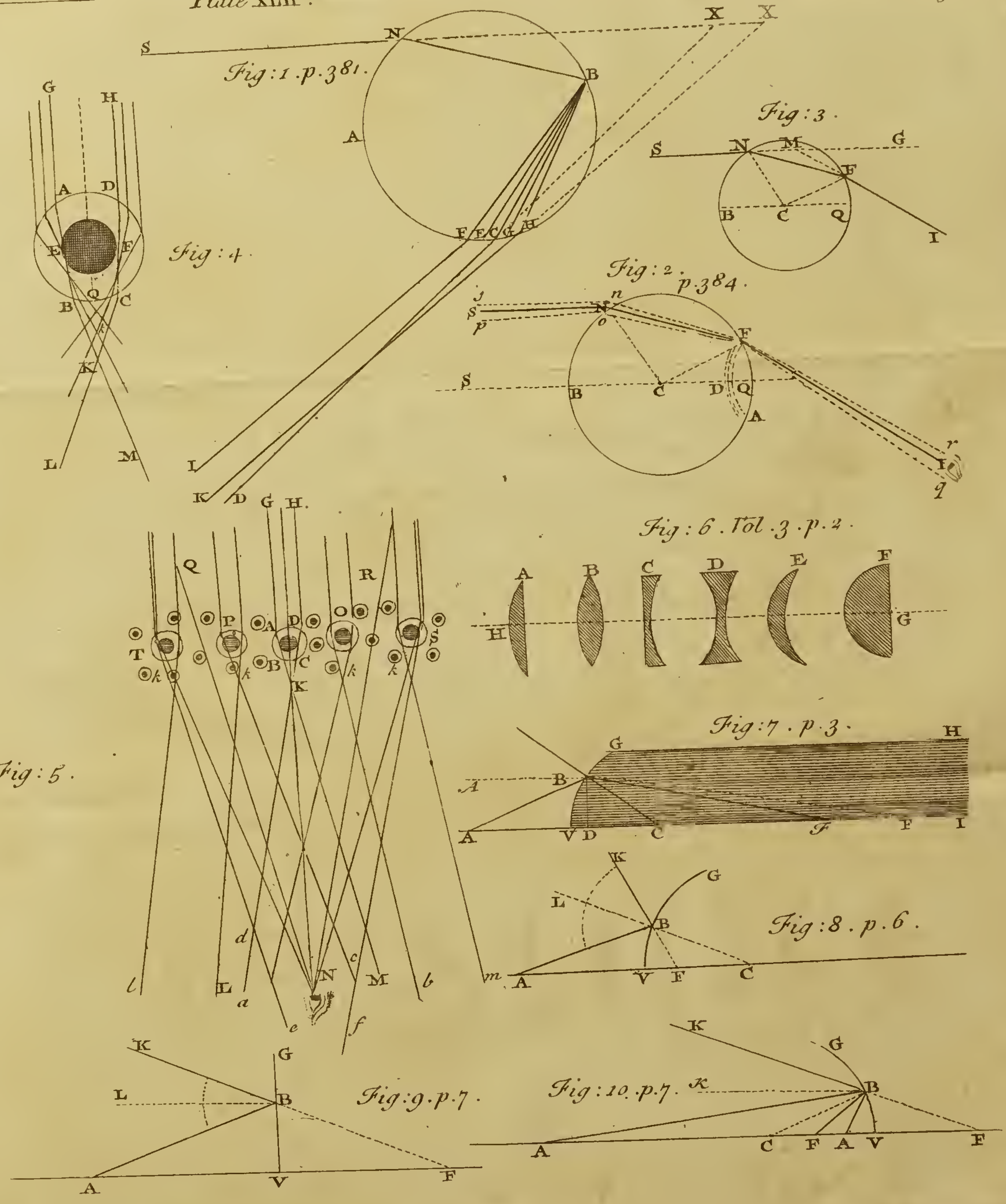

Fig: 1. p. 381.

Fig: 2. p. 384.

Fig: 3.

Fig: 4.

Fig: 5.

Fig: 6. Vol. 3. p. 2.

Fig: 7. p. 3.

Fig: 8. p. 6.

Fig: 9. p. 7.

Fig: 10. p. 7.

cauſed, it ſeems ſufficient to produce the Appearance now mentioned.

41. Let A B repreſent a Globule of Water, B the Point from whence the Rays of any determinate Species being reflected to C, and afterwards emerging in the Line C D, would proceed to the Eye, and cauſe the Appearance of that Colour in the Bow which appertains to this Species. Here ſuppoſe, that beſides what is reflected regularly, ſome ſmall Part of the Light is irregularly ſcatter'd every Way; ſo that from the Point B, beſides the Rays that are regularly reflected from B to C, ſome ſcattered Rays will return in other Lines, as in B E, B F, B G, B H, on each Side of the Line B C. Pl. XLII. Fig. 1.

42. Now it has been obſerved, (*Annotat.* CXXI.) that the Rays of Light in their Paſſage from one Superficies to another, in any refracting Body, undergo alternate *Fits of eaſy Tranſmiſſion and Reflection*, ſucceeding each other at equal Intervals, inſomuch that if they reach the farther Superficies in one of thoſe Fits, they ſhall be tranſmitted; if in the other, they ſhall be reflected back. Whence the Rays that proceed from B to C, and emerge in the Line C D, being in a Fit of *eaſy Tranſmiſſion*, the ſcatter'd Rays that fall at a ſmall Diſtance without theſe on either Side (ſuppoſe the Rays B E and B G) ſhall fall on the Surface in a Fit of *eaſy Reflection*, and ſo will not emerge; but the Rays next to theſe, *viz.* B F and B H, ſhall arrive at F and H in a Fit of *eaſy Tranſmiſſion*, and ſo be refracted in the Rays F I and H K.

43. Now theſe Rays will emerge, ſo as to contain a leſs Angle with the incident Beam S N than the Ray C D, which was ſhewn to make the greateſt Angle therewith of all others whatſover: (See *A. t.* 5, 22, 23.) The Colours therefore which they exhibit muſt appear within thoſe of the primary Bow. And if we ſuppoſe other ſcatter'd Rays without theſe to emerge (having the intermediate Rays intercepted by Reflection) they will contain Angles ſtill leſs with the incident Ray S N, and will therefore form colour'd Arches ſtill within the former: And this may be conceived for divers Succeſſions.

44. Now as the ſcatter'd Rays by various *Reflections* and *Refractions* form Arches variouſly mixed together, ſome of theſe by the lighter Colours may be loſt in the inferior Part of the primary Bow, and may contribute to the red Tincture which the Purple of that Bow uſually has. The darker Colours of thoſe refracted ſcatter'd Rays form the Arches which reach below the Bow, and are ſeen diſtinct; of which the firſt has a *light Green*, *dark Green*, and *Purple*; the ſecond has a *Green* and *Purple*; the third a *faint Green* and *vaniſhing Purple*.

45. The Diſtances between the Bow and theſe ſecondary Arches depend on the Size of the Drops; to make them in any Degree ſeparate, 'tis requiſite the Drops ſhould be exceeding ſmall. It is therefore moſt likely, that they are formed in the Vapour of the Cloud, which the Air, being agitated by the Rain, may carry down with the larger Drops; and this may be the Reaſon why they never appear but under the upper Part of the Bow only, this Vapour not deſcending very low. As a Confirmation of this, theſe Arches are ſeen ſtrongeſt when the Rain falls from very black Clouds, which cauſe the fierceſt Rains, and therefore produce the greateſt Agitation of Air. Thus far Dr. *Pemberton*.

46. But to return: The *Tenth Phænomenon* is, the *Appearance of the Bows in that Part of the Heavens oppoſite to the Sun.* This neceſſarily happens from the incident and emergent Ray being both on one Side of the Drop, for 'tis evident that in order to ſee the Colours, we muſt look to that Part againſt which the Sun ſhines.

47. The *Eleventh Phænomenon* is, *that they never appear but when and where it rains.* This is becauſe Rain affords a ſufficient Plenty of Drops, or aqueous Spherules, proper to reflect and refract the Light fit for this Purpoſe, which cannot be done without a requiſite Size, Figure, and Diſpoſition of the Particles, which the Vapour of the Cloud does not admit, and therefore Clouds alone exhibit no ſuch Appearance.

48. The *Twelfth Phænomenon* is, *the Dimenſion of the Bows.* This is determined eaſily, for continuing the Axis A P to Q the Centre of the Bows, we have the Semidiameter of each Bow in the Angle Q A g, or Q A G; the double of which gives the Angles which the whole Diameters

Pl. XLI.
Fig. 2.

Diameters of the Bows ſubtend, and are therefore the Meaſure of their Magnitude.

49. The *Thirteenth Phænomenon* is, *the Altitude of the Bow above the Horizon or Surface of the Earth.* This is equal to the Angle G A T, which may be taken by a Quadrant, or it may be known for any Time by having given the Sun's Altitude, which is equal to the Angle T A Q; which therefore ſubducted from the conſtant Angles Q A F, or Q A Y, will always leave the Angle of the apparent Height of the Bow.

50. Hence it follows, that when the Sun is in the Horizon, the Lines Q A and T A will coincide, and therefore the Points Q and T; whence, in this Caſe, the Bows will appear compleat Semicircles; as on the other Hand, when the Altitude of the Sun is equal to the Angle $QAF = 42^\circ\ 02'$, or to $QAY = 54^\circ\ 10'$, the Summits of the Bows will be depreſs'd below the Horizon, and therefore within a certain Interval in many Days, in the Summer-Time, no Rainbow can appear.

51. We have hitherto conſidered the Bows, and given their Dimenſions, ſuch as they would have were the Sun but a Point; but becauſe the Sun ſubtends an Angle of half a Degree, or 30 Minutes at a Mean, therefore the Breadths of the Bows will be increaſed, and their Diſtance decreaſed by half a Degree, and ſo the Breadth of the interior Bow will be 2° 15′, and that of the exterior one 3° 42′, and their Diſtance 8° 26′; alſo the greateſt Semidiameter of the interior Bow 42° 17′, and the leaſt of the exterior Bow 50° 43′.

52. For let S F A be the Angle of any one particular colour'd Ray coming from the Centre of the Sun, and reflected from the Drop to the Eye at A. In the Ray S S take any Point S at Pleaſure, and make the Angles F S N, F S M, each equal to 15′, as alſo the Angles F A M and F A N; then will S N be Part of a Ray nN coming from the lower Limb of the Sun, S M a Part of a Ray mM coming from the upper Limb; and ſo the whole Angle $NSM = mSn = 30'$, the Sun's apparent Magnitude.

53. Join S A; and ſince the Sums of the Angles at the Baſe S A of the ſeveral Triangles A S N, A S F,

A S M,

A S M, are equal among themſelves, their vertical Angles at N, F, M, are alſo equal to each other. Wherefore the Angle S M A will be that which the emergent Ray makes with the incident Rays S M of the ſame Colour, as before, coming from the higheſt Point *m* of the Sun, and S N A of that which comes from the loweſt Point of the Sun *n*. Therefore, if all the Rays of the Sun were of one Sort, the apparent Breadth of the Bow, meaſured by the Angle M A N, would be but 30′ or half a Degree.

54. But ſince the Rays of the Sun are differently refrangible, conceive the Drop F to be placed any where in the inward or outward Verges of the Bows (above deſcribed) and then it is manifeſt that the Angle F A M muſt be added to the Inſide, and F A N to the Outſide of the Angles, which the Breadths of thoſe Bows ſubtend at A, to obtain their apparent Breadths; which therefore will be ſuch as are defined in *Article* 51.

55. I deſigned here to have added Dr. *Halley*'s Method of diſcovering the Ratio of the Sine of Incidence to that of Refraction, by having given the Angle which an efficacious Ray, as S N, contains with its emergent Part G A; but as this Angle is determined only by Experiment, and the Calculation brings us to a Cubic Equation, I think it a Matter of too much Intricacy to trouble the Reader with in this Place, eſpecially as it is ſo eaſy to determine the refractive Power of any tranſparent Bodies, by the experimental Methods before deliver'd. (See *Annotat.* CXVII.)

56. I have often taken Notice (with Mr. *Whiſton*) of the Silence of Authors concerning the Reaſon why the *Iris*, or rather a ſtrong and deeply colour'd *Corona* does not appear about the Sun in the falling Drops of Rain, at the Diſtance every way of about 25 Degrees; becauſe at that Diſtance from the Axis, the efficacious
Pl. XLII. Fig. 2. Rays S N, *s n*, *p o*, after Refraction into the Drop are refracted a ſecond Time at F towards the Eye at I. For if S B Q be the Axis of the Drop, we have ſhewn the Angle B N = 59° 23′ (*Art.* 21.) and the Arch N F = 99° 36′; therefore the Arch F Q = 21°, wherefore in a Glaſs Globe of Water, held in the Sun's Light in a dark Room, we ſee a colour'd Circle or *Corona* A D F of

of about 42 Degrees in Diameter, and the Superficies within it extremely luminous, as containing all the Sun-Beams that fall on the fore Part within 60 Degrees all round the Axis.

57. But becauſe the Rays are there promiſcuouſly blended together, they produce only a white Light; whereas on the Circumference F D A, where the efficacious Rays fall, there all the Colours of the Bow appear; and from thence many have wondered why we ſee not a Circle coloured with ſtronger Tints than even the primary Bow itſelf, from this Refraction of the efficacious Rays in all the Drops of Rain between us and the Sun from the Circle F D A. But we are to obſerve, with Mr. *Whiſton*, that the efficacious Rays S N, *s n*, *p o*, which are parallel when incident on the Drop, are not ſo when refracted at their Emergence at F; for being there refracted to one Point, they are not parallel within the Drop, and therefore cannot be ſo after their Emergence, but will proceed diverging to the Eye at I in the ſeveral Directions F I, F *r*, F *q*, and therefore will not be ſufficiently denſe, and at the ſame time too much blended with others to excite any Senſation of Colours.

58. But why it ſhould be ſaid, this variegated Circle ought to appear at the Diſtance of about 26 Degrees from the Sun, I do not ſee; for the refracted Ray F I contains an Angle F M G with the incident Ray S N Pl. XLII. Fig. 3. (produced to G) of 38° 22′; for 'tis plain that $MN = MF$, and therefore the Angle $MNF = MFN = CNM - CNF = 59° 23' - 40° 12' = 19° 11'$; but the external Angle $IMG = MNF + MFN = 38° 22'$, and conſequently this is the Angle of Diſtance at which ſuch a Bow muſt appear all around the Sun.

SCHOLIUM.

59. In *Article* 5. it is aſſerted, that the efficacious Ray S N makes the Arch QF a *Maximum* by its refracted Part N D. To prove this, let Radius $CN = CB = 1$, Pl. XLI. Fig. 3. the verſed Sine $BA = x$, and $CD = z$, and by the Nature of the Circle $QB : BN :: BN : AB$; therefore $BN = \sqrt{2x}$. Again, let $a =$ Ratio of the

Incidence and Refraction; then becauſe I : R :: N D : C D, we have $ND = \frac{I}{R} CD = az$; likewiſe from the ſimilar Triangles B N D and F Q D, we have N D : N B :: Q D : Q F; that is, $az : \sqrt{2x} :: z - 1 : \frac{z-1}{az}\sqrt{2x} = QF$.

60. This Value Q F is to be determined to a *Maximum*; in order to this we neglect the given Part $\frac{\sqrt{2}}{a}$, and take the variable Part $\frac{\overline{z-1} \times \sqrt{x}}{z} = \frac{z\sqrt{x} - \sqrt{x}}{z} = \sqrt{x} - \frac{\sqrt{x}}{z} = x^{\frac{1}{2}} - \frac{x^{\frac{1}{2}}}{z}$; then making its Fluxion $\frac{1}{2}x^{-\frac{1}{2}}\dot{x} - \frac{1}{2}z^{-1}x^{-\frac{1}{2}}\dot{x} + z^{-2}x^{\frac{1}{2}}\dot{z} = 0$; or (multiplying by $2z^2x^{\frac{1}{2}}$) $z^2\dot{x} - z\dot{x} + 2x\dot{z} = 0$. Whence $2x\dot{z} = z\dot{x} - z^2\dot{x}$, and $\dot{z} = \frac{z\dot{x} - z^2\dot{x}}{2x}$.

61. We muſt now find another Value of $\dot{z}$ in order to exterminate it, which we find from the right-angled Triangle N A D, where $ND^2 = AN^2 + AD^2$; that is, $a^2z^2 = z^2 + 2z - 2x\dot{z} + 1$, which in Fluxions is $2a^2z\dot{z} = 2z\dot{z} + 2\dot{z} - 2\dot{x}z - zx\dot{z}$; therefore $\dot{z} = \frac{z\dot{x}}{\overline{z - a^2z - x - 1}} = \frac{z\dot{x} - z^2\dot{x}}{2x}$; whence $z^2 - a^2z^2 - xz + a^2z + 3x - 1 = 0$; now by Means of this and the preceding Equation $a^2z^2 = z^2 + 2z - 2xz + 1$, if we throw out x, we ſhall get this cubic Equation $z^3 - a^2z^3 - a^2z^2 + z^2 + 3z + 3 = 0$; whoſe Roots will be found $z = -1$, $z = -\sqrt{\frac{3}{a^2 - 1}}$; $z = \sqrt{\frac{3}{a^2-1}}$; of which the two firſt being negative, are of no Uſe; therefore the Arch Q F is a *Maximum* when $CD = \sqrt{\frac{3}{a^2-1}}$:

62. By

therefore, entering at the Point N, are called *Efficacious Rays*, to diſtinguiſh them from the reſt which are ineffectual (CXXIV.)

62. By inſerting this Value of z, into the Equation $a^2 z^2 = z^2 + 2z - 2xz + 1$; we get this Equation $x = \frac{1}{\sqrt{\frac{3}{a^2-1}}} + 1 + \overline{\tfrac{1}{2} - \tfrac{1}{2}a^2} \times \sqrt{\frac{3}{a^2-1}}$. And ſince in Caſe of the leaſt refrangible Rays, we have $a = \frac{4}{3}$, therefore $\sqrt{\frac{3}{a^2-1}} = 1{,}964 = CD$; and $AB = 0{,}4908$ the verſed Sine of the Angle $BCN = 59°\ 23'$, the ſame as was found before in *Article* 21. Alſo if we put $a = \frac{109}{81}$ (See *Art.* 19.) we ſhall have the Angle $BCN = 58°\ 40'$ for the moſt refrangible Rays, as in *Article* 23.

63. Alſo we get the Value of $QF = \frac{z-1}{az}\sqrt{2x} = 0{,}3648$, which is the Chord of $21°\ 02' = QCF$, as was before ſhewn, *Article* 56. Hence all the Particulars relating to the *principal Bow* are eaſy to be underſtood; and this is an egregious Inſtance of the extreme Uſefulneſs of the *Fluxionary Calculus* in Natural Philoſophy. This noble Theorem was firſt given us by Mr. *Stewart* in his Comment on Sir *Iſaac Newton's* Quadratures.

64. If BNQ were a Globe of Glaſs, then $a = \frac{3}{2}$, $CD = z = 1{,}5492$, and the Arch $BN = 49°\ 48$; alſo the Arch $QF = 11°\ 22'$. Some other conſiderable Uſes, which may be made of this Theorem, will be conſidered in the next Lecture of *Optics*.

(CXXIV.) 1. Concerning the Production of HALO's, our illuſtrious Author has left us to make the beſt Shift we can in accounting for it; having ſaid nothing of this Phænomenon that can be of any Service to help us in this Diſquiſition. He intimates, indeed, *that Halo's are formed by the Light which comes through the Drops of Rain by two Refractions* (viz. *at* N *and* F) *without any Reflection*; but how this can be is not eaſy to conceive. Pl. XLII. Fig. 3. We have ſhewed that a Rainbow or deeply coloured Ring might have been expected at the Diſtance of about 38 Degrees from the Sun, and alſo why it cannot happen.

2. For the ſame Reaſon we ſhould alſo not expect an Halo to be formed by the ſame refracted Rays, *viz.* on Account of their not being refracted parallel to the Eye, and conſequently not entering it denſe enough to render that Part of the Heavens more luminous than the reſt, or to produce the lucid Ring we call by this Name. Again, Sir *Iſaac* ſays, *it ought to appear ſtrongeſt at the Diſtance of about* 26 *Degrees from the Sun (viz* when the Angle I M G = 26°) *and to decay gradually both ways.* But though our Author did not undoubtedly aſſert any Thing without very great Reaſon, yet this does not appear to us.

3. For that the Angle I M G may be 26 Degrees, the Angle of Incidence B C N muſt be about 46, and then the Angle of Refraction C N F will be near 33 Degrees; but why ſuch an Incidence and Refraction ſhould cauſe the Rays to be refracted in greater Plenty to the Eye than any other, does not appear to me, nor can I find it by any Experiment. On the contrary, as the Angle I M G increaſes with the Angle of Incidence, and conſequently with the Angle of Refraction, it is evident that with reſpect to heterogeneal Light, the greater the Angle I M G is, the more will it be refracted and ſcattered; and conſequently, the farther the Drops are ſituate from the Sun, the leſs denſe will be the Light tranſmitted by Refraction to the Eye, which therefore ought to decreaſe as the Diſtance from the Sun increaſes.

4. As Sir *Iſaac Newton* has ſaid but little, ſo his Expoſitors, D. *Pemberton* and Dr. *s'Graveſande*, have thought fit to be abſolutely ſilent on this Head. Mr. *Hugens* has advanced an Hypotheſis by which the *Phænomenon* may be ſolved, if we grant him all his Petitions. And ſince none of our great Philoſophers, not even Sir *Iſaac* himſelf, have undertook to diſprove it,

but on the contrary ſeem rather to approve of it, as Sir *Iſaac* in his *Optics*, and Dr. *Smith*, in his has adopted the ſame entirely; I think upon theſe Accounts, and conſidering the Character of the great Author, the Reader will be pleaſed to have the ſame in a very conciſe Manner repreſented to him.

5. His *Poſtulatum* is, *That there are certain Globules in the Atmoſphere conſiſting of a Coat or Shell of tranſparent Ice or Water, containing an opake Nucleus or Kernel within*; and that theſe are made from Particles of Snow, (which is in itſelf opake) attracting the aqueous Particles in the Vapour or Exhalation by which it is ſuſtained, which gathering together form the pellucid Shell of Water, or are frozen into a cryſtalline Shell of Ice; and this he thinks is proved to be Matter of Fact by the Hail-ſtones which fall to the Eearth, for theſe (ſays he) when broken do diſcover ſome Snow at the Centre.

6. Theſe Things premiſed, he addreſſes himſelf to the Solution as follows: Let A B C D repreſent ſuch a Globule, with the opake Nucleus E F in the Middle of it; and let us ſuppoſe the Rays coming from, G, H, to fall on the Side A D. It is manifeſt they will be refracted inwards from the Surface A D; from whence it follows, that a great Number of them muſt ſtrike upon the Kernel E F. Plate XLIV. Fig. 4.

7. Let G A and H D be the Rays which after Refraction touch the Sides of the Kernel E F, and let them be refracted again at B and C, emerging in the Lines B K, C K, croſſing each other in the Point K, whoſe Diſtance from the Globule is ſomewhat leſs than its Semidiameter.

8. Wherefore, If B K and D K be produced towards M and L, it follows, that no Light coming from the Sun through the Globule can proceed to the Eye any where placed within the Angle L K M, or rather in the Cone which that repreſents, ſuppoſing that the Obliquity of the incident Rays H D and G A is ſuch as ſhall make the Arch Q C and Q B the greateſt poſſible; (ſee the laſt Note, *Art.* 5.) for then all the Rays exterior to H D, G A, will be refracted nearer to Q, and after Emergence croſs each other in a Point *k* nearer the Globule than the former, and therefore cannot come at the Eye placed within the ſaid Cone L K M.

Plate XLIV. Fig. 5. 9. Suppoſe now the Eye placed at N; and let N R, N Q, be drawn parallel to L K and M K; then 'tis plain, none of the Globules (the ſame as A B C D) within the Cone R N Q can come to the Eye at N. Thus the Globules at O and P have their refracted Rays *a k d* and *c k d* including the Eye in the Cone of Obſcurity: But other Globules, which lie without the Cone Q N R, as S and T, do not involve the Eye N by the ſhady Cones *l k e* and *f k m*; and therefore ſome of thoſe Rays, which are more refracted than *k e* or *k f*, will fall upon the Eye, and produce a luminous circular Ring or *Corona*, including a dark Area within, and whoſe Light outwardly decreaſes as it is more remote from the Centre.

10. Much after a like Manner this great Man undertook to account for the Appearances of *Mock-Suns* and *Mock-Moons*, called *Parhelia* and *Paraſelenæ* or *Coronæ*; which I ſhall not here detain the Reader with, becauſe I cannot help thinking the Whole is but too much like a mere (though ingenious) Hypotheſis; having never obſerved in any Hail-ſtones any ſuch opake Kernels, ſo regularly formed, and ſurrounded by ſuch regular Shells of pellucid Ice as is here ſuppoſed. However the Reader will be agreeably entertained by peruſing *Mr. Hugens's learned Diſſertation on this Subject*, printed at the End of his Dioptrics in his *Opuſcula Poſtuma*.

N. B. I have let the Account of the *Abberration of Light in the fixed Stars* ſtand here, both in this and the following Volume, as it was in the firſt Edition; tho' the Methods of Reaſoning and Demonſtration by which it is ſupported are ſuch as upon a more nice Examination are by no Means ſatisfactory to myſelf, at leaſt I cannot reconcile them to the Notions I have of the Nature of Viſion, the Laws of Optics, nor the Accounts which have been given of Experiments made on this Subject by Perſons of no ſmall Character, who ſeem to be as little inclined to take upon Credit, the Truth of this univerſally celebrated Doctrine *(vel jurare in Verba)* as myſelf. (See Annot. CXII. and CXLV. Art. 19, &c.) The Arguments which render this Diſcovery ſuſpected, I may take an Opportunity one time or other, to declare.

END of the SECOND VOLUME.

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